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BIOMETRIKA

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FOUNDED BY

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VOLUME XVI MAY, 1924 NOS 1 AND 2

BIOMETRIKA

A STUDY OF CERTAIN ORIENTAL SERIES OF CRANIA INCLUDING THE NEPALESE AND TIBETAN SERIES IN THE BRITISH MUSEUM (NATURAL HISTORY).

BY G. M. MORANT, M.Sc., Crewdson-Benington Student in Craniometry.

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1. INTRODUCTORY.

No suggested classification of the races of India and the Orient has hitherto gained universal acceptance, for none could be considered satisfactory or final which was based on an imperfect knowledge of the anthropometric characters of the several peoples. The physical anthropologist has not neglected to draw far-reaching conclusions from comparisons of the cranial cephalic and nasal indices, or more frequently from similar measurements on the living head, and from the available evidence relating to stature and tegumentary characters, but the systems which have been founded on such meagre data can be no more than provisional and their importance may be easily over-estimated. The present paper provides measurements of the ninety Nepalese and Tibetan skulls which were presented by Brian Houghton Hodgson to the British Museum over fifty years ago. This material, and that contained in a previous paper dealing with the Tibetan crania at the Royal College of Surgeons, fills a gap in our knowledge of the craniometry of the Far East, and we are enabled to form some true conception of the relations of the

so-called "Mongolian" peoples and the aboriginal races of India. The measurements of Burmese skulls furnished by Miss Tildesley and those of neighbouring races dealt with by other—chiefly German—craniologists have been collected together and compared with the Tibetan and Nepalese data. The races of man inhabiting South-East Asia appear to be inter-related in a very simple way, and it was only the lack of adequate data which previously made it somewhat difficult to obtain a just conception of their connections.

Comparison has been made between the mean characters of the several races in the manner already adopted by biometric craniologists. The one method of estimating the similitude of two racial series is by comparing mean or type contours, and the other, and far more important one, is that which deals with mean direct measurements and estimates degrees of resemblance by computing "Coefficients of Racial Likeness." Very great use has been made in this paper of that criterion which was originally suggested by Professor Karl Pearson. Previously it had not been used to any great extent, so an attempt was made to discover in what ways and with what restrictions it could be best employed in practice. In general the conclusions resulting from its use were fully confirmed by more direct methods of comparison. With the aid of the Coefficient it is possible to estimate at the same time all the inter-relations of a considerable number of races: by the very process of its deduction it enables us to pick out those characters which differ most significantly between the racial types and those which are constant for all. Caution is needed when comparing Coefficients derived from several pairs of series of unequal length, and a description of the methods of comparison which may be legitimately used in such cases is given below. The Coefficient of Racial Likeness is admittedly a provisional criterion only and it is based on some assumptions which are not precisely justified by experience, but there is nothing at present which can take its place and it should prove a potent and invaluable aid to all craniologists.

The majority of the skulls of the Hodgson Collection are those of natives of Nepal, but several crania had been procured in Tibet and a few are those of Hindus and Dravidians. In collecting the material Hodgson appears to have been careful to include no specimens which could be thought to be of Western origin or admixture. In his day it was possible to make a clear distinction between the aborigines of the country and the invading Gorkhas and hybridised tribes (Khas); in his researches into the history and ethnography of the country Hodgson was concerned only with the aboriginal race. By dividing the people into three sections it has been possible to show that the Central and Eastern Nepalese tribes, together with the Sikkimese and Bhutanese, form a racially homogeneous population while the natives of West Nepal are distinct in type. It must not be supposed that this difference is in any way due to the effect of Aryan influences, for the Western Nepalese form quite a pure branch of the Tibetan A race which extends over the whole of Tibet except the Eastern Province of Khams. The latter district is the home of a race—the so-called Tibetan B—which is clearly differentiated from all neighbouring Asiatic peoples. The greater number of the crania contained in the Hodgson Collection have been divided into two groups—the Nepalese and Tibetan

A—and the measurements of the Tibetans were pooled with those of the A type skulls in the Royal College of Surgeons.

A comparison of the mean cranial measurements of the principal races of man inhabiting South-East Asia has shown that there is a gradual transition in head form as we pass from the Veddahs of Ceylon to the Mongolian peoples of Burma and Southern China. In order the races comprising this series or sequence are: Veddah—Dravidian (Maravar)—Hindu (Bengalee)—Nepalese—Tibetan A—Southern Chinese—Annamese—Siamese—Malayan—Burmese A; and the relations between them are such that:

(i) When arranged in the above order, the degree of resemblance between any two races, when estimated from all the more important cranial characters, is proportional to their distance apart in the series.

(ii) A certain number of cranial characters—actually fifteen of the thirty-one from which Coefficients of Racial Likeness are calculated—are reasonably constant for all the races.

(iii) The characters which show significant differences between race and race, and particularly those for which the differences are most significant, place the races, when arranged in order of magnitude, in the same or in very similar orders; the direct measurements being least for the Veddahs and greatest for the Burmese A.

We could not have argued from the first of these observed conditions alone that the races conforming to it were of necessity bound by any close tie. It would no doubt be possible to pick out a group of totally unallied races from different parts of the world which would display a similar hierarchy of affinity, though in such an artificial case the bonds between adjacent races in the series could not be so intimate as they are in the case of the Asiatic races. The extremes of our series are as unlike as a European and an Asiatic race picked at random might be, but all pairs of neighbouring races are very similar in skull form. The Annamese and Southern Chinese data are so alike that it would be reasonable to consider that the two series of skulls were random samples drawn from the same population, but there are no other cases of such intimate relationship. The second criterion of affinity which was found to be applicable to the ten races is of a more striking nature than the first. That there should be fifteen cranial characters which show no appreciable differences between all the members of such a group of races is obviously a fact of peculiar significance since the intra-racial correlations between those characters are all insignificant or small. Comparison with a considerable number of races from all parts of the world—principally European, African and unallied Asiatic races—suggests that four of the constant measurements, viz. the foraminal length and index and the nasal and alveolar angles of the fundamental triangle, are almost identical for all modern races of man. The other characters that are reasonably constant for the ten Southern Asiatic races are the facial ($100\,G'H/GB$, $100\,O_2/O_1$, and $100\,NB/NH$) and palatine ($100\,G_2/G_1$) indices, the profile angle, calvarial length and arc, nasio-basional length, occipital index, least forehead breadth

and palatine length. Thus the faces of the types are all very similar in shape though differing in size, and the sagittal sections of the calvariae are all very similar in both shape and size. In making such wide generalisations it must not be forgotten that the available data relating to the Veddah, Maravar and Hindu types are particularly meagre. If their facial indices should prove to be the same as those of the Chinese and Burmese, as the present figures suggest, that would be a most noteworthy fact. In the third place it was observed that when the races were arranged in order of magnitude of any of the characters which differed significantly from race to race, then those orders were the same as, or very similar to, that arrangement which had been deduced from a generalised criterion of affinity—the Coefficient of Racial Likeness. If this third condition had been found to hold initially, then the first might have been inferred *a priori*. It has been observed that the characters which differ most significantly also give, in general, orders most like that suggested by the Coefficients of Racial Likeness. We may suppose that the disturbances in the order are due primarily to the errors introduced as a result of random sampling and their effect would thus be inversely proportional to the significances of the differences of the several characters. The more profound racial differences are found between all the breadth characters of the cranium with the exception of the least forehead breadth, and between facial characters. From a comparison of the values tabulated below it will be clear that the cephalic index is influenced enormously by the skull breadth and hardly at all by the length.

Race	Maximum Breadth	Maximum Length	Cephalic Index	Zygomatic Breadth
Veddah	128·8	179·1	71·9	124·5
Maravar	131·4	175·6	74·6	124·4
Hindu	132·3	175·4	75·8	126·8
Nepalese... ...	132·6	176·9	75·1	127·2
Tibetan A ...	138·7	175·7	79·2	131·0
Southern Chinese	139·4	177·6	78·6	131·8
Annamese ...	140·3	177·0	79·3	132·7
Siamese* ...	143·9	177·9	80·9	136·3
Malayan	142·2	174·7	81·7	133·2
Burmese A ...	143·7	173·5	82·7	134·0

The position of any race in the series might be determined with some precision, but not with certainty, from the value of its mean cephalic index; but when comparison is made with only partly allied races or with races foreign to the group the cephalic index alone is a most untrustworthy guide in estimating racial affinities. Between the ten Southern Asiatic races the differences in the case of the facial heights (nasal and nasio-alveolar) are hardly less significant than those between the maximum breadths; and the orbital heights are also racially distinctive. As we pass from the Veddah end of the scale to the Burmese, the face tends to become

* The Siamese measurements are based on fewer than ten crania, but all the other figures are more reliable.

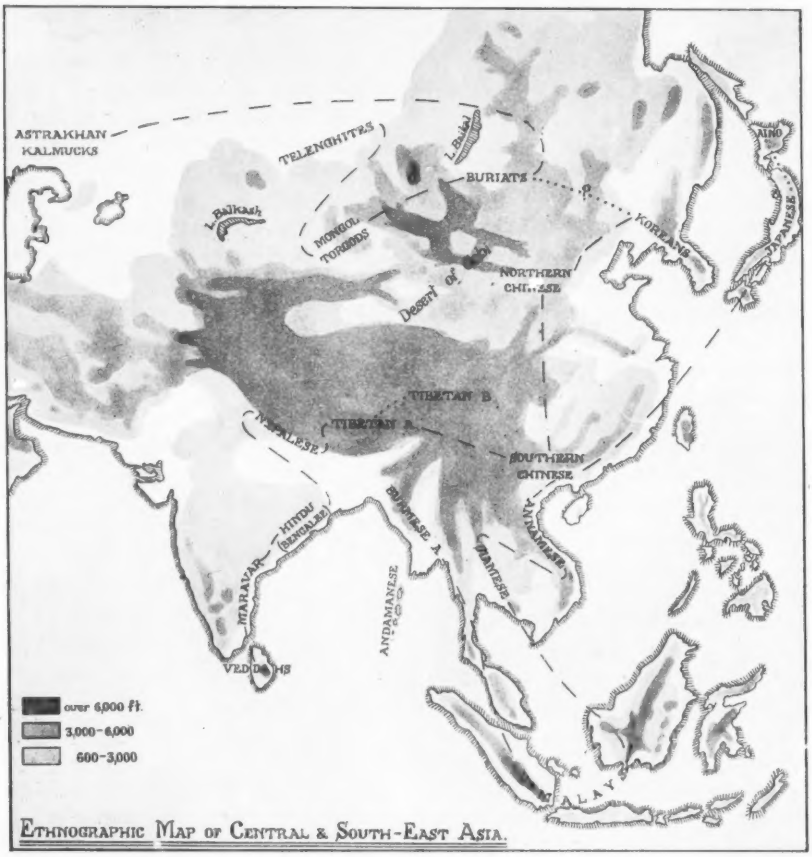
longer and broader but does not alter in shape, while the calvaria becomes broader, but does not alter in length. But again it is advisable to emphasise the fact that it would be unwise to base any far-reaching theories on evidence deduced from the inadequate material relating to the Indian races. Of the Nepalese, Tibetans and Far-Eastern races we can speak with more certainty.

It is clear that no hard and fast line can be drawn which will divide the aboriginal races of India from the so-called "Mongolian" people to the North; the transition from one type to the other is perfectly gradual. But nevertheless it will be convenient, though somewhat arbitrary, to class the Nepalese with the Tibetans, Chinese, Malays and Burmese, and it is suggested that that group of races may be termed the Oriental family or series. The extreme terms of this series—the Nepalese and Burmese A—show very marked differences in skull form, but their relationship can be clearly traced through the intermediate terms. The Oriental races must not be thought of as radiating from a common centre but rather as forming links in a straight chain, and accordingly it will be convenient to speak of them as constituting a series and not a group of races. Doubtless there are many missing links in the present classification. There appears to have been no "short-circuiting" between terms of the series separated by one or more intermediate terms; the relationship of the Tibetans and Burmese, to take a concrete example, must be traced through the Chinese and Malays, and there is no evidence which suggests that direct contact of one with the other has modified either of the former types. There are undoubtedly some hybrid races of the Orient, such as the Assamese, which have resulted from the crossing of two or more divergent terms of the series, and a study of their craniology would be of peculiar interest, but as yet we have hardly any exact knowledge of them. The hybrid Burmese B type*, which is unfortunately represented by no more than eight ♂ skulls, was found to be predominantly of Burmese stock, but there was also a suggestion of admixture with Hindu elements. Another short series of Burmese (Karen) skulls, provided by Tildesley, apparently represented a pure term of the Oriental series intermediate between the Chinese and Malays.

Having considered all the available data relating to races which were likely to be unhybridised members of the Oriental group it was thought that a study of the relations between that group and other races of Asia would be of considerable interest. With that end in view material was collected and compared with that already described. The Andamanese were found to be of a distinct type which is clearly differentiated from that of all other Orientals. The Tibetan B type is also segregated from all the neighbouring races though it appears to have been modified somewhat by admixture with the compatriot A type so that no other neighbouring race bears a closer resemblance to it. The Aino again stand quite apart from the Oriental series, and there can be no suggestion of affinity between the aborigines of Japan and either the Tibetan B or Andamanese stocks. There are thus three alien and unique types in close proximity to the Orientals: two are represented by island

* See M. L. Tildesley, "A First Study of the Burmese Skull," *Biometrika*, Vol. XIII. pp. 217, 218.

peoples and the third is probably shut in by mountain barriers and has had little direct communication with its neighbours. The Japanese are apparently an intermediate type between the Aino and Southern Chinese strains and not between the Aino and Malayan as is generally supposed; they are predominantly Oriental in type. Measurements of Chinese skulls were collected from various sources and it was possible to arrive at a fairly reliable conception of the Northern Chinese type prevailing in the Pekin district. This was so similar to that of the Southern Chinese



The lines connecting races merely indicate craniometric linkage and it is not suggested that they coincide with, or are very similar to, the routes of migration*. The broken lines denote closer affinity, as measured by the Coefficient of Racial Likeness, than the dotted. Races for which no adequate craniometric material is at present available are not shown on the map.

* The lines of the principal Asiatic migrations have been indicated by Dr A. C. Haddon on a map appended to *The Wanderings of Peoples*.

that the two peoples might almost have been considered to belong to the same homogeneous population. A discordant series of Northern Chinese was found however, and this, though still predominantly Oriental in type, had apparently been modified somewhat by admixture with an unknown people. The remarkable fixity of the Chinese type is a fact of very peculiar significance to ethnography. The Chinese is but one term in the Oriental series and as such it does not occupy a more prominent position than any other, and yet it is probable that a greater population conforms to it than to all the other types together. As far North as Korea the Oriental type still prevails mixed with some other strains of which we have no exact knowledge, but the peoples of South Siberia and North-West Mongolia are quite distinct from the Southern races. The Chinese have undoubtedly extended into those parts in considerable numbers, but it is unlikely that they intermixed with or at any rate modified the type of the nomadic tribes. We are able to use the mean measurements of series of Buriat, Torgod, Telenghite and Western Kalmuck skulls. These four races are evidently closely inter-related and it is reasonable to suppose that they are all members of the same family of races, though we are unable to estimate the range and peculiarities of that larger division while the craniometric material is so meagre. It is probable that there are some forms of man intermediate in type between the Oriental and Southern Siberian peoples, and, if there be such, they almost certainly inhabit the region immediately to the North of China. The vast uninhabited Desert of Gobi stretches between the Altai races and the Tibetans, and we have no reason whatever to suppose that there has been any direct communication between the peoples inhabiting regions on opposite sides of it. There is thus a clear line of division between the Oriental peoples and the so-called "Central-Mongols," and this line coincides almost precisely with the Northern boundary of the Oriental region of the earlier zoogeographers: see accompanying map. Furthermore, the other boundaries of the region occupied by the Oriental group of human races, with the exception of the Western, are approximately conterminous with those of that division of the world fauna which has been designated by the same name. Both the animal and the human region includes almost the whole of China and Tibet together with Burma, Assam, Siam, Sumatra, Borneo and Java, but whether the human division should include the Philippine Islands as the animal does, I am not as yet prepared to say. The islands of Japan stand somewhat apart and cannot be definitely included in either case. I shall lay no greater stress at present on the agreement between the areas occupied by these more or less unique divisions of man and other animals: but it is certainly a noteworthy correspondence which can hardly be satisfactorily explained as the result of blind chance.

It is apparently not possible to find any single cranial character which will serve to differentiate the members of the Oriental series from all unallied races; the combination of characters is alone unique. In general they are characterised by round faces and palates, broad nasal apertures and long and narrow orbits, associated with short calvariae and very rounded sagittal contours. The latter point was brought out by a comparison of their type contours with those of other

races. Very few of these were available for the Oriental peoples, but they confirmed the general conclusions which had been arrived at from a comparison of direct measurements except that the Tibetan A horizontal and vertical contours, but not the sagittal, were more like those of the Tibetan B than we should have anticipated.

2. Measurements and Methods of Measurement.

The direct measurements taken were precisely those described in "A First Study of the Tibetan Skull" (*Biometrika*, Vol. XIV.); for a fuller account of them the reader is referred to that paper (pp. 196–200) and to *Biometrika*, Vol. I. pp. 412–419, where descriptions are also given of the angular and indicial measurements.

The majority of the measurements are in accordance with those of the *Frankfurter Verständigung*.

F=Flower's Ophryo-occipital length. L'=Glabellar projective length. L=Glabellar-occipital length. B=Maximum parietal breadth. B'=Least forehead breadth. H'=Basio-bregmatic height. H=Basion to point vertically above it with skull on craniophor. OH=Craniophor auricular height. LB=Basion to nasion. Q=Craniophor transverse arc through "apex" and terminating at ear-rods. Q'=Transverse arc terminating at auricular points. S=Arc from nasion to opisthion. S_1=Arc nasion to bregma. S_2=Arc bregma to lambda. S_3=Arc lambda to opisthion. S_2'=Chord lambda to opisthion. U=Horizontal circumference. PH=Alveolar point to tip of anterior nasal spine. $G'H$=Nasion to alveolar point. GB=Distance between points where zygomatic-maxillary sutures cross lower front ridges of zygomatic arches. J=Zygomatic breadth. NH, R and L=Nasion to lowest edge, right and left, of pyriform aperture. NB=Greatest breadth of pyriform aperture. DS=Shortest subtense from bridge of nose to dacryal chord with Mérejkowsky's simometer. DC=Chord dacryon to dacryon. DA=Arc dacryon to dacryon. SS=Shortest subtense from nasal bridge to simotic chord with simometer. SC=Minimum chord between naso-maxillary sutures. O_1=Breadth of orbit R and L using curvature method*. O_1'=Breadth of R orbit from dacryon. O_2=Height of orbit R and L. G_1=Point of *spina nasalis posterior* to imaginary line tangential to inner rims of alveoli of middle incisors. G_1'=Similar to G_1 but from base of spine. G_2=From inner alveolar walls at second molars. GL=Basion to alveolar point. fml=Basion to opisthion. fmb=Greatest breadth of *foramen magnum*.

The weights of the skulls could not be determined and it was not possible to estimate the capacities by the method usually employed in the Biometric Laboratory, because no chemical balance was available. The usual method of packing the skull with mustard seed and then packing the contents into a measuring cylinder was adopted. The readings may be supposed to give fair approximations to the true capacities, but the double packing is always subject to personal equation, one half

* See *Biometrika*, Vol. I. p. 430, and Vol. VIII. pp. 311, 312.

of which may be supposed to be eliminated when the capacity is deduced from the weight of the seed*.

The Occipital Index (*Oc. I.*), which is defined to be

$$100\,\frac{S_3}{S_3'}\sqrt{\frac{S_3}{24(S_3-S_3')}},$$

was found with the aid of Tildesley's table of the function†.

Some of the index letters of the measurements have not been used by different workers in the Biometric Laboratory with precisely the same significance, and special care has to be taken when abstracting their material for comparative purposes. An account of all the differences will be found in *Biometrika*, Vol. XIV. pp. 199 and 200. The definitions of cranial "points" given in that paper (pp. 200 and 201) were made use of in the present enquiry.

A description of the mandibles in the Hodgson Collection and of the sexing of the whole material will be found in later sections of this paper.

3. "REMARKS" ON INDIVIDUAL CRANIA.

Remarks relating to the conditions of the individual skulls of the Hodgson Collection and to their outstanding cranial anomalies will be found in the Appendix. The numbers dealt with are for statistical purposes so small that no racial signifi-

PERCENTAGES OF CONFORMATIONS OF NASAL APERTURE OF NEPALESE AND TIBETAN A.

Nepalese.

Edge of pyriform aperture.

Slope of floor.	Sharp	Blunt	Rounded	Totals
Downwards ...	16·1 (9)	12·5 (7)	3·6 (2)	32·2 (18)
Flat	16·1 (9)	23·2 (13)	5·3 (3)	44·6 (25)
Upwards ...	8·9 (5)	10·7 (6)	3·6 (2)	23·2 (13)
Totals	41·1 (23)	46·4 (26)	12·5 (7)	100·0 (56)

Tibetan A.

Edge of pyriform aperture.

Slope of floor.	Sharp	Blunt	Rounded	Totals
Downwards ...	4·8 (2)	11·9 (5)	14·3 (6)	31·0 (13)
Flat	19·0 (8)	23·8 (10)	4·8 (2)	47·6 (20)
Upwards ...	21·4 (9)	—	—	21·4 (9)
Totals	45·2 (19)	35·7 (15)	19·1 (8)	100·0 (42)

Figures in brackets are the number of crania in each group.

* See *Biometrika*, Vol. III. pp. 203–206.

† *Biometrika*, Vol. XIII. p. 261.

cance can generally be attached to the proportions possessing or lacking any particular character. In general the remarks relate to the age of the skull, the condition of the teeth, the presence of bony ridges across the palatine grooves, pre-condyles, tympanic perforation, inter-parietal and wormian bones and complete or partial horizontal sutures across the malar bones*. Where no comment is made the skull is adult with sutures joined but not obliterated. There were five young skulls and their measurements were, of course, not pooled with the others when calculating the racial means. The teeth, on the whole, were in a very good state and hardly any were found to be diseased.

Base of the Pyriform Aperture. This feature is indicated by the letters *PB*. The floor is first described as "flat"—meaning that it is also parallel to the Frankfurt plane—"sloping upwards and outwards," or "sloping downwards and outwards"; and the edge is described as sharp, blunt or rounded. The frequencies for these characters are shown in the tables on p. 9.

Asymmetry. In addition to calvarial asymmetry a record was kept of the relative sizes of the Sylvian depressions and jugular foramina on the right and left sides of each skull, denoting those features by *SR* or *SL* and *JR* or *JL* according as each was more marked on the right or left side. The resulting frequencies are:

	Sylvian Depression	Jugular Foramen
Greater on right side	47	52
Equal	29	18
Greater on left side	8	13

The balance in favour of the right side has been observed in the case of several other series.

There was one case of an almost completely fused atlas and one (1 *k.k.k.*) exhibiting a pathologically enlarged condition of the calvaria. Four plates of this latter skull will be found included in this paper (Plates IX to XII) and a detailed description need hardly be given. The cranium belonged to a member of the Kocch tribe which inhabits the Sikkimese and Bhutanese Terai and may be assigned to the Nepalese race†. In shape the calvaria was remarkable for its high vertical and square forehead with a complete lack of development of the frontal sinuses—the skull was that of a young adult—and a flat vault. The outer surface affected by disease had a rough mottled appearance due to the small cavities in its surface; it was bounded laterally (see Plate X) by the well-marked temporal lines, occipitally by the *lineae nuchae supremae* and there were clear markings on the frontal bone as low as the ophryon and a few small cavities below that point. The roofs of the orbits were corrugated in the same way as the vault of the skull, but

* See *Biometrika*, Vol. XIV. pp. 201–3, for a more detailed account of these anomalies.

† See pp. 17 and 22 below.

the facial and basial bones were all normal in appearance. The coronal, sagittal and parieto-temporal sutures were completely closed, but the lambdoid was open for the whole of its length; a well-marked depression extended along the line of the sagittal suture. On removing the skull-cap it could be seen that the vault was thickest in the region of the frontal bosses (21 mm.) and thinnest in the sagittal plane. The cerebral markings were somewhat larger and more clearly defined than in the normal skull, but otherwise the internal surface was not in any way peculiar. Another skull (1 *t.t.t.*) of the Dhimal tribe, which is closely related to the Kocch, bore slight traces of the same disease. The external surface of the parietal bones had been affected in the region of the obelion and the sagittal groove was well marked. This specimen—like the former—was characterised by a high square forehead and flat vault. The Brothers Sarasin* have provided photographs of a Veddah skull which is almost identical in appearance with the Kocch specimen in the Hodgson Collection, though the roofs of the orbits of the former were not affected by the disease. Its age was estimated to be from twenty to twenty-five years. There is no very similar specimen in the Museum of the Royal College of Surgeons, and the writer is not aware of the existence of such an one in any other collection. We may label the condition acrocephaly, osteitis or hypertrophy, but the fact that the three examples referred to above resemble each other remarkably closely suggests that they represent some more specific pathological state†. Measurements of the Veddah skull are unfortunately not available, but the Kocch ($LB = 92{\cdot}9$) and Dhimal ($LB = 94{\cdot}4$) specimens are apparently characterised by shorter nasio-basional lengths than the mean Nepalese ($LB = 98{\cdot}0$ (47)), although their facial and basial measurements other than LB do not appear to be in any way peculiar.

4. The Use of the Coefficient of Racial Likeness.

Before a really adequate and final comparison of the mean cranial characters of two races can be made the inter-racial standard deviations and correlations deduced from the mean measurements of at least fifty to a hundred distinct races must be known, and each of the mean measurements for those fifty or more races should be based on fifty or more crania. The comparative material at present available is far too meagre to provide those inter-racial constants or even an approximation to them. But provisionally intra-racial standard deviations, i.e. measurements of the variabilities of the individual racial populations, may be adopted in estimating the significance of inter-racial differences, and for that purpose Professor Karl Pearson has suggested that a single coefficient, known as the Coefficient of Racial Likeness, which takes into account a large number of mean characters, may be used.

* *Ergebnisse naturwissenschaftlicher Forschungen auf Ceylon*, Bd. III. 1892–3, p. 257.

† The Brothers Sarasin (*loc. cit.* Table of individual measurements at end of their Atlas volume) say that the Veddah skull is an example of *leontiasis ossea*—Virchow's term for *osteitis deformans*—but Sir Arthur Keith, who was good enough to examine my plates, affirms that the Kocch specimen does not exhibit the traits of that disease.

Let m characters be measured and let the sth character in the first race have M_s for mean and σ_s for standard deviation, those two constants being based on n_s individual measurements. Let the corresponding constants for the second race be M_s' and σ_s' based on n_s' individual measurements. Then, if the two races were really samples from the same population, the following expression would be sensibly zero:

$$\frac{1}{m} S \left\{ \frac{(M_s - M_s')^2}{\frac{\sigma_s^2}{n_s} + \frac{\sigma_s'^2}{n_s'}} \right\} - 1,$$

and the probable error of its deviation from zero would be $\cdot 67449/\sqrt{2m}$.

The value of this expression computed from a selected number of the mean characters of two races is the Coefficient of Racial Likeness between them and it is thus a measure of the probability of the two being random samples from the same population. A Coefficient which differs sensibly from zero may be taken to indicate a difference of type, and it is suggested that in such cases the Coefficient may still be used as an index of the degree of resemblance between the races compared although all the values found may not be compared directly together. Given two random samples each of ten individuals drawn from the same homogeneous population, the Coefficient of Racial Likeness deduced from the mean characters of the two samples will not differ significantly from zero, and if two samples each of a hundred individuals are drawn from the same population then their Coefficient will also be of the same order. But if two random samples each of ten individuals are drawn from two different populations and then two samples each of a hundred individuals are drawn from the same differing populations it will be found that the Coefficient between the first pair will be very distinctly less than that between the two samples of a hundred individuals each. The difference in this case is merely an expression of the rather obvious fact that it is more probable that the small samples were in reality drawn from the same population than that the larger samples were. It is for this reason that Coefficients of Racial Likeness may not be compared directly by estimating differences in terms of probable errors only as may be done when dealing with the majority of statistical constants in use. Reference has to be made constantly to the numbers of crania in the several racial series used. Statistical comparison would be greatly facilitated if all racial means were based on the same number of crania—say 50 or 100—but at present we are obliged to deal with series of unequal length. In practice direct comparison may be made between the numerical values of Coefficients of Racial Likeness in the cases when:

(i) All the racial means are based on the same or approximately the same numbers of crania;

and (ii) When one series of racial means is compared with a number of others, the latter being based on the same or approximately the same numbers of crania which may differ from the number in the first series.

The use of these methods will be sufficiently illustrated in later sections of the paper.

Intra-racial correlations between characters not connected in some obvious way, such as the various measurements of head length, are known to be small; otherwise the Coefficient should not theoretically be used. The measurements chosen should show little correlation among themselves, and at the same time they should be as representative as possible of all regions of the skull. The sagittal arc S is one of the selected characters and hence the component arcs S_1, S_2 and S_3 and the chord S_3' cannot be employed; two angles of the fundamental triangle may be used but not three. Thirty-one measurements were chosen, such that all the correlations between them were fairly small, and the majority did not differ sensibly from zero, If the Coefficients computed by different workers are to be comparable, some such list must be adopted and adhered to*. Unfortunately all these mean measurements are not given for some of the comparative material it is desirable to use, and when there are many missing an element of uncertainty is introduced which detracts appreciably from the value of the results. Another source of possible error has to be introduced in practice which is perhaps of greater moment. The standard deviations σ_s and σ_s' cannot de determined with at all sufficient accuracy from the short series of skulls with which the craniologist is usually concerned. Accordingly the assumption has to be made that the variabilities are equal to each other and are also equal to the standard deviation of the longest homogeneous series of crania available. That there should be any very near approach to equality in all racial populations is unlikely, but in the present state of our knowledge we are bound to postulate it. The majority of the races inhabiting the Orient appear to be fairly equally variable, though they are more variable than European and African peoples, and thus no great error will be introduced as long as the Coefficients of Racial Likeness between them are only compared with each other. An unpublished series (E) of Dynastic Egyptian crania is the longest available for which standard deviations are given. Through the kindness of Professor Karl Pearson I am able to reproduce the female values here (p. 14); the data for male skulls have already been given†. These values have been used in calculating all the Coefficients of Racial Likeness given in this paper.

Whenever possible the thirty-one characters shown in Table I are used in calculating the Coefficient of Racial Likeness, but if necessary the place of some of them may be taken by similar measurements: thus L or L' may be used in place of F; H or H' in place of OH; Q' in place of Q and G_1' in place of G_1. When dealing with short racial series a criterion of racial resemblance which can take cognizance at one time of a considerable number of cranial measurements and express their relations as a single number does in a great measure compensate for the indefiniteness of conclusions drawn from a comparison of single measurements. But the more detailed comparison is not neglected, for in the process of computation of the

* The Coefficient of Racial Likeness was first used in a paper by M. L. Tildesley entitled "A First Study of the Burmese Skull" (*Biometrika*, Vol. XIII. p. 176), but the list of characters selected was not identically the same as ours. The Coefficients given in *Biometrika*, Vol. XIV. pp. 210–227, were computed in precisely the same way as in the present enquiry.

† *Biometrika*, Vol. XIV. p. 208.

Coefficient an estimate is obtained of the significance of the differences between individual characters.

TABLE I.

Standard Deviations of Female Egyptian Skulls of the 26th to 30th Dynasties (Series E).

Character	100 B/L'	100 H/L'	100 B/H	*Oc. I.*
Standard Deviation ...	2·57 ± ·05	2·86 ± ·06	4·07 ± ·08	3·20 ± ·06
Character	100 $G'H/GB$	100 O_2/O_1, R	100 NB/NH, R	100 fmb/fml
Standard Deviation ...	4·48 ± ·09	4·59 ± ·09	3·77 ± ·07	5·62 ± ·11
Character	100 G_2/G_1	$P\angle$	$N\angle$	$A\angle$
Standard Deviation ...	6·58 ± ·15	2·95 ± ·06	3·26 ± ·07	3·41 ± ·07
Character	F	B	B'	OH
Standard Deviation ...	4·77 ± ·09	4·52 ± ·09	3·79 ± ·07	3·65 ± ·07
Character	LB	Q	S	U
Standard Deviation ...	3·52 ± ·07	8·72 ± ·17	10·49 ± ·21	11·76 ± ·23
Character	$G'H$	J	NH, R	NB
Standard Deviation ...	3·76 ± ·08	4·38 ± ·09	2·60 ± ·05	1·64 ± ·03
Character	O_1R	O_2R	G_1	G_2
Standard Deviation ...	1·56 ± ·03	1·84 ± ·04	2·96 ± ·06	2·53 ± ·06
Character	fml	fmb	C	
Standard Deviation ...	2·19 ± ·04	2·00 ± ·04	92·06 ± 2·02	

5. An Account of the Skulls of the Hodgson Collection in the British Museum (Natural History).

At the beginning of the nineteenth century Nepal had not been explored by any European and the facts known concerning the country were few and of very doubtful authenticity. Several Englishmen had crossed the border and a few had been to Katmandu on political missions, but the natives would not allow them to penetrate further and prevented them, as far as possible, from acquiring any knowledge of the local civilisation and geography. The first scientific accounts of Nepal were given by Brian Houghton Hodgson. He became Assistant Resident at Katmandu in 1820 and Acting Resident in 1829, a post which he held until retiring from office in 1844*. During those 24 years he had been prosecuting researches into the history, ethnography, zoology and geography of the country. Hodgson quickly acquired a European reputation as the collector and donor to the museums of England and France of enormous numbers of Sanskrit manuscripts and zoological specimens. The latter were usually presented to the British Museum and by 1858 they totalled 10,499. Among them were more than 90 human crania and that series forms the subject of the present paper†.

* The dates and other particulars of Hodgson's life are taken from the *Life* by Sir W. W. Hunter (1896).

† I am indebted to the authorities of the British Museum (Natural History) for permission to measure these crania, and I have also to thank them for the invariable courtesy they have shown and the help they have given during the course of my work, especially in the matter of photographing selected crania.

A catalogue of the Hodgson Collection* was issued in 1846, giving records of 16 skulls, and in 1863 this was reprinted with the addition of all later donations including 79 skulls which had been presented to the Museum in 1858. Sir Richard Owen was at that time the Superintendent of the Natural History Departments of the British Museum, and in a paper communicated by him to the British Association in 1859 he gave a brief account of the skulls. The only measurements given were those of the length and breadth of a few selected crania. Of one tribe it was said, "Upon the whole these Lepcha skulls are to be referred to a low, uneducated and undersized family of the Indo-European race; but one approaches the Æthiopian type, another the Australian type; whilst a third shows almost the Greek model save in a slight prognathism." Owen arrived at no very definite conclusions when comparing the several tribes represented, but he appeared to be of the opinion that the series was a heterogeneous one. There has been no other published account of the crania. There can nowhere be found either the record of a single measured character for all the skulls, or even a list of their most outstanding anomalies. It cannot be urged in explanation of such neglect that sufficient is already known of Nepalese crania for, though not so rare as Tibetans, they are by no means common in Europe, their scarcity being partly due to the fact that many of the tribes cremate their dead. There were several skulls from Nepal and neighbouring regions in the private collection of Barnard Davis and he has described these†, but the characters measured are few in number and many of them do not correspond with those of other craniologists. The measurements of several Nepalese skulls are to be found in the German Anthropological Catalogues, and there are a few skulls in French museums, but there appears to be no series of any length in either country. It may seem surprising that Hodgson should have collected so many specimens when the craniologists of his day would draw conclusions as readily from 5 skulls as from 50. The explanation is that he sought to represent several tribes which were supposed to be more or less racially distinct; there are skulls of all the principal tribes inhabiting Nepal.

"My favourite amusements of the sedentary kind," Hodgson wrote in 1834, "are researches into the origin, genius and attainments of the various singular races of man inhabiting Nepal." Though realising the importance of the then infant science of physical anthropology‡, his interest was rather in the philology, history and cultural status of the people. His ethnographic essays, the majority of which were written between 1845 and 1858, are of great importance because they furnish the fullest and most authentic records that we have of the condition of the country in the opening decades of the nineteenth century.

From the twelfth century downwards numerous Rajputs from Rajputana fled from the victorious Mohammedan invaders of their country. They settled for a

* *A Catalogue of the Specimens and Drawings of Mammalia and Birds of Nepal and Tibet, presented by B. H. Hodgson, Esq., to the British Museum.* This is merely a catalogue, without any measurements or adequate descriptions of the human crania.

† *Thesaurus Craniorum* (1867), pp. 158–169.

‡ Measurements of several natives are to be found in Hodgson's essays.

time in the hills of Kumaon, became dominant there and gradually extended their dominions over the native tribes of Western Nepal. From the children of these Brahmins begotten on native women has sprung the now extensively ramified tribe of Khas, or Parbatias as they are sometimes called since they use the Parbatirja language, a dialect of Sanskrit. A considerable number of these Khas with some of the natives they had subjugated settled at Gorkha, a town sixty miles West of the capital Katmandu, and subsequently they became known as Gorkhas or Gorkhalis. In 1768 they completed the conquest of the country by overrunning the Valley of Nepal and they settled there as the dominant race. The Gorkha rule had been firmly established when Hodgson went to Katmandu in 1820 and the invaders had spread further to the East, though still to be found most numerously in the Valley itself. Before the invasion of the Khas the principal native hill tribes of Nepal were, from West to East, the Sunwar, Gurung, Magar, Murmi, Newar, Kiranti and Limbu. The first three of these inhabited the country to the West of the Valley; as they became Hinduised and adopted the customs and language of the Khas, they also were known as Parbatias or Gorkhas. The Murmi and Newars were the denizens of the Valley, and the Kiranti and Limbu extended to the East of that densely populated region as far as the border of Sikkim. To the North of these tribes are scattered nomadic communities of Bhotias or Tibetans. Hodgson's investigations extended over Sikkim and Bhutan as well as Nepal, those three countries comprising what he called the Central Himalayas. We have skulls of Lepchas of Sikkim and of Lhopas or Bhutanese.

Each of the tribes and races enumerated above had a language or at least a dialect of its own—the Limbus and Northern Bhotias speaking Tibetan—and it is clear that they must have been segregated in their respective valleys for several hundred years, having little or no communication with any people but their immediate neighbours. They were nomadic cultivators but their wanderings were within narrow limits, since the spurs projecting southwards from the main line of the Himalayas make transit from East to West extraordinarily difficult.

The little trade of the country was exclusively in the hands of the Newars. Hodgson's interest was in the native tribes of Nepal and his studies were confined to the status and languages of the people as they had been before the advent of the Hindus. He was able to show that the several dialects were closely related to each other and to Tibetan and must have sprung at no very distant date from the same parent stock*. Hodgson appears to have collected skulls with the same object in view as when collecting vocabularies. In both cases they relate to the original native tribes and there is no skull labelled Khas, Parbatia or Gorkha. It is reasonable to suppose, though unfortunately there is no direct evidence to support the contention, that none were included which were thought to be of Aryan origin or admixture. It seems to have been possible to make the distinction a century ago. The Eastern tribes had been most subjected to foreign influence; of

* Hodgson went further and sought to establish a link between the sub-Himalayan tribes and the aborigines of Central India—the Dravidians or Tamulians, as they were then called. Later philologists have discredited this view.

the Magars and Gurungs, Hodgson wrote in 1833, "they have retained in vivid freshness their original languages, physiognomy and to a less degree habits*." They were still more or less segregated from the Khas and probably buried their dead apart. Before 1830 the Magar and Gurung had spread extensively over the whole of Nepal, since they participated in the successful invasions of the Gorkhas, but originally they were confined to the Eastern part of the country†.

The races of which mention has as yet been made inhabited the northern and middle regions of the sub-Himalayas. At the foot of the hills and stretching along the southern frontiers of Nepal, Sikkim and Bhutan is a narrow strip of low, marshy country known as the Terai. This has been inhabited since early times by several primitive tribes, known collectively as Awalias, which are able to withstand the malarious climate, though it is fatal to both the Hindus of the plains and the hillmen to the North. There are skulls of the Kocch, Bodo and Dhimal tribes of the Sikkimese and Bhutanese Terai. These are of peculiar interest because they are said to represent a mixture of Mongolian and Dravidian strains; they are at any rate non-Aryan and non-Hindu. Hodgson, in his well-known essay‡, said that they were indistinguishable in physical appearance from the decidedly Mongolian hill races to the North, but to-day the Dravidian traits are said to predominate §. Unfortunately we are not able to make any reliable determination of type from the very few skulls of these tribes in the collection. There are also two Uraon (Hodgson) or Oraon (Risley) skulls. These belong to a Dravidian people which inhabits north-west Chota Nagpur and is said to show very little Mongolian affinity. The series includes eight skulls labelled Lowlanders, without indication of tribe, a Bengal Fakeer and a man of the Ganges. Since Hodgson's interests were in general confined to Nepal and Tibet and there are no other skulls representing the races of the Nepal Terai, it is very probable that these Lowlanders came from that region‖. Finally we have seven trans-Himalayan Bhotias or Tibetans proper and these came from several localities widely separated geographically and none belonged to the sparsely populated regions immediately adjacent to the northern boundary of Nepal. There is one of the Sakya tribe of north-eastern Tibet (Koko

* "On the Military Tribes of Nepal," *Journal Bengal Asiatic Society*, Vol. II. (1833). Reprinted in *Essays* (1874). Part II. pp. 37–44. Quotation on p. 39.

† It is important to bear in mind that the skulls of the Hodgson Collection represent the native tribes as they were until, say, 1850, but since that time they have probably become hybridised to a certain extent with Hindus and other foreigners. Rapidly increasing trade has established quite a large alien population at Katmandu: in 1830 the total value of the imports and exports of Nepal was estimated at Rs. 3,000,000, but by 1890 the trade with India alone was Rs. 33,000,000 (see Hunter's *Life of Hodgson*, pp. 114, 115). Changes in the West have affected the natives more profoundly. The Gorkhas overran Sikkim in 1814; in 1835, when the population did not number more than 200, part of Sikkim was ceded to the British and the country developed very rapidly. The establishment of a sanitarium at Darjeeling and the introduction of the tea-plant attracted thousands to the country, chiefly Hindus of the plains. It is said that the original Lepchas are fast dying out; at any rate, they are disappearing from Sikkim (see L. A. Waddell, *Among the Himalayas*, pp. 38, 149, 243).

‡ *Essay on the Kocch, Bodo and Dhimal Tribes.* Calcutta (1847). Reprinted in *Essays*, 1880. Vol. I. pp. 1–160.

§ See the articles in H. H. Risley's *The Tribes and Castes of Bengal. Ethnographic Glossary* (1891).

‖ One of the "Lowlander" skulls has "Terai" marked on the frontal bone.

Nor district) and three from the Eastern province of Khams. These are brachycephalic and in other ways very different from the large, muscular type represented by the skulls, reputed to be those of Khams Tibetans, in the Museum of the Royal College of Surgeons*. The area of Khams is so large and communication within it is so difficult that it may well contain two distinct races of man. But possibly either the Royal College skulls or the three in Hodgson's Collection came from a neighbouring locality.

With representatives of several racial groups, we should not be justified in pooling any of the material and then reducing means unless it can be shown that the groups are so similar that they may reasonably be supposed to be samples of the same population. The measurements made on the Nepalese skulls and the methods of measurement are precisely those which have been described above. The records of sex provided by Hodgson were accepted in all cases. More than half had been marked with m. or f. on the frontal bone when originally classified, probably when the specimens were first procured. In addition to these marks there is evidence relating to sex in the records kept by Hodgson and reproduced in the catalogues of the collection; these give the number of male and female skulls for each tribe, but without particularising individual skulls. Since the majority were said to be male there was little difficulty in selecting the few females†. The division of the skulls made in this way was quite in accordance with the sexing which a survey of the appearance of the skulls might have suggested, so there is good reason to suppose that it is correct. The sexes of the eight skulls of Lowlanders were not marked on them or given in the catalogues; it was decided that three of them were female. The skulls available for measurement were those of seventy-two male adults, ten female adults, four young boys and one young girl—eighty-seven in all.

We may anticipate that the various groups into which the collection of Nepalese skulls is naturally divided will be closely allied, and before comparing them it will be well to consider under what circumstances we may be justified in pooling together two series to give a population which will be as homogeneous as that with which the craniologist usually has to deal. If two series of skulls are in reality random samples from the same population then the Coefficient of Racial Likeness between the two sets of mean characters should not differ significantly—by more than three and a half times its probable error, say—from zero. But single collections of crania are seldom, if ever, random samples of the inhabitants of a country and the various intimately related types of people which the physical anthropologist is obliged to class together under the name of one racial type have not the uniformity of the ideal statistical population. Accordingly we shall be justified in practice in combining series of skulls with the object of obtaining a racial type, even though the series cannot theoretically be considered samples from the same population. It is suggested that that may be done whenever

* See "A First Study of the Tibetan Skull," *Biometrika*, Vol. xiv. pp. 193 *et seq.*

† This task was rather more difficult than it might have been owing to the fact that five skulls were transferred to the Smithsonian Institute in 1912.

a Coefficient does not exceed unity by more than three times its probable error, but it must be remembered that when the difference between two types is just significant the Coefficient will in general become greater according as the types are represented by larger numbers of crania and an arbitrary rule such as that suggested would tend to overrate the identity of very short series of skulls and would not be applicable in such cases. The following table gives the values of the least Coefficients that have yet been calculated:

TABLE II.

Some low Coefficients of Racial Likeness. (All Male.)*

	All Characters			Indices and Angles		
	C. L.	Probable Error	Number of Characters	C. L.	Probable Error	Number of Characters
Moriori, Thomson (33·8)† and Scott (31·2)	0·41	±·10	21	0·25	±·21	5
Tibetan A (16·5) and Burmese B (7·5)	0·80	±·09	31	0·33	±·14	12
Greenland (25·2) and Archipelago Eskimos	1·20	±·11	18	0·21	±·21	5
Moorfields (24·2) and Whitechapel (87·2) English	2·05	±·09	26	1·73	±·16	9

In the case of the two Moriori series, both of which contained a sufficient number of crania to make the determination of type fairly reliable, we can have little hesitation in saying that the means may be legitimately pooled, that is to say the resulting population would be as homogeneous as others with which comparison might be made. But it would be extremely rash to combine the Tibetan A and Burmese B in the same way, although the Coefficient between them is less than unity; the number of crania in each of the series is so small that if they were true representatives of the same population we should have anticipated that the Coefficient would not differ significantly from zero. The Eskimo means are more reliable and it would be just legitimate to pool them, but the Moorfields and Whitechapel English must be supposed to represent distinct though very similar types. It will be reasonable to pool two racial series of means if they both contain fewer than 20 crania and have a Coefficient of Racial Likeness which is not signi-

* The mean measurements of the series compared in this table were taken from the following sources: Moriori (Thomson)—*Biometrika*, Vol. XI. p. 90, and Scott's measurements are quoted on p. 93 of the same paper. The transverse arcs (Q) and orbital measurements were not included when computing the Coefficient, as the two writers had almost certainly not used the same methods of measurement for those characters (see Thomson, *loc. cit.* p. 92). Burmese B—*Biometrika*, Vol. XIII. p. 219; Tibetan A and Eskimo—*ibid.* Vol. XIV. pp. 206 and 222 respectively; Moorfields English—*ibid.* Vol. V. p. 92, and Whitechapel English—*ibid.* Vol. III. p. 208. The capacities were omitted when comparing the collated Eskimo material.

† The numbers in brackets after the races denote the mean number of skulls in the series for the characters used in computing Coefficients of Racial Likeness, and will be used in that sense throughout this paper.

ficantly greater than 0·5, or if they each contain more than 20 crania and have a Coefficient which is not significantly greater than unity.

In the Hodgson Collection there are 7 skulls of Trans-Himalayan Bhotias or Tibetans proper and 5 of Cis-Himalayan Bhotias or Tibetans resident in Nepal. These two types may be compared with each other and with the Tibetan skulls in

TABLE III.

Mean Characters of various Series of Tibetan Male Crania.

	100 B/L'	100 H/L'	100 B/H	*Oc. I.*	100 $G'H/GB$	100 NB/NH, (R)
Trans-Himalayan Bhotias	76·1 (6)	72·7 (6)	104·8 (6)	61·3 (7)	71·0 (7)	50·0 (7)
Cis-Himalayan Bhotias...	77·8 (6)	74·7 (6)	104·2 (6)	62·7 (6)	66·0 (6)	51·4 (6)
Tibetan A	79·6 (17)	75·2 (17)	105·7 (17)	61·7 (17)	70·4 (15)	49·3 (16)
Tibetan B	75·2 (14)	72·3 (14)	104·0 (14)	59·2 (15)	76·0 (15)	49·5 (15)
	100 O_2/O_1, (R)	100 *fmb/fml*	100 G_2/G_1	*F*	*B*	*B'*
Trans-Himalayan Bhotias	79·8 (7)	81·1 (7)	82·6 (7)	175·7 (7)	136·3 (6)	91·4 (7)
Cis-Himalayan Bhotias...	78·6 (6)	84·6 (6)	82·5 (5)	173·4 (6)	137·6 (6)	92·3 (6)
Tibetan A	84·3 (17)	84·3 (17)	86·5 (15)	174·4 (17)	139·4 (17)	92·6 (17)
Tibetan B	83·0 (15)	84·1 (15)	84·1 (12)	184·5 (14)	139·4 (14)	94·3 (15)
	OH	*LB*	*Q'*	*S*	*U*	*G'H*
Trans-Himalayan Bhotias	114·5 (6)	97·2 (7)	296·8 (6)	355·9 (7)	497·7 (6)	68·3 (7)
Cis-Himalayan Bhotias...	115·8 (6)	97·0 (6)	304·1 (6)	362·6 (6)	497·3 (6)	66·0 (6)
Tibetan A	113·2 (17)	95·7 (17)	310·9 (17)	360·9 (17)	503·6 (17)	69·4 (15)
Tibetan B	115·5 (15)	99·2 (15)	312·9 (14)	378·6 (14)	525·6 (14)	76·5 (15)
	J	*NH*, (R)	*NB*	O_1, (R)	O_2, (R)	G_1
Trans-Himalayan Bhotias	127·8 (6)	51·0 (7)	25·5 (7)	41·9 (7)	33·4 (7)	48·8 (7)
Cis-Himalayan Bhotias...	131·8 (6)	50·6 (6)	25·7 (6)	41·5 (6)	32·5 (6)	49·7 (5)
Tibetan A	130·4 (17)	51·7 (16)	25·3 (17)	41·7 (17)	35·0 (17)	47·7 (17)
Tibetan B	137·5 (15)	54·9 (15)	27·1 (15)	44·0 (15)	36·5 (15)	51·5 (13)
	G_2	*fml*	*fmb*	$P \angle$	$N \angle$	$A \angle$
Trans-Himalayan Bhotias	40·2 (7)	37·0 (7)	30·0 (7)	84·4 (6)	66·0 (7)	72·2 (7)
Cis-Himalayan Bhotias...	41·1 (6)	35·1 (6)	29·6 (6)	84·7 (6)	68·1 (6)	71·8 (6)
Tibetan A	41·0 (15)	35·7 (17)	30·1 (17)	87·4 (15)	64·7 (16)	71·8 (16)
Tibetan B	43·6 (14)	37·4 (15)	31·4 (15)	85·7 (14)	65·5 (15)	68·8 (15)
	C					
Trans-Himalayan Bhotias	1418·3 (6)					
Cis-Himalayan Bhotias...	1471·7 (6)					
Tibetan A	1433·0 (17)					
Tibetan B	1537·7 (14)					

the Royal College of Surgeons*. Table III gives a select list of mean characters, viz. those from which the Coefficient of Racial Likeness is calculated, representing all important regions of the skull.

It is evident that the Tibetan B type, which is supposed to be representative of the natives of the Eastern province of Khams, differs as markedly from the two Bhotia series as it was found to differ from the Tibetan A, but the means of the three more Western groups resemble each other closely. The Coefficients of Racial Likeness are:

	C. L. 31 characters	C. L. Indices and Angles 12 characters
Tibetan A (16·5) and Trans-Himalayan Bhotias (6·7)	+0·48 ± ·09	+0·92 ± ·14
Tibetan A and Cis-Himalayan Bhotias (5·9) ...	+0·39 ± ·09	+0·87 ± ·14
Trans-Himalayan and Cis-Himalayan Bhotias ...	−0·34 ± ·09	−0·19 ± ·14

Since not one of these Coefficients is significantly greater than ·5 we can feel justified in clubbing the three series together and the resultant means may still be said to be those of the Tibetan A type. The Tibetans of Nepal are evidently quite pure representatives of that stock as is generally supposed. The Coefficient between the Royal College A type skulls and all the Tibetans in the Hodgson Collection is 0·83 ± ·09 for 31 characters and 1·47 ± ·14 for 12 indices and angles. This last value is decidedly higher than the others, but it differs from unity by very little more than three times its probable error and we can still feel justified in combining the three series of means, resulting in the A type of Table VII (page 26). Comparing these 30 skulls with the B type (14·0) gives the Coefficient 8·71 ± 0·9 for 31 characters and 3·44 ± ·14 for 12 indices and angles. The corresponding values for the original seventeen skulls of Type A and those of Type B were 6·46 ± ·09 and 3·34 ± ·14, so the addition of the Bhotia skulls has not made the A and B types more similar. Since the skulls of the Trans-Himalayan Bhotias came from very different parts of the country, three belonging to tribes of the East residing 1,000 miles away from the Nepalese frontier, it is reasonable to suppose that the greater part of Tibet is inhabited by a homogeneous and pure race of man, though much more ample material will be required before any rigorous proof of such a statement can be given. There is a distinct race (Type B) in one of the Eastern provinces—probably that of Khams.

Having separated the Tibetan skulls we can now compare the other racial groups represented. At this stage in the enquiry the Lowlander group had to be kept distinct from the Kocch, Bodo and Dhimal, and since the number of skulls remaining was adequate it was thought that a comparison of a few divisions of the Nepalese skulls would be of considerable interest. A division into three groups suggested itself: a Western, comprising the tribes to the West of the Valley of

* The mean measurements of these latter have been provided in *Biometrika*, Vol. XIV. p. 206. Two very aberrant types were distinguished: a Tibetan A to which the vast majority of the inhabitants of the country probably conform, and a B type found in the eastern province of Khams.

Nepal; a Central, of the inhabitants of the Valley itself; and an Eastern group of the tribes to the East of the Valley together with the Sikkimese and Bhutanese. More particularly the divisions are:

Tribes	Divisions	Number of Male Crania
Gurung (2) ... ... Magar (1) ... ... Sunwar (4)	Nepalese Western ...	7
Murmi (4) ... ... Newar (13)	Nepalese Central ...	17
Kiranti (6) ... ... Limbu (5) ... ... Lepcha (Sikkim) (9) ... Lhopa (Bhutan) (2) ...	Nepalese Eastern ...	22
Lowlanders (5) ... Bengali (2)	Lowlanders	7
Kocch (2) ... ... Bodo (1) ... ... Dhimal (1)	Kocch, Bodo and Dhimal	4

There are so few crania in the smaller of these divisions that results deduced from them can be of no permanent value whatever, but we may be able to detect the more outstanding differences of type. A list of their mean characters is given in Table IV and the Coefficients of Racial Likeness between the various divisions are given in Table V.

From a comparison of the mean characters of the divisions given in Table IV it is quite evident that the five types are all very similar for nearly all measurements, but the Nepalese Western differs quite markedly from the others in the case of the indices B/L' and B/H and the breadth B. The Coefficients of Table V show how remarkably similar many of the divisions are. If the Coefficients with the first (Nepalese Western) are excluded, no one of the others is found to differ significantly from unity and the majority do not differ significantly from zero. We shall be fully justified in clubbing the last four divisions together, but the Western Nepalese* cannot be included with them. The Lowlanders are so similar to both the hillmen to the North (Nepalese Western and Central) and to the Kocch, Bodo and Dhimal tribes that the supposition that they are the natives of the Nepal Terai becomes more probable than ever. The Coefficients between the Maravar (Dravidian) (26·7) tribe of the Madras Presidency† and the Kocch, Bodo and Dhimal are $1{\cdot}67 \pm {\cdot}10$ for 21 characters and $-0{\cdot}51 \pm {\cdot}19$ for indices and angles. No stress whatever can be laid on this last Coefficient since it is based on no more than six characters, and accordingly it becomes probable that the Kocch, Bodo and Dhimal tribes are primarily of "Mongolian" rather than Dravidian origin. We should require many more than four skulls to prove such a contention, which is at variance with the views of Risley, but at any rate we may safely include the skulls

* Probably the natives of the Western border province of Ladak are of a more or less hybrid stock, though predominantly of our A type.

† The Maravar means are taken from *Biometrika*, Vol. XIII. p. 239.

TABLE IV. *Mean Characters for Various Divisions of the Hodgson Collection.*

Character	Nepalese Western	Nepalese Central	Nepalese Eastern	Lowlanders	Kocch, Bodo and Dhimal
$100\,B/L'$	78·7 (6)	73·8 (17)	75·3 (21)	73·1 (7)	71·7 (4)
$100\,H/L'$	74·5 (6)	74·7 (17)	75·4 (21)	73·4 (7)	74·3 (4)
$100\,B/H$	105·6 (6)	98·7 (17)	99·7 (21)	99·5 (7)	95·4 (4)
Oc. I.	61·8 (7)	60·5 (17)	60·2 (21)	60·7 (7)	59·8 (4)
$100\,G'H/GB$	70·3 (5)	69·4 (15)	68·6 (22)	72·4 (4)	71·4 (3)
$100\,NB/NH, R$	51·5 (6)	51·9 (17)	51·4 (22)	51·3 (7)	52·6 (4)
$100\,O_2/O_1, R$	78·8 (7)	79·0 (17)	79·5 (22)	77·4 (7)	79·6 (4)
$100\,fmb/fml$	80·4 (7)	83·2 (17)	82·4 (21)	83·4 (6)	81·4 (4)
$100\,G_2/G_1$	86·5 (7)	85·0 (13)	86·7 (22)	79·9 (3)	78·6 (3)
$P\angle$	89·9 (5)	85·8 (16)	85·6 (22)	83·7 (4)	79·9 (3)
$N\angle$	64·5 (6)	66·2 (16)	66·3 (21)	67·8 (4)	69·5 (3)
$A\angle$	73·7 (6)	73·1 (16)	72·8 (21)	71·9 (4)	69·5 (3)
C	1512·1 (7)	1439·1 (17)	1453·8 (21)	1355·0 (7)	1447·5 (4)
F	177·0 (7)	176·9 (17)	175·4 (21)	174·6 (7)	179·9 (4)
B	140·3 (6)	132·1 (17)	134·1 (21)	129·9 (7)	131·6 (4)
B'	91·7 (6)	92·7 (17)	89·8 (22)	89·7 (7)	91·7 (4)
OH	116·3 (6)	115·2 (17)	114·0 (22)	113·4 (7)	117·1 (4)
LB	99·4 (7)	98·2 (17)	98·2 (21)	98·2 (7)	97·6 (4)
Q'	306·1 (6)	301·1 (17)	299·4 (21)	296·5 (7)	312·1 (3)
S	366·0 (6)	365·9 (17)	363·7 (20)	361·2 (7)	375·9 (4)
U	507·8 (6)	497·8 (17)	497·5 (21)	493·0 (7)	502·9 (4)
$G'H$	70·7 (6)	67·4 (16)	67·6 (22)	68·1 (4)	70·5 (3)
J	133·6 (6)	127·1 (15)	128·0 (22)	124·5 (4)	125·7 (4)
$NH, (R)$	52·1 (6)	49·3 (17)	50·2 (22)	49·3 (7)	49·5 (4)
NB	26·6 (7)	25·5 (17)	25·8 (22)	25·2 (7)	26·0 (4)
$O_1, (R)$	43·4 (7)	41·9 (17)	41·6 (22)	42·3 (7)	42·5 (4)
$O_2, (R)$	33·6 (7)	33·1 (17)	33·0 (22)	32·7 (7)	33·8 (4)
G_1	49·2 (6)	48·4 (15)	47·7 (22)	49·4 (3)	48·9 (3)
G_2	41·5 (7)	41·3 (15)	41·9 (22)	39·6 (7)	38·4 (3)
fml	36·8 (7)	35·4 (17)	35·0 (21)	36·8 (7)	37·6 (4)
fmb	29·2 (7)	29·4 (17)	29·6 (21)	30·4 (6)	30·6 (4)

TABLE V. *Coefficients of Racial Likeness between Various Divisions of the Hodgson Collection*.*

		Nepalese Western (6·4)	Nepalese Central (16·5)	Nepalese Eastern (21·5)	Lowlanders (6·1)	Kocch, Bodo and Dhimal (5·4)
Nepalese Western (6·4)	31 characters ...	—	1·58 ± ·09	1·25 ± ·09	1·89 ± ·09	1·71 ± ·09
	12 indices and angles	—	2·05 ± ·14	1·38 ± ·14	2·02 ± ·14	3·75 ± ·14
Nepalese Central (16·5)	31 characters ...	1·58 ± ·09	—	−0·45 ± ·09	−0·23 ± ·09	0·19 ± ·09
	12 indices and angles	2·05 ± ·14	—	−0·55 ± ·14	−0·41 ± ·14	0·65 ± ·14
Nepalese Eastern (21·5)	31 characters ...	1·25 ± ·09	−0·45 ± ·09	—	0·15 ± ·09	0·74 ± ·09
	12 indices and angles	1·38 ± ·14	−0·55 ± ·14	—	0·16 ± ·14	1·22 ± ·14
Lowlanders (6·1)	31 characters ...	1·89 ± ·09	−0·23 ± ·09	0·15 ± ·09	—	−0·12 ± ·09
	12 indices and angles	2·02 ± ·14	−0·41 ± ·14	0·16 ± ·14	—	−0·39 ± ·14
Kocch, Bodo and Dhimal (5·4)	31 characters ...	1·71 ± ·09	0·19 ± ·09	0·74 ± ·09	−0·12 ± ·09	—
	12 indices and angles	3·75 ± ·14	0·65 ± ·14	1·22 ± ·14	−0·39 ± ·14	—

* The numbers in brackets immediately after the races in this table and throughout the paper indicate the mean number of skulls available for the characters used in computing the Coefficients of Racial Likeness.

we have with the very similar Nepalese. Before doing this it will be well to enquire for what characters the differences between the types of Table IV are most significant and in what ways the Nepalese Western differs from all the other groups. We are able to compare individual characters readily by considering the values of the function $\frac{n_s n_s'}{n_s + n_s'}\left(\frac{M_s - M_s'}{\sigma_s}\right)^2$, which has been denoted by the letter α and is determined when computing Coefficients of Racial Likeness. If, when comparing two types, the value of α for any particular character is between 0 and 2·7 then there is no significant difference between the types for that character; if between 2·7 and 6·1 the difference is probably significant and when α is greater than 6·1 the two characters are extremely unlikely to represent samples drawn from the same population. The following table gives all the values of α greater than four which have been found between the groups of Table IV, and we are able to pick out at once from it all the single measurements which differentiate the various types.

TABLE VI.

Values of $\frac{n_s n_s'}{n_s + n_s'}\left(\frac{M_s - M_s'}{\sigma_s}\right)^2$ *between various Groups of Nepalese Crania.*

	B/L'	*B/H*	*F*	*B*	*B'*	*J*	*NH, (R)*
Nepalese Western and Nepalese Central ...	15·23	11·64	0·00	13·39	0·28	8·67	4·16
" " Nepalese Eastern ...	7·57	8·78	0·41	7·90	1·04	7·07	1·99
" " Lowlanders	14·21	6·50	0·62	15·40	0·79	9·51	2·97
" " Kocch, Bodo and Dhimal	16·49	10·98	0·65	8·00	0·00	7·17	1·90
Nepalese Central and Nepalese Eastern ...	2·97	0·51	0·64	1·65	4·92	0·35	0·91
" " Lowlanders	0·34	0·17	0·80	0·73	2·72	1·02	0·00
" " Kocch, Bodo and Dhimal	2·00	0·93	0·89	0·04	0·20	0·30	0·02
Nepalese Eastern and Lowlanders	3·56	0·01	0·11	4·08	0·01	1·98	0·50
" " Kocch, Bodo and Dhimal	6·11	1·98	2·07	0·92	0·74	0·86	0·19
Lowlanders and Kocch, Bodo and Dhimal ...	0·70	1·32	2·18	0·32	0·62	0·14	0·01

	O_1, (R)	*C*	*Q'*	G_2	*P* ∠	*N* ∠	
Nepalese Western and Nepalese Central ...	4·00	2·05	1·16	0·03	6·10	1·15	
" " Nepalese Eastern ...	6·16	1·39	2·14	0·12	7·17	1·38	
" " Lowlanders	1·52	6·70	3·04	1·83	8·13	2·39	
" " Kocch, Bodo and Dhimal	0·74	0·82	0·74	2·92	17·86	4·56	
Nepalese Central and Nepalese Eastern ...	0·31	0·16	0·05	0·46	0·04	0·02	
" " Lowlanders	0·28	2·46	1·07	2·00	1·34	0·75	
" " Kocch, Bodo and Dhimal	0·42	0·01	3·15	3·04	8·37	2·51	
Nepalese Eastern and Lowlanders	0·93	3·91	0·45	4·06	1·16	0·69	
" " Kocch, Bodo and Dhimal	0·98	0·01	4·33	4·67	8·17	2·45	
Lowlanders and Kocch, Bodo and Dhimal ...	0·04	1·69	5·22	0·44	2·36	0·45	

The most striking feature of this table is the absence from it of the majority of facial measurements. The nasal and orbital indices in particular appear to be quite incapable of providing a criterion which will distinguish between the various closely related groups represented. The nasal height of the Western Nepalese is just significantly greater than those of the other types, but the orbital length is rather less characteristic. The angles of the fundamental triangle and palatine measurements are no surer guides. Evidently the characters which most markedly differentiate the Western Nepalese from their neighbours to the South and East are the profile angle ($P\angle$), the zygomatic breadth (J), the maximum breadth (B) and the indices B/L' and B/H. The values of α for these five characters alone are the cause of all the higher Coefficients of Racial Likeness of Table V. The very marked differences in the case of the two indices are entirely due to the breadth; the heights and lengths of all the types are almost identical. The cephalic indices and the characters B/H, B and J arrange the races in very suggestive and almost identical orders and the profile angles do the same. Thus:

	B/L'	B*	$P\angle$
Nepalese Western ...	78·7	140·3	89·9
„ Eastern ...	75·3	134·1	85·6
„ Central ...	73·8	132·1	85·8
Lowlanders	73·1	129·9	83·7
Kocch, Bodo and Dhimal	71·7	131·6	79·9

It cannot be said that this comparison provides any evidence in favour of the use of the cephalic index as a measure of racial affinity, for the large difference between the extremes shown above would have suggested that two distinct racial types were being dealt with, which is by no means the case. The skulls of the natives of Western Nepal differ from those of the inhabitants of other parts of the country in having greater maximum and zygomatic breadths and a more vertical and probably flatter face. It should be noted that the difference in breadth is associated with very similar forehead breadths (B') and transverse arcs (Q'). Now it is not conceivable that admixture with Hindus from Rajputana could have produced differences such as these. Comparison with the Tibetans is more possible. The Coefficient of Racial Likeness between the Western Nepalese and the Tibetans A (30 skulls) is $+0{\cdot}30 \pm 0{\cdot}09$ for 31 characters and $+0{\cdot}11 \pm {\cdot}14$ for 12 indices and angles. Hence it is highly probable that the native tribes originally inhabiting the West of Nepal were of a pure Tibetan stock: the skulls representing those tribes in the Hodgson Collection show no signs of Hindu admixture. If they are pooled with the Tibetan A, the standard deviation of the cephalic index (B/L') for 35 skulls is $4{\cdot}04 \pm {\cdot}48$, a low value when compared with standard deviations which have been found for other Asiatic races†. For the combined groups of Table IV, with the exception of the Western Nepalese, the value of the standard deviation based

* With these values may be compared the breadths of 29 Tibetans of the A type, $B=138{\cdot}4$; of 38 Dravidians, $B=131{\cdot}4$, and 69 Hindus (Bengali), $B=132{\cdot}3$.

† Cf. several values given in *Biometrika*, Vol. XIV. p. 205.

on 47 skulls is 4·62 ± ·48. The means of this general Nepalese type are provided in Table VII together with those of the Western Nepalese and the 30 Tibetan skulls of Type A*. The Coefficients between the first and second rows are 1·89 ± ·09 for 31 characters and 2·47 ± ·14 for 12 indices and angles; the characters which differ most significantly are in order of magnitude B/L' ($\alpha = 15{\cdot}11$), B, B/H, J and $P \angle$ ($\alpha = 9{\cdot}42$). Between rows 1 and 3 the Coefficients are 4·37 ± ·09 and 7·42 ± ·14; the most divergent characters in this case being B/L' ($\alpha = 46{\cdot}50$), B/H, B, and J ($\alpha = 7{\cdot}53$), but not the profile angle.

TABLE VII.

Mean Characters of Tibetan and Nepalese Crania.

	100 B/L'	100 H/L'	100 B/H	*Oc. I.*	100 $G'H/GB$	100 NB/NH
Nepalese ...	74·2 (47)	74·6 (47)	99·0 (47)	60·4 (47)	69·5 (42)	51·7 (48)
Nepalese Western	78·7 (6)	74·5 (6)	105·6 (6)	61·8 (7)	70·3 (5)	51·5 (6)
Tibetan A ...	78·5 (29)	74·6 (29)	105·2 (29)	61·8 (30)	69·6 (28)	49·9 (29)
	100 O_2/O_1	100 *fmb/fml*	100 G_2/G_1	$P \angle$		
Nepalese ...	79·3 (48)	82·4 (46)	85·2 (40)	85·2 (43)		
Nepalese Western	78·8 (7)	80·4 (7)	86·5 (7)	89·9 (5)		
Tibetan A ...	82·1 (30)	83·6 (30)	84·7 (27)	86·1 (27)		
	$N \angle$	$A \angle$	C	F	B	
Nepalese ...	66·5 (42)	72·8 (42)	1436·2 (47)	176·1 (47)	132·6 (47)	
Nepalese Western	64·5 (6)	73·7 (6)	1512·1 (7)	177·0 (7)	140·3 (6)	
Tibetan A ...	65·7 (29)	71·9 (29)	1438·0 (29)	174·5 (30)	138·4 (29)	
	B'	OH	LB	Q'	S	
Nepalese ...	90·9 (48)	115·1 (48)	98·0 (47)	300·7 (46)	364·9 (46)	
Nepalese Western	91·7 (6)	116·3 (6)	99·4 (7)	306·1 (6)	366·0 (6)	
Tibetan A ...	92·3 (30)	114·0 (29)	96·3 (30)	306·6 (29)	360·1 (30)	
	U	$G'H$	J	NH, (R)	NB	O_1, (R)
Nepalese ...	497·4 (47)	67·9 (43)	127·2 (44)	49·7 (48)	25·7 (48)	41·8 (48)
Nepalese Western	507·8 (6)	70·7 (6)	133·6 (6)	52·1 (6)	26·6 (7)	43·4 (7)
Tibetan A ...	501·1 (29)	68·4 (28)	130·2 (29)	51·3 (29)	25·4 (30)	41·7 (30)
	O_2, (R)	G_1	G_2	*fml*	*fmb*	
Nepalese ...	33·1 (48)	48·6 (42)	41·3 (45)	36·0 (47)	29·6 (46)	
Nepalese Western	33·6 (7)	49·2 (6)	41·5 (7)	36·8 (7)	29·2 (7)	
Tibetan A ...	34·1 (30)	48·3 (29)	40·8 (28)	35·9 (30)	30·0 (30)	

* In Table VIII below the means are given of the combined Nepalese Western and Tibetan A series shown in Table VII, and the means based on 37 skulls are always referred to hereafter when dealing with the Tibetan A type.

6. A COMPARISON OF THE SKULL MEASUREMENTS OF CERTAIN ORIENTAL RACES.

It has been shown that the majority of the crania of the Hodgson Collection belong to one or other of two types which are so far unlike each other that they may be considered racially distinct. The first of these groups includes skulls of seven male adult Tibetans proper (Trans-Himalayan Bhotias), six Nepalese Tibetans (Cis-Himalayan Bhotias) and seven skulls of natives of the Gurung, Magar and Sunwar tribes of West Nepal. These twenty skulls are of the Tibetan A type to which all the natives of Tibet appear to conform with the exception of the dolichocephalic, massive and large headed people of the North-East—the Tibetans of the B type. The measurements of the seventeen male Tibetan A skulls at the Royal College of Surgeons may be pooled with those of the British Museum skulls and we may suppose that the resulting population is at least as homogeneous as the neighbouring racial groups with which it may be compared. The mean measurements of the thirty-seven Tibetan A skulls and of the other assemblages mentioned below will be found in Table VIII*. There are three ♀ skulls of the Tibetan A type and two young ♂'s. The means of these are not provided but the individual measurements will be found, together with those of all the other skulls of the Hodgson Collection, in the Appendix to this paper. The Nepalese group includes all the representatives of the native tribes of the country—except the Magars, Gurungs and Sunwars—together with the Lepchas of Sikkim, the Lhopas of Bhutan and the Kocch, Bodo and Dhimal tribes of the Sikkimese and Bhutanese Terai. The most Northerly hillmen and the inhabitants of the low malarious swamps abutting on the plains of Bengal appear, from the similarity of their crania, to belong to the same race. Of this Nepalese type we have forty-eight male adult skulls, six female adult, one young male and one young female. The remaining skulls are those of two male Hindus of Bengal, and two male and one female Uraon of Chota Nagpur which are said to be Dravidian.

In choosing the geographical boundaries of a country as conterminous with ethnic divisions it is quite possible that we are grouping together several closely allied but yet distinct sub-races. The foregoing comparison of the main groups into which the population of Nepal naturally falls has shown at least that there are no tribes in the country which differ very markedly from the others except those of the West which appear to be of fairly pure Tibetan stock. If we were provided with nine hundred instead of ninety Nepalese crania we should be justified in examining in detail the relationship of the various tribes, but at present the means of all the skulls of the Nepalese type are the only statistical constants of any permanent value, and we may anticipate that that population is at least as homogeneous as the other Asiatic ones with which we are able to compare it.

The craniometric material relating to the races of Indo-China and the Orient, and which has also been measured by methods sufficiently like those described above to make comparison possible, is now quite considerable. But there are many lacunae and the data from the densely populated regions of Eastern and Central

* The mean measurements of the ♀ skulls are in Table IX.

TABLE VIII.

First Comparative Table of Male Means.*

Character	Maravar	Hindu	Nepalese	Tibetan A	Tibetan B	Chinese	Malayan	Burmese C	Burmese B	Burmese A
Capacity	[1289·7 (17)]	[1319·9 (34)]	1436·2 (47)	1452·4 (36)	1537·7 (14)	[1467·6 (46)]	[1424·4 (76)]	1442·2 (5)	1415·0 (4)	1406·9 (27)
Weight in grs.	—	—	—	525·4 (17)	673·1 (15)	—	—	578·0 (6)	595·6 (7)	656·8 (38)
F	175·6 (17)	175·9 (45)	176·1 (47)	174·9 (37)	184·5 (14)	179·6 (16)	—	175·5 (8)	172·4 (8)	172·4 (44)
L'	—	173·2 (9)	179·1 (47)	176·8 (35)	185·7 (14)	179·2 (4)	—	177·2 (8)	176·7 (7)	174·2 (44)
L	175·6 (21)	175·4 (33)	176·9 (47)	175·7 (37)	185·5 (14)	177·1 (84)	174·7 (78)	176·7 (8)	173·8 (8)	173·5 (44)
B	131·4 (38)	132·3 (69)	132·6 (47)	138·7 (35)	139·4 (14)	139·5 (102)	142·2 (77)	140·4 (8)	141·1 (7)	143·7 (45)
B'	93·2 (21)	92·4 (10)	90·9 (48)	92·2 (36)	94·3 (15)	93·9 (49)	93·4 (77)	90·4 (8)	89·7 (8)	94·3 (44)
H	132·5 (38)	132·1 (43)	134·2 (47)	131·9 (35)	134·8 (15)	136·6 (37)	136·2 (76)	140·1 (8)	136·6 (7)	136·8 (43)
H'	—	131·5 (10)	132·8 (47)	130·9 (35)	134·1 (15)	136·9 (69)	137·4 (76)	139·1 (8)	134·7 (7)	136·0 (43)
OH	—	111·4 (10)	115·1 (48)	114·4 (35)	115·5 (15)	119·2 (38)	118·1 (77)	116·9 (8)	116·7 (7)	117·7 (44)
LB	98·8 (38)	99·2 (44)	98·0 (47)	96·9 (37)	99·2 (15)	99·1 (66)	99·5 (76)	100·5 (7)	98·8 (8)	98·5 (43)
Q	—	—	301·4 (47)	306·3 (35)	312·1 (15)	—	—	317·8 (8)	319·2 (7)	323·7 (44)
Q'	—	302·9 (10)	300·7 (46)	306·5 (35)	312·9 (14)	321·2 (39)	319·0 (77)	319·4 (8)	319·5 (7)	325·8 (43)
S	—	363·5 (33)	364·9 (46)	361·2 (37)	378·6 (14)	370·3 (78)	365·7 (73)	367·4 (8)	367·0 (7)	363·7 (44)
S_1	127·0 (21)	127·1 (33)	124·2 (46)	125·6 (37)	129·3 (15)	126·6 (57)	—	127·9 (8)	124·9 (7)	128·6 (44)
S_2	—	126·5 (33)	126·6 (46)	123·8 (37)	126·0 (14)	126·0 (56)	—	122·5 (8)	124·4 (7)	124·0 (45)
S_3	—	110·1 (33)	113·7 (47)	111·9 (37)	122·7 (15)	116·1 (56)	—	117·0 (8)	117·7 (7)	111·2 (44)
S_3'	—	92·3 (9)	94·5 (47)	94·1 (37)	100·3 (15)	98·7 (19)	98·0 (73)	99·7 (8)	98·1 (7)	94·7 (44)
U	488·7 (38)	493·5 (69)	497·4 (47)	502·2 (35)	525·6 (14)	508·5 (82)	505·7 (77)	503·1 (8)	497·7 (7)	505·7 (44)
PH	—	—	19·5 (43)	18·2 (36)	22·0 (15)	—	—	20·1 (7)	17·2 (7)	19·9 (38)
G'H	—	63·8 (9)	67·9 (43)	68·7 (36)	76·5 (15)	71·2 (49)	70·1 (73)	74·5 (7)	68·2 (8)	71·4 (39)
GB	—	95·2 (9)	97·6 (47)	99·0 (36)	100·7 (15)	99·9 (38)	99·3 (76)	94·3 (7)	100·9 (8)	101·9 (40)
J	124·4 (21)	126·8 (32)	127·2 (44)	131·0 (35)	137·5 (15)	131·8 (65)	133·2 (75)	126·7 (7)	131·7 (8)	134·0 (40)
NH, (*R*)	46·5 (38)	49·0 (45)	49·7 (48)	51·3 (36)	54·9 (15)	53·1 (54)	51·9 (75)	55·1 (7)	52·3 (8)	53·4 (41)
NH, (*L*)			49·7 (48)	51·0 (37)	55·1 (15)			55·2 (7)	52·4 (8)	53·5 (41)
NB	24·0 (38)	24·2 (43)	25·7 (48)	25·7 (37)	27·1 (15)	25·5 (60)	26·1 (73)	25·5 (7)	26·6 (8)	28·1 (41)
DS	—	—	10·1 (47)	9·3 (35)	8·8 (15)	—	—	11·6 (7)	10·6 (8)	10·3 (42)
DC	—	—	21·2 (48)	22·4 (36)	22·7 (15)	—	—	18·7 (7)	21·3 (8)	22·8 (43)
DA	—	—	31·1 (47)	30·2 (35)	29·4 (15)	—	—	30·7 (7)	30·9 (8)	32·6 (41)
SS	—	—	3·0 (47)	2·5 (35)	2·6 (15)	—	—	3·6 (7)	3·5 (8)	3·0 (41)
SC	—	—	8·1 (48)	8·0 (35)	7·7 (15)	—	—	7·2 (7)	7·8 (8)	9·0 (42)
O_1, (*R*)	—	42·6 (10)	41·8 (48)	42·0 (37)	44·0 (15)	—	—	44·4 (7)	44·3 (8)	44·2 (41)
O_1, (*L*)	—		41·1 (48)	41·5 (37)	43·4 (15)	—	—	44·1 (7)	43·2 (8)	43·6 (41)
O_2, (*R*)	31·7 (38)	32·4 (45)	33·1 (48)	34·0 (37)	36·5 (15)	33·8 (54)	33·5 (74)	35·6 (7)	33·6 (8)	35·0 (41)
O_2, (*L*)			33·0 (48)	34·3 (37)	36·7 (15)			35·9 (7)	34·3 (8)	35·0 (41)

* The capacities in square brackets were determined by methods which might give results only approximately the same as those given by the method described above. The angles and indices in curled brackets were determined from the mean measurements of the component lengths instead of from the measurements of the individual skulls.

TABLE VIII—(*continued*).

Character	Maravar	Hindu	Nepalese	Tibetan A	Tibetan B	Chinese	Malayan	Burmese C	Burmese B	Burmese A
O_1'	36·6 (17)	37·6 (46)	39·7 (48)	39·6 (37)	41·2 (15)	38·3 (54)	39·1 (73)	39·4 (7)	39·1 (8)	39·6 (41)
G_1	50·5 (21)	46·4 (5)	48·6 (42)	48·5 (35)	51·5 (13)	54·5 (4)	—	50·0 (7)	49·2 (8)	49·9 (37)
G_1'	—	45·2 (7)	45·4 (42)	45·1 (35)	47·7 (13)	45·8 (32)	47·3 (69)	46·2 (7)	44·8 (8)	45·4 (37)
G_2	—	35·9 (9)	41·3 (45)	41·0 (37)	43·6 (14)	39·2 (35)	41·2 (70)	40·6 (7)	40·1 (6)	39·6 (40)
GL	96·0 (36)	95·1 (39)	94·5 (42)	92·9 (36)	97·2 (15)	97·7 (58)	98·2 (76)	95·9 (7)	93·8 (8)	96·6 (39)
EB	—	—	41·2 (45)	41·2 (37)	43·7 (14)	—	—	—	—	—
EH	—	—	12·4 (45)	12·1 (36)	14·2 (14)	—	—	—	—	—
fml	33·9 (21)	35·5 (8)	36·0 (47)	36·0 (37)	37·4 (15)	36·0 (23)	35·5 (73)	37·3 (8)	35·4 (8)	36·7 (43)
fmb	28·2 (21)	27·4 (8)	29·6 (46)	29·8 (37)	31·4 (15)	30·9 (23)	30·0 (74)	30·8 (8)	30·6 (8)	30·7 (42)
$100\ B/L'$	—	76·2 (9)	74·2 (47)	78·5 (35)	75·2 (15)	77·8 (13)	—	79·4 (8)	80·0 (7)	82·4 (44)
$100\ H/L'$	—	75·5 (9)	74·6 (47)	74·6 (35)	72·3 (14)	{76·2 (4)}	—	78·7 (8)	76·3 (7)	78·2 (43)
$100\ B/L$	74·6 (21)	75·8 (33)	75·1 (47)	79·2 (35)	75·3 (14)	78·9 (73)	81·7 (77)	79·5 (8)	80·4 (7)	82·9 (44)
$100\ H/L$	75·2 (21)	75·6 (9)	76·0 (47)	75·2 (35)	72·4 (14)	77·4 (21)	78·2 (76)	78·9 (8)	76·7 (7)	78·5 (43)
$100\ B/H$	{99·1 (21)}	{100·0 (43)}	99·0 (44)	105·3 (35)	104·0 (14)	103·2 (37)	104·6 (76)	101·0 (8)	104·9 (7)	105·8 (43)
$100\ H'/L$	—	75·8 (10)	75·3 (46)	74·7 (35)	72·1 (14)	77·4 (35)	—	{78·7 (8)}	{77·5 (7)}	{78·4 (43)}
$100\ B/H'$	—	99·8 (10)	99·6 (46)	106·1 (35)	104·5 (14)	102·4 (46)	{103·5 (76)}	{100·9 (8)}	{104·8 (7)}	{105·7 (43)}
$100\ (B-H')/L$	—	{0·5 (10)}	−0·2 (46)	4·5 (35)	3·5 (14)	{1·5 (84)}	{2·7 (76)}	{0·7 (8)}	{3·7 (7)}	{4·4 (43)}
$100\ G'H/GB$	—	68·0 (8)	69·5 (42)	69·7 (33)	76·0 (15)	71·1 (36)	70·7 (73)	79·0 (7)	67·8 (8)	69·8 (38)
$100\ NB/NH,\ (R)$	51·7 (38)	49·7 (43)	51·7 (48)	50·2 (36)	49·5 (15)	48·9 (53)	50·4 (73)	46·4 (7)	51·2 (8)	52·8 (41)
$100\ NB/NH,\ (L)$			51·8 (48)	50·5 (37)	49·3 (15)			46·3 (7)	51·0 (8)	52·7 (41)
$100\ O_2/O_1,\ (R)$	—	77·2 (9)	79·3 (48)	81·5 (37)	83·0 (15)	—	—	80·3 (7)	75·9 (8)	79·1 (41)
$100\ O_2/O_1,\ (L)$	—		80·5 (48)	83·1 (37)	84·6 (15)	—	—	81·5 (7)	79·4 (8)	80·0 (41)
$100\ O_2/O_1'$	84·4 (17)	86·3 (45)	85·3 (47)	86·2 (37)	88·6 (15)	88·4 (54)	86·0 (73)	{91·1 (7)}	{87·7 (8)}	{88·4 (41)}
$100\ fmb/fml$	83·3 (21)	77·3 (8)	82·4 (46)	83·0 (37)	84·1 (15)	85·9 (23)	84·6 (73)	82·9 (8)	86·6 (8)	83·8 (42)
$100\ DS/DC$	—	—	48·3 (47)	42·3 (35)	39·1 (15)	—	—	61·9 (7)	49·8 (8)	45·4 (42)
$100\ SS/SC$	—	—	37·7 (47)	32·9 (35)	34·6 (15)	—	—	54·0 (7)	45·2 (8)	32·7 (41)
$100\ G_2/G_1$	—	77·8 (5)	85·2 (40)	85·1 (34)	84·1 (12)	78·4 (4)	—	81·3 (7)	82·2 (6)	79·8 (37)
$100\ EH/EB$	—	—	30·2 (45)	29·6 (36)	31·7 (14)	—	—	—	—	—
$P\angle$	—	86°·1 (9)	85°·2 (43)	86°·7 (34)	85°·7 (14)	84°·9 (17)	—	84°·0 (7)	85°·9 (8)	86°·0 (89)
$N\angle$	—	{67°·2 (9)}	66°·5 (42)	65°·5 (36)	65°·5 (15)	{67°·3 (49)}	{68°·2 (73)}	64°·4 (7)	65°·2 (8)	66°·8 (38)
$A\angle$	—	{74°·8 (9)}	72°·8 (42)	72°·1 (36)	68°·8 (15)	{70°·1 (49)}	{70°·4 (73)}	71°·2 (7)	73°·4 (8)	70°·5 (38)
$B\angle$	—	{38°·0 (9)}	40°·7 (42)	42°·4 (36)	45°·7 (15)	{42°·6 (49)}	{41°·4 (73)}	44°·5 (7)	41°·4 (8)	42°·6 (38)
θ_1	—	{26°·7 (9)}	27°·6 (42)	27°·4 (34)	28°·5 (14)	{27°·8 (17)}	—	31°·7 (7)	28°·8 (8)	27°·2 (38)
θ_2	—	{11°·3 (9)}	12°·6 (42)	14°·8 (34)	17°·7 (14)	{14°·8 (17)}	—	12°·8 (7)	12°·9 (8)	15°·4 (38)
Oc. I.	—	60·5 (9)	60·4 (47)	61·8 (37)	59·2 (15)	{62·1 (19)}	—	62·8 (8)	61·5 (7)	62·8 (44)

TABLE IX.

Comparative Table of Female Means.*

Character	Andamanese	Veddahs	Nepalese†	Annamese	Siamese	Northern Chinese	Koreans	Japanese	Aino
Capacity	[1130·0 (17)]	[1177·6 (26)]	1249·2 (6)	[1367·9 (7)]	[1356·3 (3)]	1371·8 (10)	1301·1 (6)	[1308·0 (30)]	1308·0 (51)
F	161·0 (17)	170·6 (15)	167·4 (6)	—	163·3 (3)	—‡	—	161·5 (2)	—
L	159·4 (5)	173·0 (18)	167·4 (6)	169·9 (7)	164·8 (4)	172·0 (10)	167·7 (9)	171·3 (32)	177·2 (62)
B	132·5 (22)	125·1 (30)	127·4 (6)	138·7 (7)	139·3 (7)	136·0 (10)	135·1 (9)	136·1 (34)	136·8 (63)
B'	89·4 (17)	90·3 (4)	89·1 (6)	92·1 (7)	90·0 (3)	87·6 (10)	90·2 (6)	91·0 (4)	92·4 (62)
H	123·0 (1)	127·6 (15)	126·2 (6)	—	138·0 (1)	133·6 (9)	137·8 (6)	133·5 (32)	135·1 (62)
H'	125·0 (19)	128·4 (13)	125·6 (6)	133·6 (7)	128·0 (3)	131·0 (10)	130·0 (3)	127·3 (4)	—
OH	—	105·4 (4)	111·6 (6)	—	—	113·2 (9)	113·8 (6)	115·0 (4)	115·0 (63)
LB	91·0 (19)	93·4 (18)	92·6 (6)	95·5 (7)	92·0 (3)	94·9 (10)	96·0 (6)	91·3 (9)	100·3 (60)
Q	277·5 (2)	290·7 (15)	290·3 (6)	—	—	308·3 (9)	312·0 (6)	309·3 (4)	317·1 (61)
S	338·2 (19)	344·3 (19)	349·0 (6)	356·6 (6)	353·7 (7)	360·4 (10)	358·2 (6)	359·2 (6)	360·7 (53)
S_1	116·6 (19)	122·8 (6)	120·3 (6)	123·8 (6)	125·0 (7)	123·1 (10)	120·2 (6)	121·8 (6)	—
S_2	117·8 (19)	117·2 (6)	120·9 (6)	123·4 (7)	121·3 (7)	122·3 (10)	124·4 (6)	120·8 (4)	—
S_3	104·2 (19)	107·7 (6)	107·8 (6)	110·4 (7)	108·9 (7)	115·0 (10)	112·3 (4)	108·0 (4)	—
S_3'	—	—	89·6 (6)	—	—	98·4 (9)	—	—	—
U	463·8 (22)	476·1 (18)	474·4 (6)	491·1 (7)	487·2 (6)	493·0 (10)	481·6 (9)	497·9 (6)	501·7 (62)
G'H	56·7 (3)	60·9 (12)	62·5 (6)	—	—	68·8 (9)	68·4 (5)	67·5 (24)	65·5 (41)
GB	—	87·5 (8)	93·0 (5)	96·0 (4)	92·0 (3)	93·4 (10)	97·0 (6)	93·8 (31)	96·7 (46)
J	118·3 (17)	115·8 (11)	118·8 (4)	124·2 (6)	127·2 (6)	126·1 (10)	126·3 (6)	125·0 (28)	129·7 (44)
NH	46·5 (1)	48·3 (3)	44·4 (6)	—	—	52·4 (9)	52·3 (6)	47·9 (31)	47·4 (46)
NH'	47·0 (3)	44·5 (8)	—	49·2 (7)	46·3 (3)	54·0 (1)	—	—	—
NB	21·9 (21)	23·3 (17)	24·9 (6)	24·8 (7)	24·0 (3)	23·6 (10)	25·0 (5)	25·3 (32)	24·7 (45)
DC	27·0 (1)	21·7 (10)	20·9 (6)	23·4 (7)	23·0 (3)	27·0 (1)	22·8 (6)	23·3 (4)	—
O_1	35·0 (1)	39·8 (8)	40·4 (5)	—	—	—	37·5 (2)	37·6 (29)	39·8 (49)

* The capacities in square brackets were determined by doubtful methods; the indices and angles in curled brackets were derived from the mean measurements of the component lengths instead of from the measurements of the individual skulls. The Burmese, Malayan, Hindu and Maravar measurements will be found in *Biometrika*, Vol. XIII. p. 240.

† Other measurements of the Nepalese series are $L'=170{\cdot}4$ (6), $Q=289{\cdot}2$ (6), $PH=18{\cdot}1$ (6), NH, $L=45{\cdot}6$ (6), $DS=10{\cdot}0$ (6), $DA=31{\cdot}0$ (6), $SS=3{\cdot}0$ (6), $O_1L=39{\cdot}5$ (6), $O_2L=31{\cdot}4$ (6), $EB=37{\cdot}2$ (5), $EH=10{\cdot}6$ (4), 100 $H/L'=74{\cdot}1$ (6), 100 NB/NH, $L=55{\cdot}1$ (6), 100 O_2/O_1, $L=79{\cdot}8$ (6), 100 $DS/DC=48{\cdot}0$ (6), 100 $SS/SC=34{\cdot}4$ (6), 100 $EH/EB=28{\cdot}9$ (4). The nasal and orbital measurements of the Nepalese given above refer to the right side.

‡ The Northern Chinese L' is $175{\cdot}0$ (9).

TABLE IX—(*continued*).

Character	Andamanese	Veddahs	Nepalese	Annamese	Siamese	Northern Chinese	Koreans	Japanese	Aino
O_2	32·4 (22)	32·7 (18)	31·9 (5)	32·5 (7)	33·0 (3)	33·8 (10)	37·0 (2)	33·3 (30)	33·9 (50)
O_1'	37·3 (3)	36·7 (10)	38·3 (5)	—	—	34·8 (9)	—	—	—
G_1	—	47·0 (5)	47·1 (6)	—	—	—	48·4 (5)	48·8 (4)	51·4 (38)
G_1'	—	—	43·4 (6)	—	—	—	—	—	—
G_2	—	—	37·2 (5)	—	—	—	39·3 (6)	37·5 (2)	37·4 (46)
GL	91·8 (17)	88·4 (17)	89·6 (6)	—	—	—	93·0 (5)	94·0 (1)	100·1 (43)
fmb	27·3 (15)	25·0 (1)	27·6 (5)	29·0 (7)	29·0 (3)	27·9 (10)	28·1 (9)	26·5 (2)	28·9 (56)
fml	32·1 (17)	35·0 (5)	34·4 (6)	33·6 (7)	34·0 (3)	33·3 (10)	34·2 (9)	36·5 (2)	33·7 (56)
SC	—	8·9 (7)	8·8 (6)	—	—	6·5 (8)	7·3 (6)	7·5 (2)	—
100 B/L'	—	—	74·8 (6)	—	—	{77·9 (9)}	—	—	—
100 B/L	{83·3 (5)}	{72·3 (18)}	76·2 (6)	81·6 (7)	84·5 (4)	{79·1 (10)}	{80·6 (9)}	{79·5 (32)}	77·6 (62)
100 B/F	82·5 (17)	73·5 (15)	{76·1 (6)}	—	84·5 (3)	—	—	78·9 (2)	—
100 H/L	79·9 (1)	{73·8 (15)}	75·4 (6)	—	85·7 (1)	{77·7 (9)}	{82·2 (6)}	{77·9 (32)}	76·7 (61)
100 H'/L	{78·4 (5)}	{74·2 (13)}	75·1 (6)	{78·6 (7)}	77·1 (3)	{76·2 (10)}	{77·5 (3)}	{74·3 (4)}	—
100 H'/F	77·9 (16)	{75·3 (13)}	{75·0 (6)}	—	78·4 (3)	—	—	75·6 (1)	—
100 B/H	106·5 (1)	{98·0 (15)}	101·1 (6)	—	106·5 (1)	{101·8 (9)}	{98·0 (6)}	{101·9 (32)}	{101·3 (62)}
100 B/H'	{106·0 (19)}	{97·4 (13)}	101·5 (6)	{103·8 (7)}	{108·8 (3)}	{103·8 (10)}	{103·9 (3)}	{106·0 (4)}	—
100 $(B-H')/L$	{4·7 (5)}	{−1·9 (13)}	1·1 (6)	{3·0 (7)}	{6·9 (3)}	{2·9 (10)}	{3·0 (3)}	{5·1 (4)}	—
100 $G'H/GB$	—	{69·6 (8)}	68·5 (5)	—	—	72·4 (9)	69·7 (5)	{72·0 (24)}	67·7 (60)
100 NB/NH	46·2 (1)	{48·7 (3)}	56·4 (6)	—	—	45·3 (9)	48·3 (6)	{52·8 (31)}	50·5 (44)
100 NB/NH'	47·8 (3)	52·8 (8)	—	{50·4 (7)}	52·2 3)	46·3 (1)	—	—	—
100 O_2/O_1	97·1 (1)	83·0 (8)	76·9 (5)	—	—	—	98·7 (2)	88·6 (29)	85·2 (48)
100 O_2/O_1'	88·7 (3)	89·4 (10)	83·2 (5)	—	—	95·5 (9)	—	—	—
100 *fmb*/*fml*	85·3 (15)	73·5 (1)	81·0 (5)	{86·3 (7)}	{85·3 (3)}	{83·8 (10)}	{82·2 (9)}	{76·3 (4)}	{85·8 (56)}
100 G_2/G_1	—	77·8 (5)	78·2 (5)	—	—	—	81·1 (5)	77·4 (2)	72·8 (38)
100 G_2/G_1'	—	—	85·0 (5)	—	—	—	—	—	—
$P\angle$	—	—	86°·3 (6)	—	—	79°·7 (9)	83°·8 (5)	83°·5 (23)	81°·0 (41)
$N\angle$	{72°·4 (3)}	{66°·0 (11)}	67°·0 (6)	—	—	—	{66°·2 (5)}	—	{70°·5 (41)}
$A\angle$	{71°·7 (3)}	{75°·3 (11)}	73°·0 (6)	—	—	—	{71°·3 (5)}	—	{71°·5 (41)}
$B\angle$	{35°·9 (3)}	{38°·7 (11)}	40°·0 (6)	—	—	—	{42°·5 (5)}	—	{38°·0 (41)}
θ_1	—	—	26°·6 (6)	—	—	—	{30°·0 (5)}	—	{28°·5 (41)}
θ_2	—	—	13°·5 (6)	—	—	—	{12°·5 (5)}	—	{9°·5 (41)}
Oc. I.	—	—	59·9 (6)	—	—	{62·8 (9)}	—	—	—

India are particularly meagre. The mean skull measurements of the following races have been compiled by Tildesley from various scattered sources*.

Malays. These means are based on seventy-eight skulls in the collection of Dr Emil Schmidt which have been measured by him, and the females are supplemented by thirty-eight skulls from the other German Catalogues and eighteen from Barnard Davis' Catalogue. The male skulls at least were said to be free from Indonesian admixture.

Chinese. Measurements were collected from the German Anthropological Catalogues and Flower's and Barnard Davis' Catalogues. The regions of China from which some of the skulls had been obtained were not known, but the means may be taken to represent most nearly the Southern Chinese type.

Hindus. Only a section of this racially heterogeneous people is represented. The material considered, compiled from Flower's, Barnard Davis' and the German Catalogues, to which was added a short series (9 ♂ and 6 ♀) of Hindu skulls in the Biometric Laboratory, relates to North-East India and Bengal in particular. Care was taken to exclude all skulls belonging to tribes of Dravidian origin or admixture.

Dravidians. The Dravidian type is represented by the means of forty skulls of the Maravar tribe (Madras). These originally formed a single collection but half were sent to Paris and measured by Callamand† and the remainder came to the Royal College of Surgeons and will be found in Flower's Catalogue.

The reliability of the mean measurements of a given race which are reduced from the figures provided by several craniologists using different methods, perhaps, and dealing with collections not made at the same time and place, should always be questioned. In the case of the four races considered above, the male data for Malays and Dravidians are not open to such criticisms since only one collection is dealt with in each case, though, indeed, the Dravidians were measured by two different workers; the measurements of the Chinese and Hindus are not so satisfactory.

Several series of Asiatic skulls have been measured in the Biometric Laboratory and the measurements provided for these are precisely the same as those described in an earlier section of this paper. The races are:

Burmese. Three Burmese series—one hundred and forty-two skulls in all—were measured by Miss Tildesley and compared by her with the above racial material which she had collected‡. The A series was of Burmans proper, the B of supposed hybrids and the skulls of the C series were thought to be of Karen origin.

Tibetan B. Measurements of fifteen skulls of this race in the Royal College of Surgeons are available§. They indicate a type which is quite distinct from the

* See *Biometrika*, Vol. XIII. pp. 235–238, for a more detailed account of this material.

† *Revue d'Anthropologie*, Vol. I. 4th Series, pp. 607–625, 1878. It is unfortunate that Callamand gives nothing but the mean measurements of the twenty skulls, so we are not able to calculate the standard deviations of the original forty.

‡ M. L. Tildesley, "A First Study of the Burmese Skull," *Biometrika*, Vol. XIII. pp. 176–262.

§ See *Biometrika*, Vol. XIV. p. 206.

Tibetan A. These so-called Khams Tibetans are only found in the North-East Provinces of Tibet.

The majority of the races of Southern and Central Asia of which mention has been made are said to be possessed of certain common cranial characters which indicate that they are members of the same stock or family of races. The most salient of these so-called Mongolian characters are brachycephaly, characteristic flattening of the facial and projection of the zygomatic bones, absence of both superciliary ridges and prognathism and more or less flattening of the nasal bones without depression at the root. The living head is said to be also characterised by the "Mongolian eye," absence of hairs on the face and a characteristic skin colour. On the basis of these tegumentary and skeletal criteria, the Burmese, Malays, Chinese and Tibetans are classed as pure representatives of the "Mongolian" type, and Hodgson was able to discern the same characteristic facial features in the native Nepalese, though associated with characters obviously derived from another stock and, in particular, a lower cephalic index. Anthropologists do not recognise any Mongoloid elements in the narrow heads and pointed faces of the Hindus of Bengal. The Dravidians to the South, too, are commonly supposed to represent a radically different stock. The Ainos of Japan, though possessing remote Mongolian affinities perhaps, are considered to be a distinct type which has no representative people on the mainland. And the Khams Tibetans, though conterminous with Mongoloid peoples to the South, East and West, are also of a unique type which resembles those of the Maoris, Morioris and even Fuegians more than those of the neighbouring races.

We may now compare the mean measurements provided for these racial series with the aid of Coefficients of Racial Likeness. The need has been emphasised above of taking into account the average number of skulls on which the mean racial characters are based when using that criterion of relationship. That number will be found in round brackets immediately after the designation of each race*.

TABLE X.

Coefficients of Racial Likeness (Males).

		Nepalese (45·8)	Tibetan A (35·9)	Chinese (46·2)	Burmese A (41·0)
Nepalese (45·8)	All Characters ...	—	5·07 ± ·09	13·39 ± ·09	27·25 ± ·09
	Indices and Angles ...	—	8·40 ± ·14	7·70 ± ·14	27·24 ± ·14
Tibetan A (35·9)	All Characters ...	5·07 ± ·09	—	5·50 ± ·09	10·09 ± ·09
	Indices and Angles ...	8·40 ± ·14	—	2·66 ± ·14	7·73 ± ·14
Chinese (46·2)	All Characters ...	13·39 ± ·09	5·50 ± ·09	—	6·24 ± ·09
	Indices and Angles ...	7·70 ± ·14	2·66 ± ·14	—	4·77 ± ·14
Burmese A (41·0)	All Characters ...	27·25 ± ·09	10·09 ± ·09	6·24 ± ·09	—
	Indices and Angles ...	27·24 ± ·14	7·73 ± ·14	4·77 ± ·14	—

* The actual numbers on which the means of the characters are based are provided in Table VIII.

The means of the four races noticed in the above table are based on approximately the same number of male crania, so the Coefficients of Racial Likeness between them may be compared directly.

The races are arranged in this table in the order in which the Coefficients of Racial Likeness for all characters place them; each race has the lowest Coefficients with its immediate neighbour or neighbours. The Tibetans are most similar to the Nepalese, but the Chinese are almost as close. Between the Chinese and Burmese there appears to be an hiatus which one or more races not represented in the table may be expected to fill. It may be noted that the Coefficients based on angles and indices only would place the Chinese between the Nepalese and the Tibetans—an order to which no significance could be given. That the Nepalese should be so similar in head form to the Tibetans is distinct evidence in favour of the view that they are primarily of so-called "Mongolian" origin and must have arrived at the border of India from the North. It is known that there has been close contact between the Chinese and Tibetans, and no bar to fusion of race, since early historic times at least, and it is highly probable that the intercourse between Tibetans and Nepalese has been of the same nature.

Turning now to a comparison of the Malayan means, based on seventy-seven skulls, with those of the four races already considered, we arrive at the following Coefficients. These may be compared directly with each other but not with the Coefficients of Table X.

TABLE XI.

Coefficients of Racial Likeness between the Malayan and other Asiatic Races (Males).

		Nepalese (45·8)	Tibetan A (35·9)	Chinese (46·2)	Burmese A (41·0)
Malayan (74·6)	All Characters ...	22·99 ± ·09	6·42 ± ·09	4·00 ± ·09	4·05 ± ·09
	Indices and Angles ...	30·89 ± ·16	7·15 ± ·16	6·13 ± ·16	2·43 ± ·16

The most noteworthy feature of this table is the low value of the Coefficients with the Chinese and Burmese A. Now the Malayan means are based on more crania than the other means, so a Coefficient with them denotes, in all probability, a closer degree of resemblance than the same Coefficient would in Table X. In other words, if the Malayan means remained unchanged in numerical value but were supposed to be based on, say, forty crania, then all the Coefficients of Table XI would be lowered. So the Malays are evidently more closely related to the Chinese and Burmese than the two latter are to each other, that is to say they take up a position intermediate between the Chinese and the Burmese. It is rather surprising, in view of the geographical positions of these peoples, to find that the Southern Chinese are more closely allied to the Malays than to the Burmese. The Burmese crania were obtained in the neighbourhood of Moulmein—400 miles from the nearest point of Southern China—but the natives of the Archipelago are all

well over 1000 miles from the Chinese boundary*. That the Malays and Southern Chinese have skulls of a very similar type is a fact on which considerable stress may be laid. The Coefficient of Racial Likeness between them—remembering that the Malayan means are based on more skulls than the means of the other races—indicates a decidedly closer bond than those between the Tibetans and either the Nepalese on the one hand or the Chinese on the other.

If the Malayan type is allowed to come between the Chinese and the Burmese A then the other races in Table XI will become arranged in the same order as in Table X.

The Hindu and Maravar data may now be compared with those of the Oriental peoples. The two series contain approximately equal numbers of crania, so the Coefficients between them and any other race may be compared directly.

TABLE XII.

Coefficients of Racial Likeness between Indian and Oriental Races (Males).

		Nepalese (45·8)	Tibetan A (35·9)	Chinese (46·2)	Malayan (74·6)	Burmese A (41·0)
Hindu (Bengalee) (25·9)	All Characters ...	4·32 ± ·09	7·97 ± ·09	14·93 ± ·09	22·68 ± ·09	25·45 ± ·09
	Indices and Angles	1·09 ± ·14	5·67 ± ·14	6·00 ± ·14	19·98 ± ·16	18·47 ± ·14
Dravidian (Maravar) (26·7)	All Characters ...	6·52 ± ·10	15·48 ± ·10	23·47 ± ·10	30·70 ± ·11	38·29 ± ·10
	Indices and Angles	−0·60 ± ·19	11·35 ± ·19	13·47 ± ·19	31·73 ± ·19	34·14 ± ·19

Again the Oriental races are arranged in identically the same order as in Tables X and XI, and the order is the same for both Hindu and Maravar series of Coefficients. When collating the Hindu (Bengalee) material Miss Tildesley included no skull which was thought to be of Dravidian origin or admixture and yet the Coefficients between the Hindus and Maravars are as low as 1·80 ± ·10 for twenty-one characters and 2·03 ± ·19 for six indices and angles. It is unfortunate that the Maravar means are available for so few measurements, but at any rate there are sufficient to show that there must be a close affinity between the natives of Bengal and the supposed aborigines of Central India. There has evidently been little if any fusion between the Bengalee and the Burmese in spite of their geographical proximity. If the Assamese are a hybrid race resulting from the crossing of these two aberrant types, then a study of their cranial measurements would be of very considerable interest; at present these measurements are conspicuously lacking. To the North, we find in the hillmen of Nepal a race which clearly bears a close relation to the Hindus. Remembering that the latter are represented by fewer skulls than are the races to the North and East, we may conclude that the Hindus are as closely allied to the Nepalese as the Nepalese are to the Tibetans and the Tibetans to the Chinese.

* Forty of the male Malay skulls were obtained in Java, six in Sumatra, fifteen in Borneo, and seventeen in the Celebes.

Turning to the Coefficients with the Maravars, each is seen to be greater than the corresponding one with the Hindus, but the order is the same for the two races.

It will be convenient to summarise at this point the main conclusions which have resulted from a comparison of the Coefficients of Racial Likeness between several races of the Orient. It appears that these races can be arranged in a sequence, with the Dravidians at one end and the Burmans at the other, such that every member of the sequence is most closely allied to the race or races adjacent to it and the measures of the affinity between any one race and all the others arrange them in the same sequence. The relationships of any two races of this series must clearly be traced through the intervening races. There are, no doubt, many missing links in the chain, but by taking broad racial classifications it has been possible to show the continuity of the whole. Geographically (see Map, p. 6) the sequence traces out roughly the path of a spiral starting in Madras and passing through Bengal, Nepal, Tibet, Southern China, the Malay Archipelago and terminating in Burma. The extremes of the series are as unlike as any European and any Asiatic races are likely to be. It will be well to glance at the Coefficients of Racial Likeness between the two short Burmese series (males) which have not yet been dealt with and the male data for all the other races, though the material is very meagre.

TABLE XIII.

Coefficients of Racial Likeness between Burmese and other Races (Males).

	Burmese B (Hybrids?) (7·5)		Burmese C (Karens?) (7·5)	
	All Characters	Indices and Angles	All Characters	Indices and Angles
Maravar (26·7)	6·80 ± ·10	5·45 ± ·19	9·84 ± ·10	7·74 ± ·19
Hindu (25·9)	4·75 ± ·09	2·51 ± ·14	6·51 ± ·09	4·25 ± ·14
Nepalese (45·8)	3·27 ± ·09	3·43 ± ·14	5·58 ± ·09	6·41 ± ·14
Tibetan A (35·9)	+0·82 ± ·09	+0·49 ± ·14	3·08 ± ·09	3·74 ± ·14
Chinese (46·2)	+0·92 ± ·09	+0·59 ± ·14	1·73 ± ·09	2·18 ± ·14
Burmese C (7·5)	1·14 ± ·09	2·24 ± ·14	—	—
Burmese B (7·5)	—	—	1·14 ± ·09	2·24 ± ·14
Malayan (74·6)	+0·28 ± ·09	1·12 ± ·16	2·98 ± ·09	4·99 ± ·16
Burmese A (41·0)	+0·78 ± ·09	+0·83 ± ·14	2·77 ± ·09	4·42 ± ·14

The Burmese series B and C contain so few crania that no reliable conclusions can be deduced from a comparison of their mean measurements with those of other races, and yet the orders in which their Coefficients arrange the Oriental races are identical in one case and almost identical in the other with that which was suggested by more trustworthy material*. The Karens and Hybrids have been

* These statements will not be literally true until allowance has been made for the differences between the numbers of crania in the racial series. We may suppose that the Burmese B shows greater affinity to the Chinese than to the Tibetan A, while the Burmese C skull is, in all probability, more like the Malayan than the Burmese A is like the Burmese C.

placed between the Chinese and the Malays in the order shown after considering also the relations of the female data, which are more ample than the male for those two races. A noteworthy feature of the table is that all the Coefficients with the Burmese C are greater than the corresponding ones with the Burmese B, although the two series of means are based on the same numbers of crania. That is a relation which we might have anticipated in view of the fact that the B series probably represents a hybrid population and possibly there has been slight admixture with Hindu elements.

The Coefficients of Racial Likeness of Table XIII will be found, if comparison is made, to be all of a lower order than those of the three previous tables, the difference being due, as has been pointed out, to the fact that the Burmese B and C means are based on far fewer crania than the means of the other series. If we had attempted to compare the Coefficients without paying due regard to that factor our conclusions would obviously have been erroneous.

We now pass to such female data as it has been possible to collect, and give, in Table XIV, the Coefficients of Racial Likeness.

TABLE XIV.

Coefficients of Racial Likeness (Females).

		Hindu (13·3)	Burmese C (15·9)	Burmese B (16·1)	Malayan (19·9)
Hindu (13·3)	All Characters ...	—	2·53 ± ·09	9·76 ± ·09	14·63 ± ·09
	Indices and Angles	—	1·52 ± ·14	13·34 ± ·14	17·33 ± ·14
Burmese C (Karens ?) (15·9)	All Characters ...	2·53 ± ·09	—	4·45 ± ·09	6·92 ± ·09
	Indices and Angles	1·52 ± ·14	—	8·43 ± ·14	9·86 ± ·14
Burmese B (Hybrids ?) (16·1)	All Characters ...	9·76 ± ·09	4·45 ± ·09	—	3·21 ± ·09
	Indices and Angles	13·34 ± ·14	8·43 ± ·14	—	3·17 ± ·14
Malayan (19·9) ...	All Characters ...	14·63 ± ·09	6·92 ± ·09	3·21 ± ·09	—
	Indices and Angles	17·33 ± ·14	9·86 ± ·14	3·17 ± ·14	—

The means of the four races compared in this table are based on approximately the same numbers of crania. With the order as shown, each race most closely resembles the race or races adjacent to it, and the Burmese B is placed between the Burmese C and the Malayan. It is rather surprising to find that the Coefficient between the Hindus and the supposed Karens is decidedly lower than that between Karens and Burmese Hybrids, but it must be remembered that we are comparing means based on small numbers of skulls. The following Table of Coefficients of Racial Likeness with the Burmans proper does not give the same order as that of Table XIII; now the Hyloid race comes between the Malayan and the Burmese A. This latter order is also given by the Coefficients with the Nepalese (Table XVI), but judged by the affinities of the Chinese the Burmese B is placed between the Malayan and Burmese C types as in Table XIV. It is probable that no unique

position can be found for the Burmese B relative to the other races because, though predominantly of Burmese stock, it is possibly a hybrid strain and the result of the crossings of several races. The other Asiatic peoples may have mixed freely with their immediate neighbours, but they cannot have been modified greatly by extraneous elements.

TABLE XV.

Coefficients of Racial Likeness between Burmese A and other Races (Females).

		Hindu (13·3)	Burmese C (15·9)	Burmese B (16·1)	Malayan (19·9)
Burmese A (35·4)	All Characters ...	18·82 ± ·09	7·93 ± ·09	1·75 ± ·09	2·76 ± ·09
	Indices and Angles ...	19·79 ± ·14	9·84 ± ·14	1·02 ± ·14	3·24 ± ·14

The data relating to Nepalese and Chinese ♀ skulls are so meagre that no results of any permanent value whatsoever can be deduced from them. In general they confirm relations which have already been observed in the case of ♂ skulls, but it would not have been surprising if no such close correspondence had been found. Only sixteen of the thirty-one characters used in calculating Coefficients of Racial Likeness are available for the Chinese and there are no angular measurements. The Coefficient between the Nepalese and Chinese is very much higher than might have been anticipated, in view of the fact that both series contain so few skulls, but no stress can be laid on this discordance.

A comparison of the female data, though it would have been insufficient in itself to have suggested any wide generalisations, has confirmed the conclusions suggested by a comparison of the racial means of male skulls. It appears that:

(i) The principal races of the Orient may be arranged in the following sequence: Dravidians (Maravar)—Hindus (Bengalee)—Nepalese—Tibetan A—Southern Chinese—Burmese C (Karens?)—Malays—Burmese A.

(ii) Each race is most closely allied to the race or races adjacent to it in the sequence, and the resemblances between all pairs of adjacent races are of approximately the same order.

(iii) The Coefficients of Racial Likeness between any one race of the series and all the other races—after allowance has been made for the disparities between the numbers of crania on which the several means are based—arrange the races in the same sequence.

It is reasonable to infer from these observed relations that there is possibly one only or more probably two fundamental types of cranium underlying the material considered and that several stages in the gradation from one type to the other are represented by the different races. It must be remembered that pairs of races which are separated by three or four others in the series will have some widely divergent characters. A comparison of individual measurements will throw more

TABLE XVI.

Coefficients of Racial Likeness between the Nepalese and Chinese and other Oriental Races (Females).

	Nepalese (5·7)		Chinese (6·2)	
	All Characters	Indices and Angles	All Characters	Indices and Angles
Hindu (13·3) ...	1·58 ± ·09	1·11 ± ·14	3·65 ± ·12	4·26 ± ·24
Nepalese (5·7) ...	—	—	4·38 ± ·12	8·22 ± ·24
Chinese (6·2) ...	4·38 ± ·12	8·22 ± ·24	—	—
Burmese C (15·9)	1·30 ± ·09	1·05 ± ·14	1·50 ± ·12	3·86 ± ·24
Burmese B (16·1)	4·78 ± ·09	7·23 ± ·14	4·31 ± ·12	9·46 ± ·24
Malayan (19·9) ...	3·18 ± ·09	4·02 ± ·14	5·82 ± ·12	15·55 ± ·24
Burmese A (35·4)	5·85 ± ·09	6·88 ± ·14	6·66 ± ·12	14·72 ± ·24

TABLE XVII.

Coefficients of Racial Likeness between Oriental and other Races (Males).*

	Tibetan B (14·0)		Aino (79·3)		Whitechapel English (87·2)		Naqada Egyptian (94·1)		Moriori (33·9)	
	All Characters	Indices and Angles	All Characters	Indices and Angles	All Characters	Indices and Angles	All Characters	Indices and Angles	All Characters	Indices and Angles
Maravar (Dravidian) (26·7)	25·72 ± ·11	3·65 ± ·21†	55·33 ± ·13	4·54 ± ·34†	42·79 ± ·12	32·59 ± ·24†	13·60 ± ·12	2·93 ± ·24†	48·75 ± ·11	21·95 ± ·21†
Hindu (Bengalee) (25·9) ...	18·34 ± ·09	4 78 ± ·16	31·58 ± ·09	4·90 ± ·15	30·90 ± ·10	14·52 ± ·16	11·43 ± ·10	4·67 ± ·16	31·84 ± ·09	9·44 ± ·14
Nepalese (45·8)	13·16 ± ·09	4·86 ± ·14	37·54 ± ·09	20·78 ± ·15	35·42 ± ·09	27·76 ± ·15	15·02 ± ·09	13·07 ± ·15	33·38 ± ·09	20·02 ± ·14
Tibetan A (35·9)	8·50 ± ·09	3·83 ± ·14	31·19 ± ·09	25·04 ± ·15	22·69 ± ·09	17·81 ± ·15	23·84 ± ·09	30·82 ± ·15	20·48 ± ·09	11·44 ± ·14
Chinese (46·2)	6·37 ± ·10	2·21 ± ·18	17·73 ± ·10	10·08 ± ·16	20·03 ± ·10	21·99 ± ·16	19·97 ± ·10	15·32 ± ·15	12·22 ± ·09	7·80 ± .14
Malayan (74·6)	11·17 ± ·10	15·99 ± ·21†	33·17 ± ·11	37·35 ± ·21†	76·64 ± ·10	126·38 ± ·18	65·46 ± ·10	110·39 ± ·18	30·75 ± ·10	36·97 ± ·17
Burmese A (41·0)	10·96 ± ·09	14·07 ± ·14	33·82 ± ·09	37·75 ± ·15	45·68 ± ·09	51·64 ± ·15	54·51 ± ·09	61·70 ± ·15	20·72 ± ·09	27·25 ± ·14

* "Oriental" is here used in the special sense of this memoir (see p. 5) and Tibetan B and Aino can only be spoken of as anomalous Orientals or "other races."

† These Coefficients are based on fewer than six indices and angles.

light on the problem; meanwhile it will be of interest to consider the Coefficients of Racial Likeness between the races already dealt with and the Tibetan B and Aino on the one hand and a few races from other continents on the other. It is not improbable that the measures of affinity between the Oriental races and an entirely foreign race would arrange the former in the order which their own inter-relations make them adopt. While estimating the significance of the Coefficients in Table XVII it must be constantly borne in mind that the Tibetan B and Moriori means are based on far fewer crania than are the means of the Aino, Whitechapel English and Naqada series, and hence the Coefficients with the two former races will be lower than the others for that cause if for no other.

Considering first columns 1 and 2 of Table XVII, we find that the Coefficients of both Tibetan B and Aino with the other Asiatic races arrange the latter in the order which was suggested by their own inter-relations, if allowance be made for the differing numbers of crania on which the several racial means are based. In both series it is reasonable to place the Malayan type mid-way between the Chinese and Burmese A and the Coefficient with the Hindu may be taken to indicate more marked divergences than those with the Nepalese. But it cannot be contended that either the Tibetan B or Aino are members of that group of allied peoples which predominates in Tibet, China and Indo-China. Bearing in mind that the Tibetan B means are based on only fifteen skulls, it becomes at once apparent that the lowest Coefficients with that type indicate a degree of dissimilarity which is very much greater than that between any adjacent pairs of the races arranged vertically in Table XVII. It is probable that the Khams Tibetans have intermingled to a certain extent with the Chinese and Tibetans of the A type, but they have preserved predominantly the characteristics of an entirely different stock which resembles the Moriori as much as any of the Asiatic races* dealt with above. The following Coefficients may be compared with those in the above table:

		Moriori (34·0)	Maori (43·0)
Coefficients of Racial Likeness with Tibetan B (14·0)	All Characters ...	4·85 ± ·09	8·14 ± ·09
	Indices and Angles	4·74 ± ·14	6·55 ± ·18

All the Coefficients with the Aino are of a higher order than the majority of those which have yet been considered and it is evident that we are dealing with a race which stands quite apart from the others. That it is most similar to the Chinese and Malayan types, as it is nearest to them geographically, is not surprising, but from this evidence alone it would be rash to conclude that there has been any intermingling or any distant relationship whatsoever between the aborigines of Japan and the Chinese for, by parity of reasoning, we might be led to infer that the seventeenth century Englishmen bear traces of relationship with Tibetans and Chinese. The Coefficients with the Aino are of the same order as

* See *Biometrika*, Vol. xiv. pp. 222–224.

those with peoples of other continents and must be regarded as indicating extremely divergent types.

7. COMPARISON OF SINGLE MEASUREMENTS.

Having considered the relations of Oriental races from the standpoint of a generalised criterion of affinity—the Coefficient of Racial Likeness—we may now turn to a comparison of individual characters. Thirty-one characters, including all the more important direct measurements and indices, take part in the computation of the Coefficients and the significances of the differences between them can be at once estimated by considering the values of the function

$$\frac{n_s n_s'}{n_s + n_s'} \left(\frac{M_s - M_s'}{\sigma_s} \right)^2,$$

which has been denoted by the letter α (p. 24). The Coefficient of Racial Likeness is equal to $\Sigma(\alpha) - 1$. Values of α for the thirty-one characters and between each pair of seven Asiatic races are provided in Table XVIII. When comparing the means of two samples which may have been drawn from similar, perhaps, but not identical populations it is necessary to take into account the number of crania (n_s) on which the individual mean measurements are based. In the case of four of the races all, or nearly all, the characters are available for each individual skull included in the racial population, but the Chinese, Maravar and Hindu means are not so uniform since they were collated from various sources. The actual number on which each mean measurement is based will be found in the Comparative Table of Means (Table VIII), and in Table XVIII the average number of skulls for the thirty-one characters dealt with is given for each race in brackets immediately after the races.

We will consider first the horizontal arrays of Table XVIII. The profile angle ($P\angle$), unfortunately only provided for five of the races, and facial index ($100\,G'H/GB$), not available for the Maravar, are evidently the same within the limits of random sampling for all the races. That the components of the index are not so constant is shown by the values of α for $G'H$ and J. The characters $100\,O_2/O_1\,(R)$ and LB are almost identical throughout and *Oc. I.*, $100\,NB/NH\,(R)$, $100\,fmb/fml$, $100\,G_2/G_1$, $N\angle$, $A\angle$, F, B', S, G_1 and fml are hardly more variable. Thus fifteen of the thirty-one characters exhibit hardly any racial distinctiveness. More marked divergences are shown for $100\,H/L'$, $100\,B/H$, U, $G'H$, G_2, fmb, C and OH, and there are still greater differences between Q', J, $NH\,(R)$, NB, O_1R and O_2R. Finally, we find extreme differences between skull breadths and cephalic indices.

The measurements of the human skull may be somewhat arbitrarily separated into three groups according as they relate to facial, basial (including palatine) or calvarial bones. Considering facial measurements first, we find that the orbital and facial ($100\,G'H/GB$) indices and the profile angles of the seven races are remarkably similar, but all the direct measurements on which the indices are based differ quite significantly. The nasal index is more variable than the others but, again,

TABLE XVIII. *Values of* $\alpha = \frac{n_s n_s'}{n_s + n_s'} \left(\frac{M_s - M_s'}{\sigma_s}\right)^2$ *between Asiatic Races**.

Character	Maravar (26·7) and						Hindu (25·9) and				
	Hindu (25·9)	Nepalese (45·8)	Tibetan A (35·9)	Chinese (46·2)	Malayan (74·6)	Burmese A (41·0)	Nepalese (45·8)	Tibetan A (35·9)	Chinese (46·2)	Malayan (74·6)	Burmese A (41·0)
100 B/L'	3·84	0·55	42·41	46·20	126·98	149·51	1·45	29·98	33·34	122·78	145·12
100 H/L'	0·12	1·07	0·00	5·88	17·15	17·78	0·71	0·67	2·36	6·29	7·24
100 B/H	0·62	0·01	27·27	12·18	26·90	34·23	1·17	29·29	11·02	31·42	39·09
Oc. I.	—	—	—	—	—	—	0·01	1·12	1·44	—	3·62
100 $G'H/GB$	—	—	—	—	—	—	0·61	0·75	2·56	2·13	0·87
100 O_2/O_1, (R)	1·81	0·41	1·54	8·44	1·43	7·85	0·94	0·00	4·25	0·11	3·71
100 NB/NH, (R)	5·53	0·00	2·85	11·88	2·90	1·34	6·21	0·33	1·04	0·92	13·81
100 *fmb*/*fml*	6·24	0·35	0·04	2·22	0·82	0·10	5·29	6·38	13·10	14·47	8·47
100 G_2/G_1	—	—	—	—	—	—	5·28	5·04	0·02	—	0·38
$P\angle$	—	—	—	—	—	—	0·57	0·24	0·81	—	0·01
$N\angle$	—	—	—	—	—	—	0·33	1·90	0·01	0·73	0·11
$A\angle$	—	—	—	—	—	—	2·48	4·38	14·03	12·96	11·24
F	0·04	0·10	0·17	4·02	0·41	3·83	0·02	0·61	4·92	0·43	8·30
B	0·88	1·34	44·02	80·04	130·73	137·31	0·11	43·45	94·00	157·13	155·93
B'	0·26	4·72	0·81	0·44	0·03	1·05	1·13	0·02	1·14	0·54	1·79
OH	0·12	3·17	0·26	12·45	13·71	14·75	6·66	4·11	28·33	23·38	19·03
LB	0·20	0·86	4·27	0·14	0·79	0·12	2·08	6·73	0·03	0·17	0·67
Q'	—	—	—	—	—	—	0·40	1·03	27·25	23·47	43·50
S	—	—	—	—	—	—	0·23	0·59	6·84	0·70	0·00
U	2·99	8·38	17·51	53·70	38·78	31·07	2·24	9·27	44·48	28·57	21·09
$G'H$	—	—	—	—	—	—	7·27	10·05	24·21	18·49	24·55
J	3·50	5·33	27·35	41·58	60·78	60·73	0·15	14·11	25·65	43·96	44·14
NH, (R)	15·08	15·55	49·93	113·91	86·23	113·28	1·32	3·96	48·38	27·73	48·70
NB	0·26	19·57	17·30	16·73	35·21	105·93	16·31	14·26	13·53	31·20	101·98
O_1R	4·44	43·24	37·58	13·40	30·89	38·77	37·14	29·41	4·37	22·74	31·09
O_2R	2·76	11·39	27·16	26·94	22·30	58·85	3·11	14·23	13·18	9·29	39·73
G_1	6·12	4·56	4·74	4·85	—	0·43	1·95	1·74	13·15	—	4·87
G_2	—	—	—	—	—	—	31·61	27·21	11·27	32·37	14·53
fml	2·44	10·49	9·69	7·94	6·85	18·13	0·28	0·27	0·24	0·00	1·76
fmb	0·80	6·11	7·42	17·32	11·47	18·94	7·14	8·20	15·74	10·56	15·84
C	0·80	20·80	23·72	33·65	19·56	11·12	20·71	23·83	33·10	19·92	8·83
C. L. All Characters	1·80	6·52	15·48	23·47	30·70	38·29	4·32	8·46	14·93	22·68	25·45
C. L. Indices and Angles	2·03	−0·60	11·35	13·47	31·73	34·14	1·09	5·67	6·00	19·98	18·47

* The numbers in brackets following the races are of the mean numbers of skulls available for the 31 characters used in calculating Coefficients of Racial Likeness.

TABLE XVIII—(*continued*).

Character	Nepalese (45·8) and				Tibetan A (35·9) and			Chinese (46·2) and		Malayan (74·6) and
	Tibetan A (35·9)	Chinese (46·2)	Malayan (74·6)	Burmese A (41·0)	Chinese (46·2)	Malayan (74·6)	Burmese A (41·0)	Malayan (74·6)	Burmese A (41·0)	Burmese A (41·0)
100 B/L'	52·02	18·51	194·08	214·32	0·33	22·96	41·58	44·86	29·78	6·10
100 H/L'	0·00	3·29	16·26	33·68	7·35	24·97	28·94	1·22	1·98	0·27
100 B/H	43·03	19·17	47·23	53·24	4·28	0·62	0·27	2·64	7·26	2·14
Oc. I.	3·73	3·59	—	12·00	0·10	—	1·85	—	0·60	—
100 $G'H/GB$	0·04	2·02	1·57	0·08	1·38	0·93	0·00	0·17	1·28	0·82
100 O_2/O_1, (R)	3·97	9·85	0·57	0·04	4·35	0·05	4·40	7·29	0·00	6·17
100 NB/NH, (R)	3·17	13·53	3·36	1·84	2·49	0·07	8·88	4·73	24·09	10·37
100 fmb/fml	0·23	5·61	4·06	1·30	3·56	1·87	0·37	0·87	1·96	0·51
100 G_2/G_1	0·00	3·65	—	12·17	3·49	—	10·79	—	0·14	—
$P\angle$	4·06	0·11	—	1·25	3·46	—	0·85	—	1·36	—
$N\angle$	1·76	1·31	7·04	0·16	6·16	16·03	2·85	2·17	0·49	4·47
$A\angle$	0·79	13·77	12·82	8·82	6·93	5·83	3·96	0·23	0·30	0·02
F	0·91	4·45	4·34	9·48	7·52	0·78	3·84	5·90	18·55	1·24
B	32·88	67·47	118·49	124·78	0·73	12·99	21·68	14·08	24·26	2·81
B'	2·12	13·31	11·27	16·18	3·65	2·16	5·33	1·26	0·23	1·37
OH	0·59	20·98	32·18	9·14	24·69	40·02	12·50	0·33	2·69	2·87
LB	1·59	2·09	4·12	0·34	7·25	10·65	3·22	0·35	0·60	1·73
Q'	6·84	90·69	98·60	143·17	40·77	38·45	73·49	1·27	4·42	13·05
S	1·78	5·38	0·11	0·20	13·28	3·19	0·80	5·13	7·82	0·71
U	2·45	18·73	10·59	8·25	5·13	1·56	1·27	1·63	1·20	0·00
$G'H$	0·73	14·50	7·60	14·56	7·53	2·75	7·94	2·05	0·04	2·49
J	13·47	26·55	47·75	46·35	0·71	5·51	8·05	3·27	5·72	0·81
NH, (R)	6·17	34·43	16·59	35·49	8·21	1·02	9·91	5·31	0·26	7·00
NB	0·00	0·35	1·48	40·69	0·30	1·25	35·79	3·79	52 61	33·55
O_1R	0·30	17·86	3·74	45·66	13·31	2·21	33·74	7·11	14·12	2·36
O_2R	4·71	3·41	1·28	21·87	0·24	1·68	5·33	0·78	9·18	16·25
G_1	0·02	11·46	—	2·99	11·65	—	3·18	—	6·89	—
G_2	0·27	12·54	0·05	8·85	8·42	0·15	5·44	13·49	0·41	9·42
fml	0·00	0·00	1·17	1·80	0·00	1·01	1·59	0·72	1·20	6·39
fmb	0·18	5·61	0·99	5·75	3·72	0·22	3·44	3·07	0·13	2·81
C	0·41	1·79	0·32	1·15	0·36	1·49	2·47	1·19	4·92	0·48
C. L. All Characters	5·07	13·39	22·99	27·25	5·50	7·25	10·09	4·00	6·24	4·05
C. L. Indices and Angles	8·40	7·70	30·89	27·24	2·66	7·15	7·73	6·13	4·77	2·43

it is far more constant than its component lengths. The following measurements are abstracted from the Comparative Table of Means (Table VIII).

Race	*NH*, *R*	*NB*	100 *NB*/*NH*, (*R*)	*J*
Maravar ...	46·5	24·0	51·7	124·4
Hindu ...	49·0	24·2	49·7	126·8
Nepalese ...	49·7	25·7	51·7	127·2
Tibetan A ...	51·3	25·7	50·2	131·0
Chinese ...	53·1	25·5	48·9	131·8
Malayan ...	51·9	26·1	50·4	133·2
Burmese A ...	53·4	28·1	52·7	134·0

The Maravar nasal height and the index of the Chinese show high values of α with all the other races, but it is well to remember that both means were based on material collected from various sources, and in such cases it is often not possible to tell definitely whether certain measurements, and nasal measurements in particular, are comparable with others or not. The nasal breadth of the Burmese A is another outstanding feature, and in that case the method of measurement was precisely the same as for the Nepalese and Tibetan A.

Attention may be attracted to a more noteworthy feature of the above lists. Both nasal direct measurements are least for the Maravar and greatest for the Burmese A and there is a more or less gradual transition between those two extremes when the races are arranged in the order shown. It is not surprising then to find that the index remains fairly constant throughout. If the Table of Means (Table VIII) be consulted it will be found that the orbital direct measurements, the facial height (*G'H*) and the facial breadth (*GB*)—a character not used when calculating Coefficients of Racial Likeness—exhibit the same gradual transition between the Maravar and Burmese A. There are some irregularities in the sequence but the general trend can be traced in every case, and it is reasonable to suppose that the order is sometimes broken owing to the errors due to random sampling, since, in general, a race hardly differs significantly in the case of these particular characters from the races adjacent to it in the series. For the zygomatic breadth (*J*), shown above, the order of the races is identically the same as that which we are considering—the order which the Coefficients of Racial Likeness suggested. We may conclude that there is a gradual diminution in the size of the faces as we pass from the Burmese to the Dravidians, but the shapes of the faces, as measured by nasal, orbital and facial indices, do not alter significantly. It may be objected that the Dravidians have obviously no "Mongolian" affinities; that they are not at all characterised by that flatness of face and projection of the zygomatic bones which are peculiar to the Oriental peoples; and that tegumentary characters clearly differentiate the two types. There may be qualitative distinctions of such kinds which elude, or, at any rate, which cannot be readily appreciated by the methods of exact measurement; nevertheless we can trace a perfectly gradual transition between the measured characters of the Maravar and Chinese crania. It is unfortunate that

several of the facial measurements taken in this Laboratory have been provided for so few of the Asiatic races that the significance of their inter-racial differences cannot yet be estimated.

The measurements of the palate and base of the skull have, on the whole, less inter-racial variation than the facial measurements. The palatine indices of five of the races—measurements for the other two not being available—and the palatine lengths (G_1) show few significant differences, but the breadth (G_2) is not so uniform. The divergences of the latter are all greatest between the Hindu and other races and it is possible that some of the Hindu material was not measured in precisely the same way as the other series. There seems to be no high inter-racial correlation between the breadths of palate and face. The length of the base of the skull (LB) is the same for all the types and the $N\angle$ and $A\angle$ of the fundamental triangle are fairly constant, the differences between alveolar angles being generally greater than the corresponding differences between nasal angles. The measurements of the *foramen magnum* have a greater significance, the Maravar possessing the smallest length and breadth and the measurements showing a tendency to increase as the races of the Comparative Table of Means (Table VIII) are passed in order from left to right.

The foraminal index remains fairly constant throughout. The values are:

Race	*fml*	*fmb*	100 *fmb*/*fml*
Maravar ...	33·9	28·2	83·3
Hindu... ...	35·5	27·4	77·3*
Nepalese ...	36·0	29·6	82·4
Tibetan A ...	36·0	29·8	83·0
Chinese ...	36·0	30·9	85·9
Malayan ...	35·5	30·0	84·6
Burmese A ...	36·7	30·7	83·8

The conclusions deduced from the direct foraminal and facial measurements are in accordance: they exhibit a gradually increasing sequence of means from Dravidians to Burmese. All the characters showing the greatest inter-racial differences have suggested such a sequence, but the more constant characters—the indices, angles, and measurements, other than foraminal, of the base of the skull—have not done so. We may now turn to a comparison of the measurements of the calvaria between several of which there are more fundamental differences than any which have yet been dealt with.

Four calvarial characters used in calculating Coefficients of Racial Likeness—the occipital index, forehead breadth, calvarial length and sagittal circumference—have no values of α greater than twenty and few greater than five. The Burmese A stands apart from the other races in having a shorter skull, but the difference is not very significant; its sagittal circumference is greater than that of the Tibetan A

* The Hindu mean foraminal measurements are based on only eight crania, so no stress can be laid on the discordance of the index.

and equal to the Hindu measurement. While the length of the cranium is not a distinguishing feature of these races, the breadth shows higher values of α than any other direct measurement, and, in consequence, there are extreme differences between the cephalic indices. Arranging the races in order of their breadths (see p. 4), we arrive at identically the same order as that which was suggested by a comparison first of Coefficients of Racial Likeness and then of facial and foraminal direct measurements. The cephalic indices give a very similar order.

It is not surprising to find that the horizontal and vertical circumferences, being characters which must be highly correlated inter-racially with breadth for a constant length of skull, arrange the races in sequences similar to that above. The height of the skull (OH, H or H') does not furnish such an orderly arrangement, although the majority of its differences are quite significant. If the Comparative Table of Means be consulted (Table VIII) it will be found that there are four calvarial characters—length, height, horizontal circumference and capacity—which have greatest mean values for the Chinese and are greater for the Malays than for the Burmese A. There appears to be a gradual increase in the values of the characters U, OH and C as we pass from the Maravar, with the least values, to the Chinese with the greatest, and then comes a fall to the Malayan means and a fall from them to the Burmese A. But no stress can be laid on the significance of the sequence since the values of α for the three characters between the Chinese and Burmese A are all less than five and they might vary sufficiently to give quite different orders if other random samples were taken from the same populations. The differences between the same two races in the case of breadth characters are far more significant and they cannot be attributed to sampling. The index $100\ H/L$ increases in value as the races from Maravar to Burmese A are passed, and in a less regular way the index $100\ B/H$ exhibits the same sequence of values.

We sum up the conclusions arrived at after a comparison of the thirty-one mean measurements from which the Coefficients of Racial Likeness are calculated as follows:

(i) Fifteen of the characters are appreciably the same for all the seven races considered.

(ii) There is a gradual transition in head form as the races from the Dravidians to the Burmese A are passed; the calvaria becomes very significantly broader and slightly higher, but does not change in length, while the face becomes broader and higher, but does not alter in shape.

(iii) There are no irregularities in this sequence which may not be attributed to random sampling.

The measurements which are not used in calculating the Coefficients confirm the relations already noted. The sagittal arcs S_1, S_2, S_3, the chord S_3' and the facial length (GL) are much the same for all the races, and the facial breadth (GB) is least for the Hindus—no measurement being given for the Dravidians—and greatest for the Burmese A, the difference between the extremes being 6·7 mm.

The Conjoint Index ($100(B - H')/L$) is chiefly affected by the skull breadth and hence it increases with that character.

We may anticipate that there are no cranial characters which show very significant differences between any one race and *all* the others. The Maravar nasal height and the palate width of the Hindus do apparently distinguish those races, but differences in methods of measurement may account for the discordances. The only other measurement which differentiates a single race from its neighbours is the nasal breadth of the Burmese A. Both males and females of that type have an unusually broad nose, but the Burmese B and C series are not characterised in the same way. The question whether there are any characters which are constant for the seven races yet considered, and which will serve to distinguish them as a group from all other groups or families of races, will be considered later.

8. FURTHER COMPARATIVE MATERIAL.

A comparison of the skull measurements of seven of the principal races of India and the Orient has shown that they may be arranged in a series such that adjacent members are intimately related while the extremes are of markedly divergent types. The question whether other races could be fitted in the same series, either between those already dealt with or at the ends, was apparently one of considerable interest. With the object of discovering such missing terms if possible, the writer collected from various sources the measurements available for several Asiatic peoples. In the majority of cases no single racial series could be found which contained enough skulls to make the means deduced from them sufficiently reliable, and accordingly data provided by different workers for the same race had to be collected and pooled. The mean measurements of the male material are given in Table XIX*, and it will be seen that some of them are based on very few skulls. But however small the number might be the figures were included so that it should be easier to add new material to that already furnished. It is not for a moment suggested, of course, that the more scanty data—such as the means based on five or fewer crania—can be considered to be a representation of the racial type on which any reliance whatever may be placed. One measurement not described in the list above (p. 8) has been included since it is used by many craniologists; that is the measure of nasal height taken in the sagittal plane from the nasion to the tip of the anterior nasal spine and denoted by the letters *NH'* in the tables. When all points on the sagittal type contours have been plotted the length in question may be read off from them with sufficient accuracy. The other measurements provided for the comparative material were made in precisely the same way as ours and all doubtful measurements were rejected.

Veddahs and Andamanese. The longest series of Veddah skulls for which measurements are available is that described by Paul and Fritz Sarasin (*Ergebnisse naturwissenschaftlicher Forschungen auf Ceylon*, Bd. III. pp. 198–307†,

* The female means will be found in Table IX above.

† Complete tables of the individual and mean measurements are provided in the supplementary *Atlas* volume.

TABLE XIX.

Second Comparative Table of Male Means.*

Character	Andamanese	Veddah	Siamese	Annamese	Southern Chinese	Formosa Chinese	Northern Chinese	Northern Chinese (Koganei)	Koreans	Japanese	Aino
Capacity	[1263·9 (18)]	[1285·0 (50)]	[1527·7 (7)]	[1501·4 (20)]	[1480·9 (64)]	1408·9 (14)	[1454·7 (40)]	1485·5 (69)	1490·2 (17)	[1503·0 (71)]	1462·0 (76)
F	168·5 (15)	179·3 (24)	174·3 (4)	—	179·6 (16)	—	—	—	—	181·3 (8)	—
L'	—	—	—	—	179·2 (4)	179·5 (14)	180·2 (27)	179·7 (70)	—	—	—
L	165·7 (7)	179·1 (32)	177·9 (7)	177·0 (27)	177·6 (102)	179·9 (14)	177·9 (46)	180·1 (70)	177·1 (20)	180·3 (69)	185·8 (88)
B	135·1 (21)	128·8 (21)	143·9 (11)	140·3 (27)	139·4 (120)	139·4 (14)	138·8 (46)	140·5 (70)	143·4 (20)	141·2 (77)	141·2 (88)
B'	92·7 (16)	90·6 (30)	95·4 (7)	94·7 (25)	93·7 (67)	92·1 (14)	92·6 (46)	91·9 (70)	92·0 (15)	94·6 (20)	96·2 (88)
H	—	133·1 (25)	145·7 (3)	—	136·6 (37)	140·5 (13)	138·5 (27)	140·5 (70)	141·6 (17)	140·0 (63)	—
H'	130·7 (19)	133·0 (37)	137·8 (5)	137·0 (25)	137·1 (89)	—	136·7 (46)	—	136·3 (3)	137·8 (16)	139·5 (88)
OH	—	116·4 (5)	125·3 (3)	—	119·2 (38)	117·1 (14)	118·1 (27)	117·0 (70)	119·2 (15)	119·5 (14)	119·3 (88)
LB	94·0 (18)	98·1 (40)	100·0 (7)	99·8 (24)	99·5 (84)	99·9 (13)	99·7 (46)	100·4 (70)	100·9 (15)	101·9 (27)	105·4 (88)
Q'	—	298·4 (31)	350·0 (1)	—	321·2 (39)	325·1 (14)	317·6 (40)	325·2 (70)	329·5 (15)	320·6 (14)	328·5 (84)
S	351·4 (16)	363·1 (30)	372·0 (9)	370·8 (14)	369·5 (96)	374·1 (14)	370·3 (45)	373·2 (69)	369·1 (17)	370·5 (25)	372·8 (77)
S_1	121·7 (16)	133·0 (9)	129·1 (8)	130·7 (14)	126·2 (75)	—	126·6 (46)	—	126·3 (17)	126·1 (24)	—
S_2	124·0 (16)	130·8 (9)	125·8 (8)	125·9 (14)	125·8 (74)	—	128·0 (45)	—	123·2 (15)	126·9 (16)	—
S_3	104·8 (17)	108·9 (26)	113·4 (8)	112·1 (23)	116·1 (74)	—	116·9 (45)	—	118·5 (15)	115·1 (16)	—
S_3'	—	—	—	—	98·7 (19)	—	100·5 (39)	—	—	—	—
U	483·0 (21)	497·3 (28)	513·4 (11)	506·1 (25)	509·3 (100)	509·5 (14)	504·6 (46)	511·6 (70)	511·1 (20)	508·8 (28)	522·5 (88)
$G'H$	61·8 (4)	61·9 (22)	71·5 (2)	—	71·2 (49)	70·2 (14)	72·5 (39)	75·2 (69)	73·0 (15)	72·2 (55)	69·8 (73)
GB	—	91·6 (20)	103·7 (7)	102·1 (9)	99·9 (56)	100·7 (14)	99·6 (45)	100·3 (69)	99·9 (17)	99·3 (59)	102·1 (76)
J	126·0 (18)	124·5 (30)	136·3 (9)	132·7 (23)	131·8 (83)	135·9 (14)	133·0 (44)	135·6 (64)	137·5 (14)	134·8 (68)	137·3 (74)
NH	—	48·3 (3)	51·3 (3)	—	53·1 (54)	53·5 (14)	54·5 (27)	55·5 (70)	54·9 (17)	51·5 (63)	50·5 (79)
NH'	46·5 (4)	47·1 (21)	51·0 (4)	51·0 (27)	—	—	51·9 (13)	—	—	—	—
NB	23·8 (18)	24·4 (41)	27·1 (7)	26·2 (26)	25·6 (78)	26·4 (14)	25·2 (46)	25·0 (70)	26·3 (17)	25·8 (76)	25·6 (78)
DC	24·5 (1)	22·2 (21)	26·0 (4)	24·8 (24)	26·0 (18)	—	24·0 (6)	—	23·8 (15)	23·7 (17)	—
O_1	—	40·8 (18)	40·3 (3)	—	—	—	41·9 (13)	—	—	39·2 (46)	40·9 (79)
O_2	32·8 (18)	33·0 (39)	32·3 (7)	32·9 (27)	33·8 (54)	—	35·8 (40)	—	—	34·9 (59)	34·9 (79)
O_1'	37·3 (4)	37·9 (21)	—	—	38·3 (54)	—	38·3 (27)	—	—	37·0 (6)	—

* The capacities in square brackets were determined by methods which might give results only approximately the same as those given by the method we employed. The indices and angles in curled brackets were determined from the mean measurements of the component lengths instead of from the measurements of the individual skulls.

TABLE XIX—(*continued*).

Character	Andamanese	Veddah	Siamese	Annamese	Southern Chinese	Formosa Chinese	Northern Chinese	Northern Chinese (Koganei)	Koreans	Japanese	Aino
G_1	—	49·5 (10)	54·5 (2)	—	54·5 (4)	47·9 (13)	50·8 (27)	48·2 (69)	50·6 (17)	49·8 (14)	53·0 (75)
G_1'	—	—	—	—	45·8 (32)	—	42·5 (11)	—	—	47·5 (6)	—
G_2	—	39·0 (1)	40·5 (2)	—	39·2 (35)	38·9 (13)	39·2 (11)	39·3 (70)	40·0 (17)	38·5 (6)	38·2 (74)
GL	96·4 (14)	93·0 (33)	—	—	97·7 (58)	—	91·8 (12)	—	97·7 (15)	100·2 (9)	104·9 (69)
fmb	27·9 (12)	30·0 (2)	31·6 (5)	30·2 (23)	30·1 (41)	30·1 (13)	29·9 (46)	30·3 (70)	31·3 (20)	30·4 (16)	30·2 (81)
fml	33·5 (16)	35·3 (8)	35·4 (5)	34·7 (22)	35·6 (41)	36·1 (13)	35·6 (46)	36·3 (69)	36·0 (20)	36·4 (16)	35·7 (76)
SC	—	7·8 (19)	—	—	7·0 (18)	—	6·9 (45)	—	—	6·8 (8)	—
100 B/L	—	—	—	—	77·8 (13)	77·7 (14)	{77·0 (27)}	78·3 (70)	—	—	—
100 B/L	{82·0 (7)}	{71·9 (32)}	{80·9 (7)}	{79·3 (27)}	{78·6 (91)}	{77·5 (14)}	{78·1 (46)}	{78·0 (70)}	{81·0 (20)}	{78·3 (69)}	76·5 (88)
100 B/F	{80·5 (15)}	{71·8 (20)}	{82·6 (4)}	—	{77·6 (16)}	—	—	—	—	{77·9 (8)}	—
100 H/L	—	{74·3 (25)}	{81·9 (3)}	—	{77·4 (21)}	{78·1 (13)}	78·0 (27)	{78·0 (70)}	{80·0 (17)}	{77·9 (63)}	—
100 H'/L	{78·5 (7)}	{74·3 (32)}	{77·5 (5)}	{77·4 (25)}	{77·2 (89)}	—	{76·8 (46)}	—	{77·0 (3)}	{76·4 (16)}	75·6 (88)
100 H'/F	{77·5 (13)}	{74·2 (24)}	{79·1 (5)}	—	{76·3 (16)}	—	—	—	—	{76·0 (8)}	—
100 B/H	—	{96·8 (25)}	{98·8 (3)}	—	{103·2 (37)}	{99·2 (13)}	{101·1 (27)}	{100·0 (70)}	{101·3 (17)}	{100·7 (63)}	—
100 B/H'	{103·2 (19)}	{96·8 (37)}	{104·4 (5)}	{102·4 (25)}	{101·7 (89)}	—	{101·5 (46)}	—	{105·2 (3)}	{102·5 (16)}	101·2 (88)
100 $(B-H')/L$	{2·7 (7)}	{−2·3 (32)}	{3·4 (5)}	{1·9 (25)}	{1·3 (89)}	—	{1·2 (46)}	—	{4·0 (3)}	{1·9 (16)}	{0·9 (88)}
100 $G'H/GB$	—	{67·6 (20)}	{68·9 (2)}	—	{71·1 (36)}	69·8 (14)	{72·0 (39)}	75·2 (68)	73·3 (15)	{72·7 (52)}	68·4 (65)
100 NB/NH	—	48·4 (3)	51·1 (3)	—	{48·2 (54)}	49·5 (14)	{46·6 (27)}	45·2 (70)	48·0 (17)	{50·7 (63)}	50·7 (78)
100 NB/NH'	51·7 (4)	52·5 (21)	52·9 (4)	51·4 (26)	—	—	{48·6 (13)}	—	—	—	—
100 O_2/O_1	—	{80·9 (18)}	81·0 (3)	—	—	—	85·0 (13)	—	—	{88·7 (46)}	85·3 (80)
100 O_2/O_1'	86·8 (4)	89·2 (21)	—	—	{88·4 (54)}	—	{93·5 (27)}	—	—	{94·4 (6)}	—
100 *fmb*/*fml*	82·9 (12)	83·3 (2)	89·3 (5)	87·0 (22)	{84·6 (41)}	83·4 (13)	{84·0 (46)}	83·5 (69)	{86·9 (20)}	{83·2 (19)}	{84·6 (76)}
100 G_2/G_1	—	77·1 (10)	74·3 (2)	—	78·4 (4)	81·4 (13)	{77·2 (11)}	81·8 (69)	78·4 (17)	{77·3 (6)}	72·1 (72)
100 G_2/G_1'	—	—	—	—	{85·6 (32)}	—	92·4 (11)	—	—	{81·0 (6)}	—
$P\angle$	—	88°·0 (1)	—	—	84°·9 (17)	83°·0 (14)	84°·6 (13)	84°·0 (69)	84°·1 (15)	84°·9 (52)	82°·0 (67)
$N\angle$	{73°·0 (4)}	{66°·8 (21)}	—	—	{67°·3 (49)}	—	{62°·3 (12)}	—	{65°·9 (15)}	{68°·2 (9)}	{70°·2 (69)}
$A\angle$	{69°·2 (4)}	{76°·4 (21)}	—	—	{70°·1 (49)}	—	{73°·8 (12)}	—	{71°·0 (15)}	{70°·1 (9)}	{71°·2 (69)}
$B\angle$	{37°·8 (4)}	{36°·8 (21)}	—	—	{42°·6 (49)}	—	{43°·9 (12)}	—	{43°·1 (15)}	{41°·7 (9)}	{38°·6 (69)}
θ_1	—	25°·2 (1)	—	—	{27°·8 (17)}	—	{33°·1 (12)}	—	{30°·0 (15)}	{26°·9 (9)}	{27°·8 (67)}
θ_2	—	11°·6 (1)	—	—	{14°·8 (17)}	—	{10°·8 (12)}	—	{13°·1 (15)}	{14°·8 (9)}	{10°·8 (67)}
Oc. I.	—	—	—	—	{62·1 (19)}	—	{63·4 (39)}	—	—	—	—

Wiesbaden, 1892–3) containing 22 ♂ and 11 ♀ specimens. Twenty-one of the skulls of the Sarasin Collection were subsequently measured by Lüthy (*Archiv für Anthropologie*, Bd. XXXIX. 1912), and that writer furnishes individual measurements and includes several that had not previously been given* (*loc. cit.* p. 70), but does not sex the skulls himself or even give the sexes which had been adopted by the brothers Sarasin. After comparing several characters I concluded that 6 ♀—viz. Nos. XXIII, XXV, XXVI, XXX, XXXIII, XXXVII—and 15 ♂ specimens were represented and the means of those groups agreed very closely with Sarasin's. The figures are:

	L	*B*	*J*	*GL*
Sarasin ♂ ...	179·2 (21)	128·2 (22)	124·8 (20)	94·2 (16)
Lüthy ♂ ...	179·3 (15)	127·9 (15)	124·6 (14)	94·3 (12)
Sarasin ♀ ...	174·4 (11)	124·0 (11)	116·2 (9?)	88·3 (8)
Lüthy ♀ ...	172·2 (6)	124·7 (6)	114·0 (6)	88·0 (6)

The correspondence is rather better between the ♂ than between the ♀ groups, but evidently there can be little harm in dividing Lüthy's series in the way suggested. Arthur Thomson's well-known paper "On the Osteology of the Veddahs of Ceylon" (*Journal of the Anthropological Institute*, Vol. XIX. p. 125, 1889) furnishes the individual measurements of skulls in the Oxford Collection (Thomson), Royal College of Surgeons (Flower), Barnard Davis' Collection† and three provided by Virchow (see below). These skulls—in all 21 ♂ and 15 ♀—have been included in the means of Tables IX and XIX, but many of our measurements are not provided for them. No description of any other series of Veddah skulls of any considerable length could be found: the following small assemblages were pooled with the material already noticed:

(i) Flower: *Catalogue of the Specimens.....in the Museum of the Royal College of Surgeons*, Part 1 (1907). A few measurements of six Veddah skulls (4 ♂ and 2 ♀) which had not appeared in Flower's 1879 Catalogue are given on p. 215. The numbers of the added specimens are 681^2, 683, 683^1, 683^2, 683^3 and 683^4.

(ii) *Die Anthropologischen Sammlungen Deutschlands*, München section, No. 4146: a ♂ ?

(iii) Virchow: *Zeitschrift für Ethnologie*, Bd. XIV. pp. 302–3: 1 ♂ and 1 ♀.

(iv) Virchow: *Zeitschrift für Ethnologie*, Bd. XVII. p. 500: 2 ♂ and 1 ♀.

* Viz. H' and O_1, and hence $100\,H'/L$, $100\,B/H'$ and $100\,O_1/O_2$.

† We cannot agree with Thomson in supposing that Barnard Davis' measures of skull length and height are equivalent to the maximum glabello-occipital length and the basio-bregmatic height. Davis' "longitudinal diameter" (see *Thesaurus Craniorum*, p. xiv) is said to be measured from the glabella, but then that point is defined to be "about an inch above the fronto-nasal suture." It would thus almost coincide with Flower's ophryon, and Davis' measurement *E* may be supposed the same as our *F*.

(v) Virchow: "Ueber die Weddas von Ceylon." *Abhandlungen der Königl. Akademie der Wissenschaften* (1881). Some of the measurements of the 1 ♂ and 2 ♀ Veddah skulls given in this paper are to be found in Thomson's table. Virchow compares his three skulls with the very meagre data which had been provided before 1881.

There are even fewer exact measurements of Andamanese than of Veddah crania. In 1880 Flower collected the material then available and it appears that no series of any length has been dealt with since. His paper "On the Osteology and Affinities of the Natives of the Andaman Islands" (*Journal of the Royal Anthropological Institute*, Vol. IX. pp. 108–135) deals with 12 ♂ and 12 ♀ specimens from various English museums and a detailed account of the sources need not be given here. In *Les Crânes des Races Humaines* (p. 197) Quatrefages and Hamy give 1 ♂ and 2 ♀ Andamanese* and seven more are to be found in *Craniological Data from the Indian Museum, Calcutta* (Ethnographic Survey of India, 1909). Four of these latter series (p. 57) are said to be male and the other three are apparently ♀ though not said to be so. Virchow (*Zeitschrift für Ethnologie*, Bd. VII. p. 68) has provided measurements of 1 ♀ skull, and in Flower's 1907 Catalogue there are five specimens not previously described. In the *Supplement to Thesaurus Craniorum* (1875) Barnard Davis gives a few measurements of 2 ♂ and 1 ♀ skull, and Duckworth has one ♂ in *Studies in Anthropology* (p. 241). Though several sources were consulted, the total number of Andamanese skulls discovered was by no means large and no final conclusions can be derived from a comparison of their mean measurements. In the case of both Veddahs and Andamanese the various collections which were ultimately pooled accorded with each other reasonably well and we may suppose that our figures give fairly good representations of the pure types, though in both cases there is a real need for the confirmatory evidence which a single long series for each could best provide.

It is probable that the Hindu (Bengalee) series of crania with which we were concerned in Section 5 of this paper contained very few individuals of Western ("Aryan") origin or admixture (see p. 32). There is good reason to suppose, too, that Hodgson's Nepalese skulls were free from Western influences, and the Dravidians were apparently of a pure type. The representatives of this last race all belonged to the same tribe—the Maravar—and there are doubtless many minor differences between the numerous tribes which it is customary to class together as Dravidian; but skull measurements of the others are lacking and we are obliged to content ourselves with the Maravar figures, and may provisionally consider them to be typical of the wider population. It appeared probable that the only pure racial type which could stand in the same relation to the Maravar type as the Maravar does to the Hindu, the Hindu to the Nepalese and so on, was that of the Veddahs or possibly of the Andamanese. Now if either of these types is to fill the position in the series of races which we may expect one of them fills, it should possess a calvaria of approximately the same length and height but of

* Measurements of these skulls had previously been given in the *Revue d'Anthropologie*, Tome 1 (1872), p. 248.

lesser breadth than the Dravidian, with facial bones narrower and less high and with certain other characters such that they do not differ significantly from the Dravidian values. It is at once obvious from a comparison of the mean measurements that the Veddah type will quite reasonably fulfil all those conditions, while the Andamanese stand quite apart. The Coefficients of Racial Likeness are:

	All Characters	Number of Characters	Indices and Angles	Number of Characters
Veddah (24·4) and Maravar (25·9) ...	2·13 ± ·11	20	3·87 ± ·19	6
Veddah (24·4) and Andamanese (12·8)	11·89 ± ·10	22	18·91 ± ·16	9
Maravar (25·9) and Andamanese (12·8)	5·52 ± ·12	17	10·76 ± ·21	5

The numbers of characters between which comparison may be made in the case of these three races are unfortunately much smaller than usual and the Coefficients based on indices and angles alone are of very little value. The Veddahs and Maravars are evidently of closely allied types, though the bond between them is hardly as close as between the Hindus and Maravars*. The characters showing the greatest values of α are in order of magnitude: $100\,B/L$ ($\alpha = 14{\cdot}12$), $100\,O_2/O_1'$, B, U, O_2, B' ($\alpha = 5{\cdot}09$), and there are no others greater than five. The significant difference shown between the orbital indices appeared at first to be somewhat disconcerting, but it is almost certainly due to a difference in method of measurement. The Veddah orbital width is that measured to the dacryon (O_1'), while the Maravar length is taken from Flower's Catalogue and the "inner point of measurement is the spot where the ridge which forms the posterior boundary of the lacrymal groove meets the fronto-lacrymal suture. This is rather behind the dacryon of Broca." The Veddah skull has a lesser calvarial breadth and cephalic index than the Maravar but does not differ significantly in length and height. The Maravar facial height ($G'H$) is unfortunately not available, but the Veddah measurement is less than the Hindu while the nasio-basial lengths and the capacities are almost identical and the zygomatic breadths are the same, too, though that would not have been anticipated. The numbers of crania for the other individual measurements are so small that we should hardly be warranted in making any comparison between them. We may conclude then from the data which are available that in all probability the Veddahs are related to the Maravars in such a way that the race may be considered to be a member of that series of races of which the Dravidians were at one extreme and the Burmese A at the other, while attention was confined to the seven races considered in the earlier section of this paper. The position of the Veddahs is at the beginning of the series and it will idently not be possible to find another people which can come before them since n Oriental or aboriginal Indonesian race has a lower cephalic index.

The Andamanese Islanders, on the other hand, are most clearly differentiated from the Veddahs, Dravidians and all the principal types of man inhabiting Indo-

* See above, p. 35. The C. L. in this case was 1·80 ± ·10 for twenty-one characters, and the mean number of skulls on which the Hindu means were based was 36·0.

nesia and the Orient. It has been shown that the differences between the Oriental and Indian races in the case of a certain number of cranial characters are not appreciable: for some of these same characters the differences shown with the Andamanese are more marked than any others. The skull length (165·7 mm.) is 10 mm. less than that of the Maravar and the length from basion to nasion is almost as distinctively less. But the Andamanese breadth exceeds the Nepalese and the cephalic index is greater than the Chinese. The highest values of α shown with the Veddahs are for the characters $100\,B/F$ ($\alpha = 97{\cdot}01$), L ($\alpha = 31{\cdot}54$), $100\,B/H'$, B, $A\angle$, LB ($\alpha = 13{\cdot}21$), and with the Maravar the most significant differences are for $100\,B/F$ ($\alpha = 42{\cdot}36$), LB, L and $100\,B/H'$ ($\alpha = 9{\cdot}07$). In comparing the other races the characters L, LB and $A\angle$ have never taken such leading positions when arrangements in order of the significances of differences were made. In absolute size and in basio-alveolar length (GL) the Dravidian and Veddah types are very similar to the Andamanese, and the facial and foraminal indices, though very unreliable since based on very few crania, are also similar; but no such similarities can lessen the significance of the wide difference in type shown in the case of the other characters. The Andamanese and Malayan indices and angles are surprisingly similar, but, while the means are so unreliable, no stress can be laid on such a correspondence. The difference in size between the two types is, of course, very great. In the following table the Andamanese ♂ and ♀ indices and angles are compared with the Malayan ♂ figures. The index $100\,B/F$ is in general slightly greater than $100\,B/L$.

Andamanese and Malayan Indices and Angles.

	$100\,B/L$	$100\,B/F$	$100\,H'/L$	$100\,B/H'$	$100\,(B-H')/L$	$100\,O_2/O_1'$	$100\,fmb/fml$	$N\angle$	$A\angle$
Andamanese ♂	82·0 (7)	80·5 (15)	78·5 (7)	103·2(19)	2·7 (7)	86·8 (4)	82·9 (12)	73°·0 (4)	69°·2 (4)
„ ♀	83·3 (5)	82·5 (17)	78·4 (5)	106·0(19)	4·7· (5)	88·7 (3)	85·3 (15)	72°·4 (3)	71°·7 (3)
Malayan ♂	81·7 (77)	—	78·6 (76)	104·3(76)	2·7 (76)	86·0 (73)	84·6 (73)	68°·2 (73)	70°·4 (73)

Annamese and Siamese. In collecting measurements of Annamese skulls from various sources, none were included which were said to be those of the aborigines of the country. The original inhabitants are classed together by the Annamese under the name of Moïs or savages and are supposed to be of Indonesian or Malay origin. The Moïs can be easily distinguished from the Annamese proper who, according to their own traditions, once inhabited South China. Zaborowski (*Bulletins de la Société d'Anthropologie de Paris*, IV^e^ Série, Tome VIII. (1897), pp. 43 and 55) has provided measurements of seven Moïs skulls having a mean cephalic index of 75·6 and for our Annamese data the index is 79·3, being the mean of twenty-seven skulls. Other measurements serve equally well to distinguish the two types, so there is little danger of confusing them. It is unfortunate that measurements of Moïs skulls cannot be found in sufficient numbers to make the task of gathering them together worth while. Data were collected from the following sources:

(i) Quatrefages and Hamy: *Les Crânes des Races Humaines*, p. 423. 6 ♂ and 4 ♀ Annamese, 2 ♂ Tongkingese and 1 ♂ Cambodian.

(ii) Zaborowski: *Bulletins et Mémoires de la Société d'Anthropologie de Paris*, V^e^ Série, Tome I. (1900), p. 327. Measurements of seven skulls are given but they are not all sexed. I have supposed that the No. II (Mondières) and the d'Enjoy skulls are ♀ and all the others ♂.

(iii) Nivard and Hovelacque: *Bulletins de la Société d'Anthropologie de Paris*, III^e^ Série, Tome IX. (1886), p. 470. 2 skulls supposed ♂.

(iv) Maurel: *Mémoires de la Société d'Anthropologie de Paris*, II^e^ Série, Tome IV. (1889), p. 459, and table of individual measurements on p. 470. These are Khmer skulls, and Zaborowski (*loc. cit.* pp. 53–59) has supposed that that people had the same origin as the Moïs though they intermixed more with Mongolian peoples; but we cannot agree with this view. The Khmer measurements are almost precisely the same as those of the acknowledged Annamese, and when both series are pooled the measurements are remarkably similar to those of the Southern Chinese type, so we can have little hesitation in stating that the Khmers are not at all closely related to the aborigines of Annam. Maurel provides the measurements of 11 ♂ and 1 ♀ skull.

The Annamese means (Tables IX and XIX) are based on 27 ♂ and 7 ♀ specimens, thus furnishing a fairly reliable racial type, but many of our measurements were not taken by the French craniologists. Descriptions of Siamese skulls are rarer. Again all were rejected which were thought to be those of the aboriginal races. The references are:

(i) Quatrefages and Hamy: *Les Crânes des Races Humaines*, p. 423. 4 ♂ and 3 ♀ (Thai skulls).

(ii) Barnard Davis: *Thesaurus Craniorum*, p. 174. 4 ♂ and 3 ♀ (Thai skulls).

(iii) *Die Anthropologischen Sammlungen Deutschlands*, Berlin (Erster Theil), No. 171, a ♀ ? and Schmidt (1886), No. 902, a ♂.

(iv) Virchow: *Zeitschrift für Ethnologie*, Bd. XX. p. 579. 2 ♂.

We may anticipate that the Annamese and Siamese will occupy positions intermediate between the Southern Chinese and Malayan types. The Coefficients of Racial Likeness are:

		Southern Chinese* (52·5)	Malayan (74·6)
Annamese (24·1)	All Characters ...	0·01 ± ·16, 17 Characters	1·67 ± ·16, 17 Characters
	Indices and Angles	0·05 ± ·24, 4 (!) „	4·85 ± ·24, 4 (!) „
Siamese (5·3)	All Characters ...	1·05 ± ·09, 26 Characters	0·90 ± ·10, 21 Characters
	Indices and Angles	1·05 ± ·17, 8 „	1·37 ± ·19, 6 „

* This series of Southern Chinese measurements is not precisely the same as that which was used in Section 6. It contains the material collected by Tildesley (see p. 32), together with another series of eighteen skulls (see p. 55).

It is surprising to find that the Coefficient between the Annamese and Southern Chinese has such a low value. Few more than half of the characters which should be used ideally are available for the former series, but the ones we have are based on a fair number of crania. No value of α is found greater than four, and we may conclude that in all probability the Annamese conform to precisely the same cranial type as the Southern Chinese. It would be rash, however, to pool the means until a comparison of the missing measurements shall have confirmed this conclusion. The Annamese and Malays appear to be of distinct though closely related types. The Siamese are somewhat closer to the Malays than to the Chinese*. With the former the only values of α greater than five are for $100\,B/H'$ ($\alpha = 5{\cdot}25$) and cranial capacity ($\alpha = 5{\cdot}10$), and with the latter the greatest divergences are for B ($\alpha = 8{\cdot}99$), J ($\alpha = 7{\cdot}87$) and $100\,O_2/O_1$ ($\alpha = 6{\cdot}10$). The Coefficient between the Annamese and Siamese is $0{\cdot}14 \pm {\cdot}11$ for nineteen characters and $-0{\cdot}19 \pm {\cdot}21$ for five indices. Though some of the data are so meagre, the gradation shown by breadth characters is fairly regular while the measures of length are reasonably constant.

Race	L	B	$100\,B/L$
Southern Chinese ...	177·6	139·4	78·6
Annamese	177·0	140·3	79·3
Siamese	177·9	143·9	80·9
Malayan	174·7	142·2	81·7

The Annamese and Siamese fit into the scheme which has been suggested after a comparison of the principal races of the Orient, and they occupy positions between the Southern Chinese and Malays, the order being that shown above.

Chinese. The measurements of Chinese skulls which were used for comparative purposes in Section 6 of this paper had been collected by Miss Tildesley from various scattered sources (see p. 32) and were said to represent the inhabitants of South China. No specimens obtained in Pekin and the North of the country had been consciously included. The numbers have been augmented by adding to them the mean measurements—individual measurements are not provided—furnished by Quatrefages and Hamy (*Les Crânes des Races Humaines*, p. 434) of 18 ♂ and 1 ♀ crania. The resulting values are given in Table XIX and they will be said to be those of the Southern Chinese type; the majority of the means are based on quite large numbers of crania. The female data in Table IX are still very scanty. It was thought desirable to obtain, if possible, measurements of the skulls of the inhabitants of North China and quite considerable numbers were found in the following publications:

(i) Koganei: *Internationales Centralblatt für Anthropologie und Verwandte Wissenschaften*, Bd. VII. 1902, p. 130.

* It should be remembered that the Malayan means are based on more skulls than the Chinese.

Mean measurements of 70 ♂ Chinese skulls from Chihli, Shantung and South Manchuria (Shingking) are given together with means of 14 ♂ Formosa Chinese from the North of the island. Though these two series had been obtained from districts very far apart, it was clear that they were remarkably similar in type. The Coefficient of Racial Likeness between them is 1·69 ± ·10 for twenty-five characters and 2·83 ± ·17 for eight indices and angles, the most significant differences being between $G'H$ ($\alpha = 16{\cdot}91$), $100\,NB/NH$ ($\alpha = 14{\cdot}77$), $100\,G'H/GB$ ($\alpha = 13{\cdot}76$), NB ($\alpha = 7{\cdot}30$), NH ($\alpha = 5{\cdot}47$) and C ($\alpha = 5{\cdot}30$). The face of the Formosa Chinese is shorter than that of their compatriots in the Pekin district, but no differences can be detected between the breadth characters—except NB—and the lengths and indices of the calvariae. The island people resemble the Southern Chinese (52·5) somewhat more closely, but they cannot be considered as representing a random sample drawn from the population of the mainland. The Coefficients are 1·17 ± ·10 for twenty-five characters and 0·91 ± ·17 for eight indices and angles. The measures of facial height do not differ significantly, but α exceeds five for the characters G_1 ($\alpha = 12{\cdot}02$), J ($\alpha = 9{\cdot}63$) and $100\,B/H$ ($\alpha = 8{\cdot}32$). In spite of the difference between the zygomatic breadths, the calvarial breadths are precisely the same; the difference in palate length may possibly be due to difference in method of measurement. If the Formosa Chinese came originally from South China it is probable that they have been just appreciably modified in type by admixture with an alien people.

(ii) Reicher: "Untersuchungen über die Schädelform der alpenländischen und mongolischen Brachycephalen," *Zeitschrift für Morphologie und Anthropologie*, Bd. XV. 1913, p. 421.

Individual measurements of 13 ♂ adult Northern Chinese skulls are given in Table 6A; there are no ♀ skulls. The value of this important paper, which gives original individual measurements of 292 skulls belonging to several races, would have been considerably enhanced if means had been provided for all characters. We have abstracted data for races other than the Chinese from it.

(iii) Haberer: *Schädel und Skeletteile aus Peking*, Bd. I. 1892. 27 ♂ and 9 ♀ adult skulls from Pekin are described. Several of them are Boxer skulls.

(iv) Quatrefages and Hamy: *Les Crânes des Races Humaines*, 1882, p. 434. Means of 6 ♂ and 1 ♀ Northern Chinese (Pekin, etc.) are given.

It was thought advisable to test in some way the homogeneity of the Northern Chinese material although all the series were said to represent the same race. To do this the whole was divided into two groups, one containing the combined means of Reicher, Haberer and Quatrefages and Hamy and the other the single series of Koganei's. Between these two groups Coefficients were found of 2·56 ± ·10 for twenty-four characters and 1·58 ± ·18 for seven indices and angles. These criteria indicate a close bond between the two groups, since both are based on substantial numbers of crania, but they are not of that order which may be expected in the case of two random samples drawn from the same population, and it was thought

that both series might profitably be compared with the Southern Chinese. The following Coefficients were determined:

TABLE XX.

Coefficients of Racial Likeness between various Groups of Chinese Skulls.

		Northern Chinese Koganei (69·4)	Other Northern Chinese (36·4)	Southern Chinese (52·5)
Northern Chinese Koganei (69·4)	All Characters ...	—	2·56 ± ·10	6·15 ± ·10
	Indices and Angles	—	1·58 ± ·18	5·45 ± ·17
Other Northern Chinese (36·4)	All Characters ...	2·56 ± ·10	—	0·59 ± ·10
	Indices and Angles	1·58 ± ·18	—	1·00 ± ·18
Southern Chinese (52·5)	All Characters ...	6·15 ± ·10	0·59 ± ·10	—
	Indices and Angles	5·45 ± ·17	1·00 ± ·18	—

It is evident that the Northern Chinese skulls, other than those described by Koganei, are so similar to those of the Southern Chinese that the two series might almost be considered to represent the same population. But Koganei's figures indicate a type which is distinctly different from the other two, though resembling what we may suppose to be the pure Northern Chinese type more than the Southern. All the values of α greater than five found between the three series of Chinese means are given in Table XXI (p. 58), so that all of the thirty-one characters employed which do not appear in the table showed insignificant differences.

Koganei's Northern Chinese are clearly differentiated from the other two series by the characters $G'H$, $100\,G'H/GB$, G_1 and to a lesser extent by J, Q' and L. Now for the majority of those characters and for several others shown in Table XXI the means furnished by Koganei are intermediate in value between those of the Chinese and Khams Tibetans. The first five measurements shown in Table XXII (p. 58) arrange the races in precisely the same order, with the Southern Chinese at one end of the scale and the Tibetans of the B type at the other. The nasal height tends in the same direction, but the palate length, nasal index and transverse circumference give rather different orders. In general it will be true to say that the measurements of Koganei's Northern Chinese skulls which diverge most markedly from the supposed pure Chinese type are also those which are intermediate between the Tibetan B and pure Chinese values. In the case of all the Indian and Oriental races previously considered except one there was evidently a high inter-racial correlation between calvarial and zygomatic breadths; the exception was the Tibetan B; Koganei's Chinese now furnish a second exception, for the calvarial breadths of the three Chinese series and the Tibetan B differ quite insignificantly from each other.

TABLE XXI.

Values of a between various Groups of Chinese Skulls.

	G'H	100 *G'H/GB*	G_1	*J*	*NH*	100 *NB/NH*	*Q'*	100 *B/H*	*OH*	*L*	*U*	*E'*
Northern Chinese, Koganei (69·4) and other Northern Chinese (36·4)	10·56	10·31	11·84	3·18	2·28	2·61	15·04	1·27	1·38	4·11	6·97	0·83
Northern Chinese, Koganei (69·4) and Southern Chinese (52·5)	26·65	16·08	13·53	24·97	20·58	16·54	4·11	13·41	7·02	7·93	1·15	6·78
Other Northern Chinese (36·4) and Southern Chinese (52·5)	2·13	0·62	4·30	1·99	4·14	8·18	2·63	3·71	1·12	0·10	3·53	2·02

TABLE XXII.

Comparison of Tibetan B and Chinese Mean Measurements.

	G'H	100 *G'H/GB*	*J*	*L*	*OH*	*NH*	G_1	100 *NB/NH*	*Q'*
Southern Chinese ...	71·2	71·1	131·8	177·6	119·2	53·1	54·5*	51·0*	321·2
Northern Chinese ...	72·5	72·0	133·0	177·9	118·1	54·5	50·8	46·6	317·6
Koganei's Chinese...	75·2	75·2	135·6	180·1	117·0	55·5	48·2	45·2	325·2
Tibetan B	76·5	76·0	137·5	185·5	115·5	55·0	51·5	49·4	312·9

* The Southern Chinese G_1 is based on only four skulls and the 100 *NB/NH* on eight. All the other means are more reliable, but those of the Tibetan B are less trustworthy than the others.

An examination of the Coefficient of Racial Likeness (see Table XXIII) at once disposes of the idea that there is any close relationship between Koganei's Chinese and the Tibetan B. The most that can be said is that a comparison of the available data suggests that the Tibetan B stock bears the same degree of relationship to the supposed pure Northern and Southern Chinese peoples and that in all probability the Chinese population of which a sample was investigated by Koganei inclines more towards the Khams Tibetans. It would be rash to say that Koganei's means represent a hybrid population, for the inclusion of only a few Tibetans in a series of pure Chinese could have modified the figures to the observed extent. We are merely told that the skulls were all procured in the provinces of North-East China and it may well be that the collection was not a particularly homogeneous one.

TABLE XXIII.

Coefficients of Racial Likeness between Tibetan B and Chinese Series.

		Southern Chinese (52·5)	Northern Chinese (36·4)	Koganei's Northern Chinese (69·4)
C. L. with Tibetan B (14·0)	All Characters ...	8·39 ± ·09	6·32 ± ·09	6·61 ± ·10
	Indices and Angles	4·65 ± ·14	6·58 ± ·14	9 52 ± 17

Koreans. Measurements of Korean skulls were found in the following publications:

(i) Koganei: *Zeitschrift für Ethnologie*, Bd. XXXVIII. (1906) p. 513. Means of 12 ♂ and 5 ♀ skulls are provided and also the individual measurements of the indices, profile angle and capacity, but not of the other characters.

(ii) Koganei: *Mittheilungen aus der medicinischen Facultät der Kaiserlich-Japanischen Universität*, Bd. I. p. 226, giving individual measurements of 3 ♂ and 1 ♀ specimen.

(iii) *Archiv für Anthropologie*, Bd. XIV. p. 279 (Referate). A few measurements of six skulls are provided but they are not sexed. The first three may be safely supposed ♂ and the remainder ♀.

(iv) Virchow: *Zeitschrift für Ethnologie*, Bd. XXXI. 1899, p. 751. 2 ♂ skulls.

TABLE XXIV.

Coefficients of Racial Likeness with the Koreans.

		Northern Chinese (36·4)	Koganei's Northern Chinese (69·4)	Southern Chinese (52·5)	Malays (74·6)
C. L. with Koreans (15·7)	All Characters ...	2·62 ± ·09	2·53 ± ·10	2·63 ± ·09	3·85 ± ·10
	Indices and Angles	3·43 ± ·16	4·69 ± ·17	2·37 ± ·15	4·33 ± ·17

The above Coefficients of Racial Likeness were found between the pooled Korean means (see Tables IX and XIX) and those of neighbouring Chinese races and the Malays.

The Koreans appear to be almost equally related to the three Chinese types, though standing somewhat closer to the Koganei series, perhaps, than to the other two, since the former contains the largest number of skulls; all the relationships are quite intimate ones. The Malays are further removed but are still very similar to the Koreans. All the values of α greater than ten that were found between the Koreans and the other four races are shown in the following table:

TABLE XXV.

Values of α greater than ten between the Korean and other Races.

		B	$100\,B/L$	J	Q'	NH	$100\,NB/NH$
Koreans (15·7) and	Koganei Chinese (69·4) ...	5·76	19·63	1·99	2·33	0·57	7·35
	Northern Chinese (36·4) .	12·19	16·45	10·29	15·80	0·20	1·40
	Southern Chinese (52·5) .	12·09	13·25	18·63	7·63	4·91	3·35
	Malays (74·6)	1·00	1·10	10·44	14·15	14·62	18·18

Practically the only significant differences between the direct measurements of the Chinese types and those of the Koreans are for those that relate to the breadth of calvaria and face; the length and height characters are all very similar. The Korean skull is broader than the Chinese, but in other respects the two types are almost identical. The Malayan has a calvarial breadth equal to the Korean, but it is less in zygomatic breadth and for the transverse circumference and nasal and facial heights. The nasal indices too are very different, the low Korean value distinguishing that race from all the Southern Mongols but not from the Northern Chinese. The actual mean measurements are:

Race	B	$100\,B/L$	J	Q'	$G'H$	NH	$100\,NB/NH$
Koreans	143·4	81·0	137·5	329·5	73·0	54·9	48·0
Northern Chinese (Koganei)	140·5	78·0	135·6	325·2	75·2	55·5	45·2
Northern Chinese	138·8	78·1	133·0	317·6	72·5	54·5	46·6
Southern Chinese	139·4	78·6	131·8	321·2	71·2	53·1	51·0
Malays	142·2	81·7	133·2	319·0	70·1	51·9	52·7
Burmese A	143·7	82·9	134·0	325·9	71·4	53·4	52·7

The Burmese A have a calvarial breadth equal to the Korean, but still the zygomatic breadth and transverse arcs are quite distinctly less. The very high values of the Korean Q' and J are somewhat discordant, but it would not be wise to lay stress on their uniqueness until more ample data shall have shown whether they be correct or not. Several cranial characters are peculiarly constant for all the races previously considered with the exception of the Andamanese: the Koreans do not differ significantly for those characters from the Oriental peoples.

Aino. We are able to use for comparative purposes the means of 88 ♂ and 55 ♀ Aino skulls which were reduced from measurements furnished by Koganei ("Beiträge zur physischen Anthropologie der Aino," *Mittheilungen aus der medicinischen Facultät der Kaiserlich-Japanischen Universität*, Tokio, Bd. II. 1894) and published in *Biometrika* (Vol. I. p. 426).

Japanese.

(i) Flower: *Catalogue of the Specimens...in the Museum of the Royal College of Surgeons*, Part I. 1907, pp. 226–8. Individual measurements of 7 ♂ and 2 ♀ skulls.

(ii) Barnard Davis: *Thesaurus Craniorum*, p. 182. 1 ♂.

(iii) Adachi: "Die Orbita und die Hauptmasse des Schädels der Japaner und die Methode der Orbitalmessung," *Zeitschrift für Morphologie und Anthropologie*, Bd. VII. (1904). Mean measurements of 49 ♂ and 45 ♀ specimens are provided on p. 403 and individual measurements in Tables I and II at the end of the paper. The writer was especially concerned with orbital measurements and he only gives a few of the many others usually taken by German craniologists.

(iv) Quatrefages and Hamy: *Les Crânes des Races Humaines*, p. 430. Means of 6 ♂ skulls.

(v) Toldt: *Archiv für Anthropologie*, Bd. XXVIII. 1903. Individual and mean measurements of 8 ♂ and 2 ♀ Japanese.

(vi) The German Anthropological Catalogues contain measurements of several Japanese skulls in the following sections: Berlin, 4 ♂, München, 1 ♂, Heidelberg, 2 ♂, Strassburg, 3 ♂ and 2 ♀, and Darmstadt, 1 ♀ (?).

The Aino of Japan possess physical characteristics, and cranial traits in particular, which obviously distinguish them from the predominant "Mongolian" types of North-East Asia. Their cephalic index (76·5) is distinctly less than the Chinese, Malayan and Korean values, and the Aino calvarial length (185·8) is not equalled by that of any of the mainland races with the exception of the Tibetan B ($L = 185{\cdot}5$). But a closer comparison shows at once that there can be no affinity between the Khams Tibetans with a facial height ($G'H$) of 76·5 and the Aino ($G'H = 69{\cdot}8$). The Coefficient of Racial Likeness between the two types is $13{\cdot}61 \pm {\cdot}10$ for twenty-three characters and $11{\cdot}94 \pm {\cdot}17$ for eight indices and angles. The only Asiatic race which is generally supposed to have assimilated Aino characters in any large proportion is the Japanese, a race which is said to have resulted from the crossing of Malayan and Aino strains and to be predominantly "Mongolian" in type. The Coefficients of Racial Likeness deduced from the Aino, Japanese, Korean and Chinese series of means are shown in Table XXVI.

The several Coefficients have been collected together in one table to facilitate comparison, but it is not for a moment suggested that their values should be compared without paying due regard to the numbers of crania on which the racial series are based. Bearing in mind that condition, we may conclude that the

TABLE XXVI.

Coefficients of Racial Likeness between Japanese and Other Races.

		Aino (79·3)	Japanese (39·2)	Southern Chinese (52·5)	Northern Chinese (36·4)	Koreans (15·7)
Japanese (39·2)	All Characters ...	8·08 ± ·09	—	2·32 ± ·09	3·69 ± ·09	2·11 ± ·09
	Indices and Angles	5·96 ± ·15	—	1·58 ± ·14	4·70 ± ·14	2·91 ± ·15
Aino (79·3)	All Characters ...	—	8·08 ± ·09	17·45 ± ·10	16·14 ± ·09	8·86 ± ·10
	Indices and Angles	—	5·96 ± ·15	8·55 ± ·16	11·78 ± ·15	10·77 ± ·16

Japanese are more closely related to both Chinese types than to the Koreans and that they resemble the Southern more than the Northern Chinese. At the same time the Japanese show a much closer bond of affinity with the mainland peoples than with the island aborigines. With the Malayan (74·6) means the Japanese show Coefficients of 7·23 ± ·09 for twenty-nine characters and 11·15 ± ·15 for ten indices and angles, those values indicating a somewhat closer resemblance than that between the Japanese and Aino types, but not such a close bond as that between the former race and the Southern Chinese. If the Japanese data under consideration be supposed to represent a random sample of the non-aboriginal inhabitants of Japan, and we have no reason to suppose that they do not, then it is highly probable that that people came originally from China and intermixed with the Aino. Their relationship with the Malays is not direct but must be traced through the Chinese. This view is not the orthodox one, but the evidence in favour of it appears to be fairly conclusive unless our means only represent a selected sample of the Japanese population. The Coefficients with the Aino are all of a higher order than the corresponding ones with the Japanese, and yet the Aino resemble the Chinese and Koreans as much as any "Mongolian" race. The following Coefficients, and others that have been given above (see Table XVII), may be compared with those of Table XXVI.

		Hindu (25·9)	Buriats (18·2)*	Altai-Telenghites (54·7)*
C. L. with Aino (79·3)	All Characters ...	31·58 ± ·09	27·05 ± ·10	70·62 ± ·10
	Indices and Angles ...	4·90 ± ·15	43·54 ± ·16	133·16 ± ·17

The Japanese race is without doubt that which is most closely allied to the Aino, and the Chinese stand next in the scale of relationship. The Korean means, though showing a Coefficient which is hardly significantly greater than the Japanese, are based on so few crania that they must be supposed to stand no closer to the Aino than the Chinese do. If the Aino inhabited Japan before the advent of the Mongols then, in view of the observed relations between the races, it is probable

* A description is given below of the Buriat and Telenghite series with which the Aino are here compared.

that the ancestors of the so-called pure aborigines were modified slightly in physical type by admixture with the invaders. Otherwise it is impossible to conceive why the Ainos should be more closely related to the Chinese than to any other Asiatic race unless that relationship is said to be merely the result of chance. In Table XXVII on the following page are given all the significant values of α found between the Japanese and Aino and the other races of the Far East. Of the thirty-one characters from which the Coefficients of Racial Likeness are calculated, seven will not be found in that table. The Occipital Index was not provided for the Japanese and Aino; the foraminal measurements—length, breadth and index*—were not considered for the Aino and they had no values of α greater than five with the Japanese; the auricular height (OH), sagittal circumference (S) and nasal breadth (NB) showed no values of α greater than five between the Japanese and Aino and other races. The Aino length (L) is very significantly greater than the Japanese, Chinese and Korean lengths, and it is somewhat surprising to find that there are no corresponding significant differences between the sagittal circumferences. The Tibetan B length exceeds the Aino by only 0·3 mm., but the arc S of the former is 378·6 mm. and of the latter 372·8 mm. The arcs S_1, S_2 and S_3 are not provided for the Aino, so we cannot judge wherein the difference in construction lies.

In addition to the seven characters not included in Table XXVII, the Japanese have insignificant differences from the Chinese, Koreans and Malays for B', U and $P\angle$ and no marked differences for $100\,H/L$, $100\,G'H/GB$, B, LB, Q', $G'H$, G_1, G_2, $A\angle$ and $100\,G_2/G_1$. The Japanese capacity differs only from the Malayan, but the cephalic index is widely removed from those of both Korean and Malayan types, and it may be remembered that the two latter diverged more from the Chinese for that character than for any other. High values of α are found with only one race in the case of J and $N\angle$, but the Japanese are more differentiated by $100\,B/H$ and $100\,NB/NH$ and still more by L, NH and the three orbital measurements. That there should be any well-marked divergences between the skull lengths is sufficient indication of the fact that the island race is not a pure member of the Oriental family but an alien race. From a comparison of seven races of South Asia, including the Southern Chinese and Malays, it has been shown that the differences between fifteen of the thirty-one characters from which the Coefficients of Racial Likeness are calculated were invariably insignificant or very nearly so. Several of those fifteen characters show as high values of α with the Aino as any recorded, indicating most profound differences of type. This is particularly true of L, LB and $N\angle$, but some of the values of α shown for $100\,G'H/GB$, $100\,NB/NH$, $100\,G_2/G_1$ and B' are also very high. The orbital index, profile angle and $A\angle$ of the Aino appear to be more like those of the mainland peoples. Of the remaining sixteen characters $100\,B/H$, $100\,H/L$, C, G_2 and B are much the same for the Aino and other races; the great differences shown between the cephalic indices and horizontal circumferences are evidently due primarily to the differences in length; the measurements Q', $G'H$, J, O_1, O_2 and G_1 show significant but rather erratic differences. The Aino nasal height is similar to the Japanese but very unlike the

* The Aino ♂ foraminal measurements are given in Table XIX and the ♀ in Table IX.

TABLE XXVII.

Values of α between Aino and Japanese and Other Oriental Races.

		100 B/L	100 H/L	100 B/H	100 $G'H/GB$	100 NB/NH	100 O_2/O_1	100 G_2/G_1	L	B	B'	LB
Japanese (39·2)	Koreans (15·7)	17·26	6·83	0·25	0·17	6·68	—	0·12	4·85	3·38	3·54	0·62
	N. Chinese (36·4)	0·14	1·40	0·17	0·45	21·75	5·44	10·75	7·62	7·22	3·40	5·21
	S. Chinese (52·5)	0·47	0·46	10·61	7·28	0·04	8·14	0·06	9·14	6·71	0·75	7·46
	Malays (74·6)	58·89	0·34	28·31	4·95	6·81	16·08	4·45	35·00	1·69	1·40	7·27
Aino (79·3)	Japanese (39·2)	19·14	6·13	8·74	21·72	0·00	6·60	3·12	35·77	0·00	2·54	16·01
	N. Chinese (36·4)	10·85	3·45	0·10	12·85	23·09	0·04	5·38	57·49	7·67	23·87	62·11
	S. Chinese (52·5)	27·69	6·36	5·63	6·86	0·44	—	3·26	96·85	7·26	14·49	94·68
	Koreans (15·7)	50·39	0·66	2·51	11·89	6·96	—	10·98	37·73	3·47	13·79	16·43

		Q'	U	$G'H$	J	NH	O_1	O_2	G_1	G_2	$P\angle$	$N\angle$	$A\angle$	C
Japanese (39·2)	Koreans (15·7)	6·03	0·33	0·44	4·05	18·14	—	1·53	0·45	1·44	0·71	2·72	0·38	0·18
	N. Chinese (36·4)	0·99	1·62	0·11	4·14	19·94	10·50	5·29	0·83	0·28	0·09	16·33	5·88	4·63
	S. Chinese (52·5)	0·04	0·02	1·50	14·32	8·72	3·27	9·36	6·20	0·36	0·00	0·56	0·00	1·28
	Malays (74·6)	0·31	1·05	8·03	4·35	0·65	8·77	17·63	0·22	5·82	—	0·00	0·64	17·58
Aino (79·3)	Japanese (39·2)	7·87	21·03	10·51	10·60	4·10	30·12	0·00	10·89	0·07	0·44	2·91	0·80	4·77
	N. Chinese (36·4)	36·11	51·05	10·78	24·42	37·75	4·00	5·90	8·66	1·39	7·01	58·20	6·80	0·10
	S. Chinese (52·5)	14·52	43·43	3·28	56·61	25·40	—	10·65	0·77	3·45	10·86	21·98	2·89	0·97
	Koreans (15·7)	0·14	11·16	7·40	0·02	31·76	—	1·56	7·19	6·49	1·73	20·79	0·04	0·08

Chinese and Korean values. Having thus obtained an estimate of the degree of the significances of the differences between the Aino and Japanese characters and those of other related races it will be well to glance at the actual mean measurements. Some of these are tabulated below and they are arranged roughly from left to right in order of the significance of the differences shown between the Aino and other types.

Race	*L*	*LB*	*N* ∠	100 *B*/*L*	*NH*	*U*	*B'*	*GL*
Aino ...	185·8	105·4	70°·2	76·5	50·5	522·5	96·2	104·9
Japanese ...	180·3	101·9	68°·2	78·3	51·5	508·5	94·6	100·2
Korean ...	177·1	100·9	65°·9	81·0	54·9	511·1	92·0	97·0
N. Chinese ...	177·9	99·7	62°·3	78·1	54·5	504·6	92·6	91·8
S. Chinese ...	177·6	99·5	67°·3	78·6	53·1	509·3	93·7	97·7
Malayan ...	174·7	99·5	68°·2	81·7	51·9	505·7	93·4	98·2

For nearly every character shown the Japanese measurement is intermediate in value between that of the Aino and those of the other races. The last measurement (*GL*) is not one used when computing Coefficients of Racial Likeness, but it apparently differentiates the Aino types as well as the other measurements. The aborigines of Japan are particularly characterised by their large nasio-basial lengths and forehead breadths.

An attempt was made to collect from various sources the skull measurements of the principal races of man inhabiting Mongolia, Chinese Turkestan and South Siberia, but many of the series cannot be dealt with here since the total numbers of skulls were in those cases too small to give any statistically reliable means. Really adequate numbers were found for only two races—the Kalmucks and Telenghites—but some of the less satisfactory material has been included. The bulk of the data was abstracted from Reicher's paper to which reference has already been made (see p. 56). In passing from the thickly inhabited plains of China to the deserts of Central Asia the ethnographic conditions become radically changed. The latter regions are as sparsely populated as any in the world, and under such circumstances a single race of man may contain very many tribes or divisions scattered over a wide area, and those tribes may be culturally more or less distinct.

It was unfortunately not possible to discover the measurements of more than five or six skulls of the Tunguses of Manchuria and Far Eastern Siberia, and any adequate data relating to Manchus proper were also lacking. The Buriats to the West have been rather more fully dealt with and we are able to give means of thirty-one skulls in the case of a few measurements. The majority of the specimens were procured in the Lake Baikal region.

Buriats.

(i) Reicher: "Untersuchungen über die Schädelform der alpenländischen und mongolischen Brachycephalen," *Zeitschrift für Morphologie und Anthropo-*

logie, Bd. xv. 1913, p. 421. Individual measurements of 15 ♂ Buriats are provided in Tables 3*a*, 3*b* and 3*c*; there are no ♀ skulls.

(ii) Fridolin: *Archiv für Anthropologie*, Bd. xxvii. p. 304. 7 ♂ and 3 ♀.

(iii) Measurements of Buriat skulls are also to be found in the Berlin (4 ♂ and 1 ♀?), Göttingen (1 ♂) and Freiburg (1 ♂) sections of the German Anthropological Catalogue (*Die Anthropologischen Sammlungen Deutschlands*).

(iv) *Archiv für Anthropologie*, Bd. x. p. 438 (Referate). 3 ♂?.

The mean collated measurements of all these skulls will be found in Table XXVIII. The Koreans are the only neighbouring people with which we are as yet able to compare them. Though immediately to the North of the two Tibetan types they are separated from them by the vast uninhabited Desert of Gobi and it is not surprising to find no suggestion of relationship. Between the Buriats (18·2) and Koreans (15·7) there are Coefficients of Racial Likeness of $7{\cdot}56 \pm {\cdot}09$ for twenty-six characters and $12{\cdot}38 \pm {\cdot}16$ for nine indices and angles. The values of α greater than five are in order of significance $100\,B/H'$ ($\alpha = 63{\cdot}27$), B ($\alpha = 36{\cdot}01$), $100\,B/L$ ($\alpha = 19{\cdot}00$), $100\,H'/L$, U, G_2, B', Q', *fml* and NB ($\alpha = 5{\cdot}75$): the majority of these high values are due to the difference in breadth characters. The Buriat skull has a much greater breadth (151·6) than the Korean (143·4) and also a lesser height, so there are great differences between the indices $100\,B/H'$. The calvarial lengths are much the same and so also are all the other characters, with the exception of the forehead breadth, which was found to be reasonably constant for the Oriental peoples. The Buriat B' is equal to the Aino value and hence significantly greater than the Chinese.

Mongol-Torgods. Individual measurements of 13 ♂ Mongol-Torgods are given by Reicher (*loc. cit.* Tables 5*a*, 5*b* and 5*c*). The people of this nomadic race, which is said to number no more than 25,000 individuals*, are found at the present day principally in West Mongolia. But it is supposed that the Torgods had the same origin as the modern Kalmucks and that they left the Volga region in 1771, and participated in the great migration of that year. Reicher very properly keeps the measurements of the Torgods quite distinct from those of the Astrakhan Kalmucks. The former are not sufficiently like the means of other tribes of Eastern Kalmucks —see a comparison below, pp. 69 and 70—to warrant the supposition that they are representative of precisely the same population.

Altai-Telenghites. Measurements of 62 ♂ and 29 ♀ skulls of this little known tribe are given by Reicher (*loc. cit.* Tables 2*a*, 2*b* and 2*c*). Like the Torgods they are said to be of Kalmuck origin, but it will be shown that they are distinctly different from the present-day people of that race inhabiting the Astrakhan region.

Kalmucks. The term "Kalmuck" is frequently used by anthropologists in such a loose manner that it includes several divisions of the human species which are at present quite racially distinct, though the primary origin of all may have been

* See Reicher, *loc. cit.* pp. 466, 467, and Deniker, *Revue d'Anthropologie*, 1883, pp. 691 *et seq.*

TABLE XXVIII.

Mean Measurements of Southern Siberian Races.*

Character	Male				Female		
	Altai-Telenghites	Mongol-Torgods	Buriats	Astrakhan Kalmucks	Altai-Telenghites	Buriats	Astrakhan Kalmucks
Capacity	1462·0 (45)	1489·0 (9)	1484·0 (15)	1498·5 (35)	1269·0 (10)	1530·0 (1)	1319·6 (7)
L	175·8 (62)	179·0 (13)	179·3 (31)	182·8 (65)	165·8 (16)	174·5 (4)	173·7 (28)
B	151·5 (59)	150·8 (12)	151·6 (31)	149·6 (65)	144·6 (15)	142·5 (4)	144·4 (26)
B'	94·6 (52)	93·7 (13)	96·0 (19)	95·9 (38)	93·8 (16)	—	93·1 (9)
LB	99·8 (62)	100·2 (13)	101·4 (25)	100·6 (35)	93·4 (16)	96·3 (6)	95·8 (13)
Q	320·1 (61)	315·7 (13)	321·7 (25)	312·6 (35)	308·0 (16)	312·3 (3)	300·7 (13)
S	357·1 (62)	356·3 (12)	362·7 (29)	363·3 (44)	343·7 (15)	357·3 (4)	346·3 (13)
S_1	126·4 (62)	125·0 (13)	127·5 (24)	127·1 (32)	122·0 (16)	125·0 (1)	120·4 (8)
S_2	118·6 (61)	121·5 (13)	122·8 (22)	119·8 (32)	114·4 (15)	115·0 (1)	112·6 (8)
S_3	112·8 (60)	110·9 (12)	113·6 (23)	116·2 (32)	106·9 (15)	122·0 (1)	111·3 (9)
S_3'	92·9 (60)	92·1 (12)	93·5 (14)	94·8 (19)	89·9 (15)	—	92·3 (6)
H	—	—	132·7 (12)	129·0 (15)	—	131·5 (4)	132·0 (2)
H'	129·8 (61)	125·5 (13)	131·5 (15)	129·7 (50)	123·9 (16)	—	122·2 (25)
OH	114·4 (62)	112·7 (13)	116·9 (15)	113·9 (23)	110·2 (16)	—	108·8 (8)
U	519·3 (62)	522·3 (13)	526·9 (27)	528·7 (62)	493·5 (16)	502·3 (3)	502·5 (26)
J	140·1 (51)	142·1 (18)	140·4 (21)	141·6 (60)	131·4 (15)	128·3 (3)	130·2 (25)
GB	100·7 (58)	101·6 (10)	102·6 (16)	103·1 (31)	95·5 (16)	—	96·2 (12)
GL	98·7 (44)	98·8 (11)	100·9 (17)	98·9 (29)	92·4 (10)	—	91·4 (10)
G'H	71·3 (44)	73·0 (11)	76·1 (17)	74·8 (38)	67·0 (10)	69·3 (3)	67·5 (12)
O_1	41·9 (62)	42·5 (12)	42·6 (18)	41·7 (23)	41·3 (16)	—	39·9 (8)
O_2	34·1 (61)	35·0 (13)	35·9 (18)	35·5 (53)	34·0 (16)	—	34·2 (27)
DC	—	—	27·0 (3)	22·5 (15)	—	26·5 (2)	20·7 (14)
SC	8·1 (60)	7·2 (13)	7·6 (15)	8·5 (18)	8·4 (15)	—	6·4 (5)
NH	—	—	53·6 (7)	56·6 (16)	—	49·7 (3)	52·1 (7)
NH'	52·4 (61)	53·2 (12)	55·3 (15)	54·4 (36)	49·6 (15)	—	50·6 (19)
NB	26·8 (60)	26·9 (12)	27·7 (20)	26·6 (62)	25·2 (16)	26·7 (3)	24·9 (26)
fmb	30·1 (59)	30·9 (12)	31·4 (15)	30·8 (19)	28·9 (15)	—	29·0 (6)
fml	36·6 (58)	36·4 (12)	38·1 (15)	37·5 (19)	35·5 (15)	—	36·7 (6)
G_1'	46·8 (40)	45·0 (10)	46·7 (9)	46·4 (18)	43·6 (9)	—	39·7 (6)
G_2	41·4 (38)	42·9 (10)	43·3 (8)	42·9 (15)	38·7 (7)	—	37·9 (8)
100 *B*/*L*	86·4 (60)	84·6 (12)	84·2 (31)	{81·8 (65)}	87·3 (15)	81·7 (4)	{83·1 (26)}
100 *H*/*L*	—	—	75·3 (13)	{70·6 (15)}	—	75·5 (4)	74·6 (2)
100 *H'*/*L*	73·7 (61)	70·0 (13)	72·6 (15)	{71·0 (40)}	74·9 (16)	—	{70·4 (25)}
100 *B*/*H*	—	—	{114·2 (12)}	{116·0 (15)}	—	108·4 (4)	{109·1 (2)}
100 *B*/*H'*	{116·7 (59)}	{120·1 (12)}	{115·3 (15)}	{115·3 (40)}	{116·7 (15)}	—	{118·2 (25)}
100 (*B*−*H'*)/*L*	{12·3 (59)}	{14·1 (12)}	{11·2 (15)}	{10·9 (50)}	{12·5 (15)}	—	{12·8 (25)}
100 *G'H*/*GB*	{68·8 (44)}	{71·8 (10)}	{74·5 (15)}	{72·5 (31)}	{70·2 (10)}	71·2 (3)	{70·2 (12)}
100 *NB*/*NH*	—	—	51·8 (7)	{47·0 (16)}	—	53·8 (3)	{47·8 (7)}
100 *NB*/*NH'*	51·4 (60)	50·7 (12)	49·9 (15)	{48·9 (36)}	49·7 (15)	—	{49·2 (19)}
100 O_2/O_1	81·6 (61)	81·9 (12)	84·3 (18)	{85·2 (23)}	82·4 (16)	—	{85·7 (8)}
100 *fmb*/*fml*	82·4 (58)	85·5 (12)	80·7 (15)	82·1 (19)	81·5 (15)	—	79·2 (6)
100 G_2/G_1'	88·6 (38)	95·9 (9)	92·1 (8)	92·5 (15)	90·0 (7)	—	95·3 (6)
P ∠	85°·1 (36)	86°·7 (10)	87°·4 (11)	85°·6 (19)	84°·2 (9)	—	86°·1 (7)
N ∠	{68°·0 (44)}	{67°·3 (11)}	{67°·3 (17)}	{66°·8 (29)}	{68°·1 (10)}	—	{65°·2 (10)}
A ∠	{70°·2 (44)}	{69°·8 (11)}	{68°·4 (17)}	{69°·4 (29)}	{70°·2 (10)}	—	{72°·4 (10)}
B ∠	{41°·8 (44)}	{42°·9 (11)}	{44°·3 (17)}	{43°·8 (29)}	{41°·7 (10)}	—	{42°·4 (10)}
θ_1	{26°·9 (36)}	{26°·0 (10)}	{25°·3 (11)}	{27°·6 (15)}	{27°·7 (9)}	—	{28°·7 (7)}
θ_2	{14°·9 (36)}	{16°·9 (10)}	{19°·0 (11)}	{16°·2 (15)}	{14°·0 (9)}	—	{13°·7 (7)}
Oc. I.	{59·0 (60)}	{59·7 (12)}	{60·0 (14)}	{58·9 (19)}	{60·9 (15)}	—	{59·6 (6)}

* The indices and angles in curled brackets were calculated from the means of the component lengths instead of from the measurements of the individual skulls.

the same. A Kalmuck skull of which a description is given but not the locality in which it was procured may belong to the people settled in the Volga Valley or to the Nomads of North-Eastern Tibet over 3,000 miles away. For these reasons it was thought advisable to separate the measurements of Kalmuck skulls collected from various sources into three groups, according as the specimens were said to have been procured in the Astrakhan region, or in the Altai region, or in an unknown locality. Data were found in the following publications:

(i) Reicher: *loc. cit.* Tables 4*a*, 4*b* and 4*c*. 19 ♂ and 6 ♀ Astrakhan Kalmucks.

(ii) *Die Anthropologischen Sammlungen Deutschlands.* Göttingen section, S. 40. 6 ♂ skulls from Astrakhan region; München, p. 94, 4 ♂ and 2 ♀, locality not given; Berlin, Erster Theil, p. 26, 1 ♂ and 1 ♀ from Sarepta and 2 ♂? from an unknown district.

(iii) Fridolin: *Archiv für Anthropologie*, Bd. XXVII. p. 304. There are 9 ♂ skulls from Manytch and Sarepta, 3 ♂ from the Siberian province of Tomsk and 3 others from an unknown locality. The majority of the 6 ♀ skulls had been procured in the Astrakhan region.

(iv) Deniker: *Revue d'Anthropologie*, 1884, pp. 297 and 298. This paper is furnished with tables of the individual measurements of ♂ and ♀ Kalmuck skulls collected from various sources and of a few described by the writer.

Excluding the specimens which are also to be found in the German catalogues, there are 11 ♂ skulls from the Astrakhan region, 2 ♂ from the Government of Tomsk and 10 ♂ from an unknown locality, and we are also able to make use of the measurements of 17 ♀ crania. The numbers are thus quite considerable, but unfortunately very few of our measurements are given by Deniker.

Dividing the whole material into three groups in the way described, there are for some characters as many as forty-six skulls representing the European type of Kalmuck, five of the Eastern horde—all from the Government of Tomsk—and nineteen which were procured in unknown parts. It is hardly likely that all the skulls in this latter group belong to a single tribe or division of the Kalmuck race, but it is quite probable, at least, that the majority were found in the Astrakhan region. Specimens from other parts are much rarer and are more likely to be assigned to some particular tribe. Between the Astrakhan Kalmucks (28·4) and those from unknown localities (10·0) there is a Coefficient of Racial Likeness of $0{\cdot}72 \pm {\cdot}10$ for twenty-three characters and $-0{\cdot}39 \pm {\cdot}16$ for nine indices and angles. This first value is slightly higher than we should anticipate if two perfectly random samples from the same population were being dealt with. The only values of α found greater than five were for C ($\alpha = 14{\cdot}00$), J ($\alpha = 5{\cdot}80$) and NH ($\alpha = 5{\cdot}32$), and in the case of the first of these at least it is more than probable that the discrepancy is due merely to differences between the methods of measurement employed. It was thought that the two series resembled one another sufficiently closely to warrant pooling, and accordingly the means were combined and the resulting type may legitimately be called that of the Astrakhan Kalmucks. The measurements of the Tomsk Kalmucks were so far unlike those of the Western group that it was thought

advisable to keep them separate and no attempt can be made to determine the relationships of the former series since it is far too short to be used for comparative purposes. The chief difference between the types was apparently for the calvarial breadths as the following figures show:

Race	*L*	*B*	100 *B*/*L*	*J*
Tomsk Kalmucks ...	182·2 (5)	155·4 (5)	85·3 (5)	144·6 (5)
Astrakhan Kalmucks ...	182·8 (65)	149·6 (65)	81·8 (65)	141·6 (60)

The Coefficients of Racial Likeness between the material described above relating to the four Siberian races are given in the following table:

TABLE XXIX.

Coefficients of Racial Likeness between South Siberian Races.

Race		Telenghites (54·7)	Torgods (11·6)	Buriats (18·2)	Kalmucks (35·5)
Telenghites (54·7)	All Characters ...	—	1·23 ± ·09	3·09 ± ·09	8·74 ± ·09
	Indices and Angles	—	2·82 ± ·14	3·23 ± ·14	12·26 ± ·14
Torgods (11·6)	All Characters ...	1·23 ± ·09	—	0·94 ± ·09	0·89 ± ·09
	Indices and Angles	2·82 ± ·14	—	1·26 ± ·04	1·90 ± ·14
Buriats (18·2)	All Characters ...	3·09 ± ·09	0·94 ± ·09	—	1·44 ± ·09
	Indices and Angles	3·23 ± ·14	1·26 ± ·14	—	1·68 ± ·14
Kalmucks (35·5)	All Characters ...	8·74 ± ·09	0·89 ± ·09	1·44 ± ·09	—
	Indices and Angles	12·26 ± ·14	1·90 ± ·14	1·68 ± ·14	–

Any very detailed comparison of these races would hardly come within the scope of the present paper, nor would it be particularly profitable while data relating to the neighbouring people to the North are lacking. We are concerned rather with the general relations of the so-called "Central-Mongols" to the Oriental peoples to the South, but a brief survey of the inter-connections of the former will aid our more ulterior object. After making due allowance for the numbers of crania on which the several racial series are based, we may conclude from the Coefficients that the Torgods and Buriats are intimately related to the Astrakhan Kalmucks and to the Telenghites and that they occupy intermediate positions between those two extreme and dissimilar types. It is surprising that the Kalmucks and Buriats should show such a close affinity, in spite of the fact that the Volga and Lake Baikal regions are 3,000 miles apart. Turning to a comparison of the individual measurements we find that there are several, viz. *Oc. I*, 100 *fmb*/*fml*, *LB*, *P*∠, *N*∠, *A*∠, *B'*, *C*, *B*, *J*, O_1, G_1, G_3, *fml* and *fmb*, which show no values of α greater than five between the four types. Now the first eight of those characters were reasonably constant for all the Southern Asiatic peoples with the exception of the Andamanese,

Japanese and Aino, but the last seven, and particularly B and J, were found previously to differ most significantly so that their values were racially distinctive. The values of α for all those characters which have any values greater than five are given in Table XXX and an attempt has been made to arrange them from left to right in order of the significances of their differences.

As with the Oriental peoples, the most significant differences between the South Siberian types are for the cephalic index, but the causes of those differences in the two groups of races are precisely the inverse of each other. For the Southern group the calvarial lengths were all very similar, while the breadths varied greatly between race and race; the Northern races have almost identical breadths but the lengths differ most significantly. The high values of α between the indices $100\,H'/L$ are due almost entirely to the variation in length and only the Torgod index $100\,B/H'$ is distinguished from the others. It is clear that the differences in skull form between any two of these four races are of a nature peculiar to that group or family of races to which they belong and are entirely different from the relations observed between pairs of Oriental races. The Siberian peoples then do not belong to that Oriental group or series of races.

A more detailed discussion of the relations between the measurements of the "Central-Mongols" need hardly be made, but a direct comparison of a few of the mean characters is of interest and it will confirm the fact that the Buriat and Torgod types are intermediate between the Kalmuck and Telenghite.

Race	L	B	$100\,B/L$	U	$G'H$
Telenghites ...	175·8	151·5	86·4	519·3	71·3
Torgods ...	179·0	150·8	84·6	522·3	73·0
Buriats ...	179·3	151·6	84·2	526·9	76·1
Kalmucks ...	182·8	149·6	81·8	528·7	74·8

If there are any characters which will serve the purpose of differentiating all the Oriental peoples from all other races, then they will almost certainly be characters which are fairly constant in value for the several races comprised in that group. Table XXXI (p. 72) was compiled with the object of discovering such measurements if possible and to aid the examination of the relations of the Oriental series as a whole to other racial groups. The fifteen cranial characters dealt with are precisely those which were most constant for the Nepalese, Tibetan A, Southern Chinese, Annamese, Siamese, Malayan and Burmese A series. No means are shown which are based on fewer than ten crania and hence all Siamese measurements had to be excluded. The constancy of the measurements of the Oriental races can be readily appreciated by comparing the greatest and least values shown in rows 1 and 2 of Table XXXI. The differences are particularly small in the case of the profile angles, facial indices and calvarial and nasio-basial lengths. Very few Hindu and Dravidian measurements may be compared, since the majority are based on fewer than ten crania. The more reliable ones shown in the table do not in any

TABLE XXX.

Values of α between South Siberian Races.

		100 B/L	L	100 H'/L	100 B/H'	$G'H$	O_2	Q'	100 $G'H/GB$	U	NH	100 NB/NH	100 O_2/O_1	100 G_2/G_1	OH	S	NB
Telenghites (54·7) and	Torgods (11·6)	4·95	3·36	16·97	6·23	1·48	2·38	2·19	2·98	0·51	0·95	0·34	0·25	8·41	1·83	0·04	0·03
	Buriats (18·2)	15·00	7·75	1·69	1·27	16·43	12·34	0·48	14·98	5·95	1·14	1·85	3·98	1·76	4·44	3·95	3·89
	Kalmucks (35·5)	100·81	47·53	20·39	2·53	14·52	15·23	13·14	10·13	14·45	10·62	9·63	8·48	3·55	0·25	6·33	0·36
Torgods (11·6) and	Buriats (18·2)	0·21	0·03	5·45	12·12	3·73	1·68	3·24	1·78	0·98	3·13	0·29	0·10	1·33	7·23	2·22	1·53
	Kalmucks (35·5)	12·13	4·79	1·14	11·49	1·60	0 71	0·96	0·15	2·32	1·28	2·00	2·42	1·41	0·71	2·95	0·28
Buriats (18·2) and	Kalmucks (35·5)	18 45	7·87	3·23	0·00	1·15	0·59	12·98	1·65	0·32	0·44	3·86	0·32	0·02	4·80	0·04	5·59

TABLE XXXI. *Comparison between Oriental and other Races*.*

		$P\angle$	$N\angle$	$A\angle$	100 $G'H/GB$	100 O_2/O_1	100 NB/NH	100 G_2/G_1	100 fmb/fml	*Oc. I.*	*L*	*LB*	*B'*	*S*	G_1	*fml*
Oriental Races†	Greatest	86°·7	68°·2	72°·8	71·1	83·1	52·7	85·2	87·0	62·8	177·6	99·8	94·7	370·8	49·9	36·7
	Least	84°·9	65°·5	70°·1	69·5	80·0	48·2	79·8	82·4	60·4	173·5	96·9	90·9	361·2	48·5	34·7
	No. of Races ...	4	5	5	5	3	5	3	6	4	6	6	6	6	3	6
Indian Races	Hindu (Bengalee) ...	—	—	—	—	—	49·7	—	—	—	175·4	99·2	92·4	363·5	—	—
	Maravar	—	—	—	—	—	51·7	—	83·3	—	175·6	98·8	93·2	—	50·5	33·9
	Veddah	—	66°·8	76°·4	67·6	80·9	—	77·1	—	—	179·1	98·1	90·6	363·1	49·5	—
Siberian Races	Greatest	87°·4	68°·0	70°·2	74·5	85·2	48·9	—	85·5	60·0	182·8	101·4	96·0	363·3	—	38·1
	Least	85°·1	66°·8	68°·4	68·8	81·6	48·9	—	80·7	58·9	175·8	99·8	93·7	356·3	—	36·4
	No. of Races ...	4	4	4	4	4	1	—	4	4	4	4	4	4	—	4
Other Asiatic Races	N. Chinese	84°·6	62°·3	73°·8	72·0	85·0	46·6	77·2	84·0	63·4	177·9	99·7	92·6	370·3	50·8	35·6
	Koganei N. Chinese.	84°·0	—	—	75·2	—	45·2	81·8	83·5	—	180·1	100·4	91·9	373·2	48·2	36·3
	Koreans	84°·1	65°·9	71°·0	73·3	—	48·0	78·4	86·9	—	177·1	100·9	92·0	369·1	50·6	36·0
	Japanese	84°·9	—	—	72·7	88·7	50·7	—	83·2	—	180·3	101·9	94·6	370·5	49·8	36·4
	Amo	82°·0	70°·2	71°·2	68·4	85·3	50·7	72·1	84·6	—	185·8	105·4	96·2	372·8	53·0	35·7
	Tibetan B	85°·7	65°·5	68°·8	76·0	84·6	49·5	84·1	84·1	59·2	185·5	99·2	94·3	378·6	51·5	37·4
	Andamanese ...	—	—	—	—	—	—	—	82·9	—	168·5‡	94·0	92·7	351·4	—	33·5
African Races	Naqada Egyptians ...	84°·4	66°·6	72°·8	70·6	74·9	51·1	—	—	—	185·1	99·3	91·1	373·0	—	—
	Theban Mummies ...	85°·5	65°·4	73°·7	72·3	86·5	50·1	88·5	85·5	—	181·9	100·6	93·8	372·4	46·3	35·1
	Modern Egyptians ...	84°·2	66°·9	72°·8	72·1	85·2	49·4	85·0	83·0	—	177·0	101·0	94·1	366·0	47·1	35·0
	Gaboon Negroes ...	—	70°·5	70°·9	69·7	84·6	55·3	76·1	83·9	66·7	179·4	100·3	96·5	364·3	52·1	35·9
	Congo Negroes ...	82°·8	69°·3	72°·8	66·9	84·1	55·2	76·8	83·5	68·8	177·9	98·2	97·5	361·8	50·3	35·9
European Races	Würtemberger ...	86°·4	—	—	78·1	85·9	46·6	74·2	86·2	—	179·5	98·6	97·2	367·2	51·7	36·2
	Altbayerisch ...	89°·1	—	—	74·5	84·5	48·7	74·4	83·2	—	180·6	100·3	103·7	365·1	44·3	37·0
	Whitechapel English	86°·1	65°·2	73°·4	76·5	77·7	47·6	76·3	—	—	189·1	101·6	98·0	377·1	48·3	—
	Moorfields English ...	84°·5	66°·6	72°·4	72·8	78·5	47·6	82·7	84·2	—	189·2	98·5	98·5	378·5	48·1	35·4
Other Races	Eskimo	—	—	—	—	89·4	42·6	—	76·4	—	188·2	104·9	94·9	373·7	—	38·1
	Fuegians	83°·4	—	—	—	—	—	—	—	—	192·0	—	97·1	—	—	—
	Moriori	84°·7	64°·8	71°·8	74·1	84·0	43·9	73·2	87·4	62·2	186·9	105·6	95·3	368·8	53·1	35·3
	Maori	—	—	—	—	86·1	47·9	71·2	—	—	185·5	103·9	95·7	378·1	52·4	35·2

* The mean measurements of the races other than Asiatic have been taken from *Biometrika*, Vol. I. p. 426 (Naqada, Theban Mummies, Modern Egyptians and Altbayerisch); *Biometrika*, Vol. VIII. p. 298 (Gaboon and Congo Negroes); *Biometrika*, Vol. II. p. 208 (Würtemberger and Whitechapel English); *Biometrika*, Vol. V. p. 92 (Moorfields English) and *Biometrika*, Vol. XIV. p. 222 (Eskimo, Fuegians, Moriori and Maori).

† viz. Nepalese, Tibetan A, Southern Chinese, Annamese, Malayan and Burmese A.

‡ This Andamanese length is *F* not *L*.

way suggest that the Indian races are of a radically different type from the Oriental series. The nasal and foraminal indices and the characters L, LB, B' and S are the same for both groups and the Maravar G_1 and *fml* do not differ appreciably from the other racial values. The Veddah measurements, however, are rather more distinctive, especially $A\angle$, $100\,G'H/GB$, $100\,G_2/G_1$ and L, but still the majority of them are very similar to the lengths, indices and angles of the Oriental races. The four Siberian races shown in rows 7 and 8 are those which have been dealt with above, viz. the Telenghite, Torgod, Buriat and Kalmuck. The differences between the Oriental and Siberian types are very marked. For the latter group the orbits are rounder, the nasio-basional lengths are greater, and the curvatures of the occipital bones are less than for any Oriental race; also the foraminal lengths, forehead breadths and calvarial lengths tend to be greater for the Southern Siberians, but it is curious that this latter character should not be correlated with greater sagittal arcs. The Oriental peoples must have the rounder frontal and parietal bones as well as occipital. The foraminal indices and nasal and profile angles are much the same for the two groups of races, but the Southern people appear, in general, to have rounder faces and greater alveolar angles than the Northern. Extending the comparison so that all the races of the table are included, we may note, in the first place, that four characters—$100\,fmb/fml$, $N\angle$, $A\angle$ and *fml*—are practically the same for all the races. The majority of the means of the unallied races fall for those characters within the limits of the Oriental group and hence they would show in all probability hardly any significant differences from each other. The Oriental group is not absolutely distinguished from all the unallied races by any single character, but, at the same time, there is no single race which has the same or any very similar combination of characters as the Orientals. The group of races with which we are primarily concerned is more or less characterised by roundness of face and palate, large nasal but small orbital indices, large profile angles and small nasio-basional and calvarial lengths. The characters B', S and G_1 are in no way peculiar.

9. COMPARISON OF TYPE CONTOURS.

Drawings of the Transverse, Horizontal and Sagittal Contours were made with the Klaatsch contour tracer in the manner described by Benington* with the additions suggested by subsequent workers in the Biometric Laboratory. But the ink-holder of the instrument originally provided was replaced by a solid brass screw-on attachment, similar in construction but having a fine rounded point vertically below the pointer. The drawing paper was placed over a sheet of duplicating paper and fixed in position. Then after adjusting the skull on the stand, in the usual way, and placing it centrally on the paper, the pointer of the tracer was moved round the circumference, while maintaining a steady downward pressure on the brass attachment. After some preliminary practice a thin and clear impress could be obtained and the cranial "points" could be clearly marked. With this

* *Biometrika*, Vol. XI. p. 129.

method, the manipulation is far easier than it was with the ink-holder and consequently the tracings can be made more quickly.

(*a*) *The Transverse Vertical Section.* A detailed account of the method of constructing transverse vertical type contours from the measurements of the individual drawings will be found in *Biometrika*, Vol. XIV. p. 227 and the mean measurements employed are provided in Table XXXII. The races represented are:

(*a*) Nepalese ♂—46 crania (Fig. 1).

(*b*) Nepalese ♀—6 crania (Fig. 2).

(*c*) Tibetan A ♂. This type is based on 35 crania, 18 of which are in the Hodgson Collection and 17 at the Royal College of Surgeons* (Fig. 3).

(*d*) Hindu ♂—10 crania, 2 of these are in the Hodgson Collection and the other 8 are at the Biometric Laboratory, University College (Fig. 4).

(*e*) Hindu ♀—7 crania, all in the Biometric Laboratory† (Fig. 5).

Contours could also be drawn of two Dravidian ♂ skulls, one Dravidian ♀, three Tibetan A ♀ and two Tibetan A young specimens. The measurements of these are given in the following table, but, of course, no types were constructed from such meagre data. The right and left of the table must be interchanged to correspond with the diagrams. The latter show the *norma occipitalis*.

With the type contours constructed from the mean measurements shown in Table XXXII we are able to compare the six Burmese diagrams (3 ♂ and 3 ♀) corresponding to the A, B and C series to which reference has been made above (p. 32) and the type contour of the Khams Tibetans. Type contours similar to ours have been drawn for no other Asiatic races. In making such comparisons, it is necessary to have some measure of the deviations from the actual contour which might be found as the result of random sampling. This possible variation has been conveniently represented by plotting points on either side of each mean ordinate and at a distance from it of twice the probable error of the ordinate. The space between the lines drawn through those two series of points is termed the type zone. Benington‡ has provided the type zone in the case of 100 English male skulls; it has a total width of 1·4 mm. at the apex and tapers down gradually to a width of 0·8 mm. on nearing the auricular points.

Assuming that the standard deviations of the Asiatic series with which we are dealing are of the same order as the English—though actually they have rather higher standard deviations than European races—we may easily calculate the widths of the type zones of the former, since each will be inversely proportional to

* The mean transverse contour measurements of this latter group are given in *Biometrika*, Vol. XIV. p. 228, and the type is shown on p. 229.

† The mean direct measurements of the Hindu skulls in the Biometric Laboratory have been provided by Tildesley and are part of the Hindu data described above (p. 32). I have not included the contours of one very asymmetrical and probably distorted skull (H 2).

‡ R. Crewdson Benington, "Cranial Type Contours," *Biometrika*, Vol. VIII. pp. 123–201. The first biometric cranial type contours were shown in this paper, which was prepared for the press by Professor Karl Pearson. The English contours are those of the Whitechapel skulls (17th century), which had been previously measured by W. R. Macdonell and described in *Biometrika*, Vol. III.

TABLE XXXII.

Mean Values of Transverse Vertical Contours.

Race	Sex	No.	*MA*	*IR=IL*	*M¼R*	*M¼L*	*2R*	*2L*	*3R*	*3L*	*4R*	*4L*	*5R*	*5L*	*6R*	*6L*
Tibetan A	♂	35	113·1	59·2	61·6	61·4	63·0	63·0	65·4	65·6	67·4	67·5	67·3	67·6	65·8	66·5
Nepalese	♂	46	113·2	56·8	58·5	58·5	59·9	59·2	61·7	61·7	63·1	63·3	63·2	63·7	61·8	62·9
Hindu	♂	10	109·6	53·4	56·5	56·0	58·2	57·7	59·7	60·7	61·6	62·6	62·2	63·4	61·0	62·7
Dravidian	♂	2	109·7	54·9	57·9	56·5	59·0	56·2	58·6	58·1	59·3	61·0	59·6	61·3	58·2	61·4
Tibetan A	♀	3	109·3	56·0	58·6	58·1	60·5	59·1	63·5	62·1	66·9	63·1	67·1	63·1	65·4	61·7
Nepalese	♀	6	109·6	53·5	56·3	55·5	58·4	58·4	60·8	58·7	62·5	60·4	62·1	60·6	60·1	59·7
Hindu	♀	7	106·1	52·3	54·4	55·0	56·9	57·0	59·2	60·4	61·7	63·4	62·6	64·0	62·4	63·4
Dravidian	♀	1	100·3	52·3	55·5	54·5	57·7	55·5	60·8	58·7	62·5	60·2	61·9	60·0	58·7	56·5
Tibetan A Young (i) ...	♂	1	112·5	52·3	57·2	54·5	60·3	59·7	63·8	63·1	66·9	65·7	67·9	67·2	68·4	67·3
Tibetan A Young (ii) ...	♂	1	99·5	48·5	51·7	51·9	55·0	55·7	58·3	60·8	62·9	62·8	63·2	63·3	59·8	62·5

Race	Sex	No.	*7R*	*7L*	*8R*	*8L*	*9R*	*9L*	*10R*	*10L*	*A ¼R*	*A ¼L*	*ZR. R*		*ZR. L*	
													y	*x*	*y*	*x*
Tibetan A	♂	35	63·3	64·1	59·0	59·7	51·2	51·2	37·9	36·5	20·8	17·1	62·0	3·3	61·6	2·9
Nepalese	♂	46	59·4	61·1	55·8	57·0	48·9	49·1	36·8	35·6	19·9	17·3	58·8	3·3	58·6	2·9
Hindu	♂	10	59·0	61·0	55·3	56·6	47·4	48·1	33 3	34·8	17·9	16·9	57·0	2·9	56·5	2·7
Dravidian	♂	2	55·4	61·0	52·0	56·7	46·4	49·1	33·2	36·8	17·9	20·1	59·1	4·2	56·7	3·2
Tibetan A	♀	3	62·9	58·7	59·5	54·0	55·0	46·8	44·4	32·4	27·3	13·1	59·0	2·5	57·9	2·2
Nepalese	♀	6	57·4	57·7	53·6	53·2	47·2	44·8	35·1	32·0	20·8	15·2	56·8	3·0	55·4	2·4
Hindu	♀	7	60·8	61·7	57·6	57·8	50·5	50·1	36·2	36·5	16·9	20·7	55·6	4·0	55·2	3·5
Dravidian	♀	1	56·7	54·5	53·4	50·8	47·0	44·8	35·7	31·1	17·0	13·1	56·1	2·0	54·6	3·1
Tibetan A Young (i) ...	♂	1	67·0	64·9	62·2	60·0	55·6	52·9	45·3	37·6	29·8	11·8	56·4	1·8	54·9	3·0
Tibetan A Young (ii) ...	♂	1	54·9	60·6	50·6	56·8	44·5	49·3	34·1	37·6	16·0	20·9	51·7	2·6	51·7	1·9

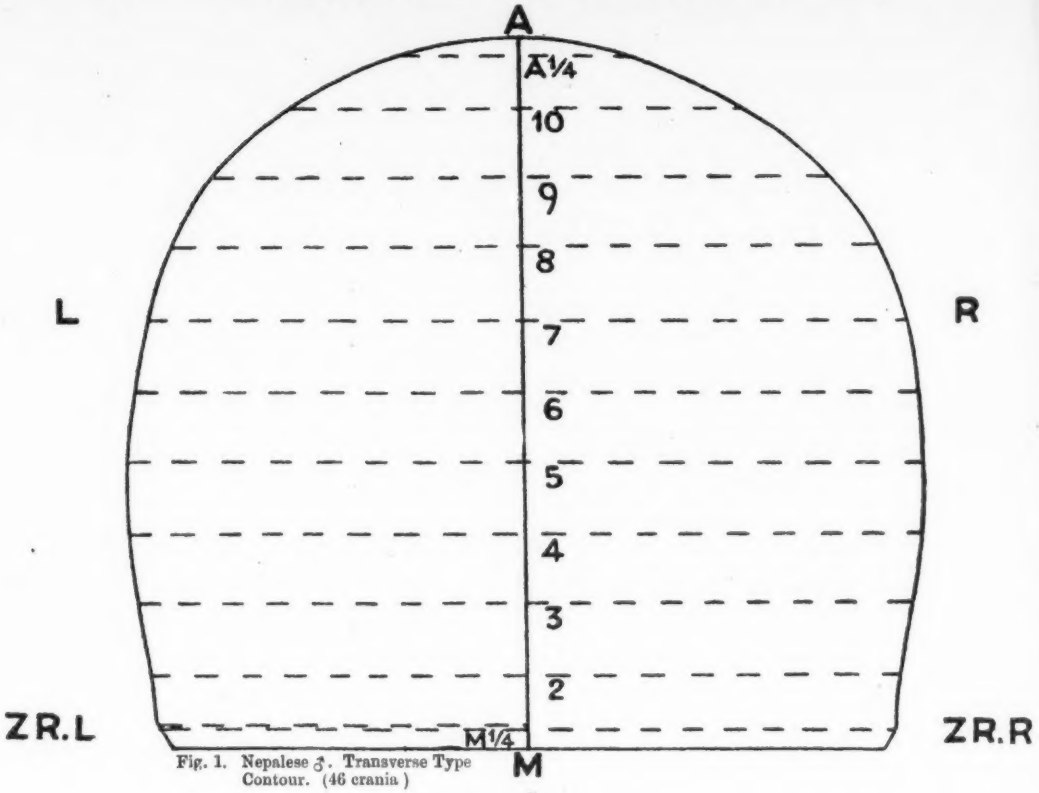


Fig. 1. Nepalese ♂. Transverse Type Contour. (46 crania)

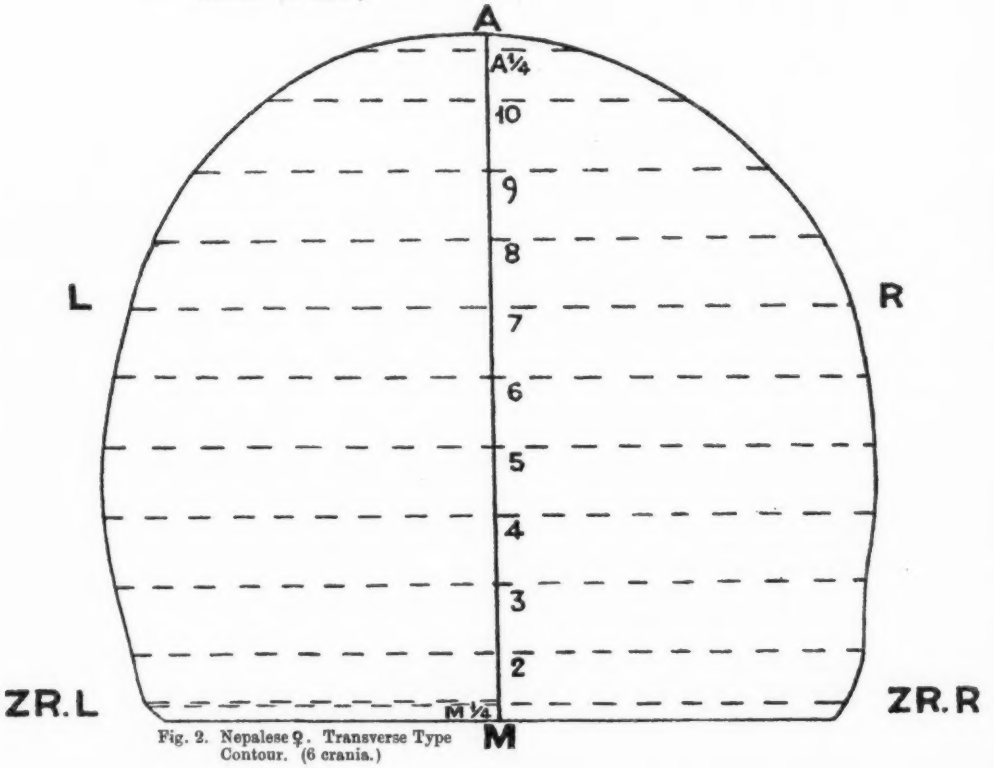


Fig. 2. Nepalese ♀. Transverse Type Contour. (6 crania.)

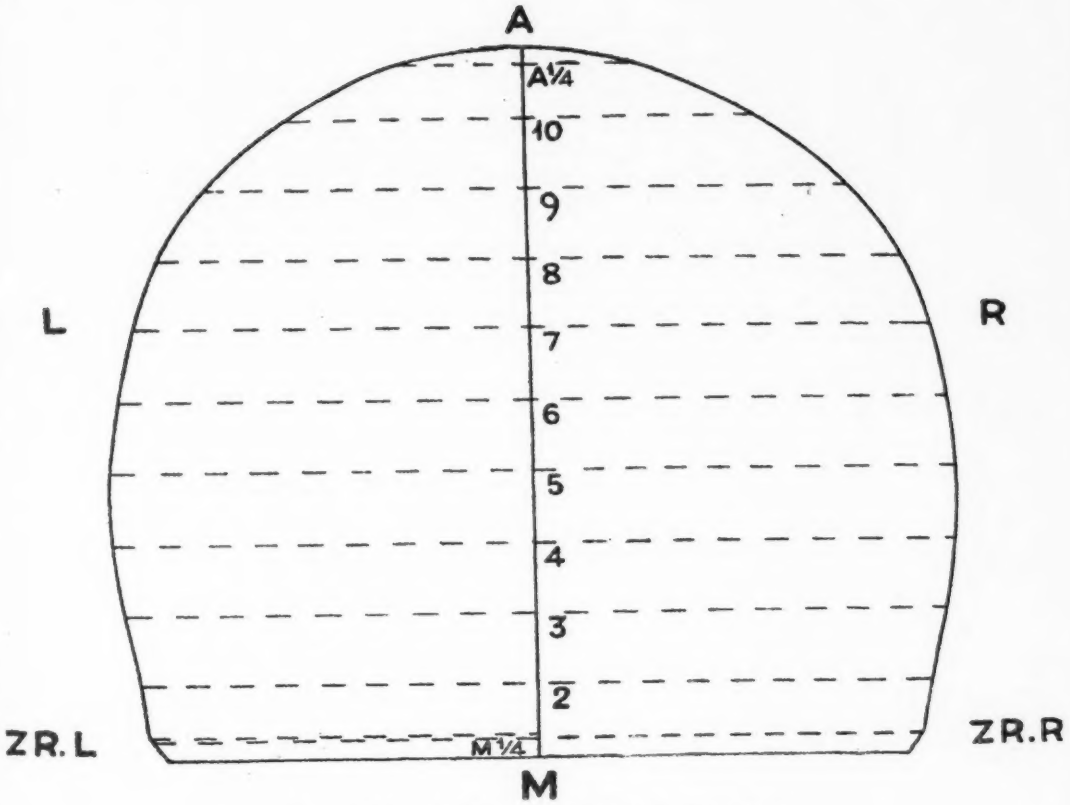


Fig. 3. Tibetan A ♂. Transverse Type Contour. (35 crania.)

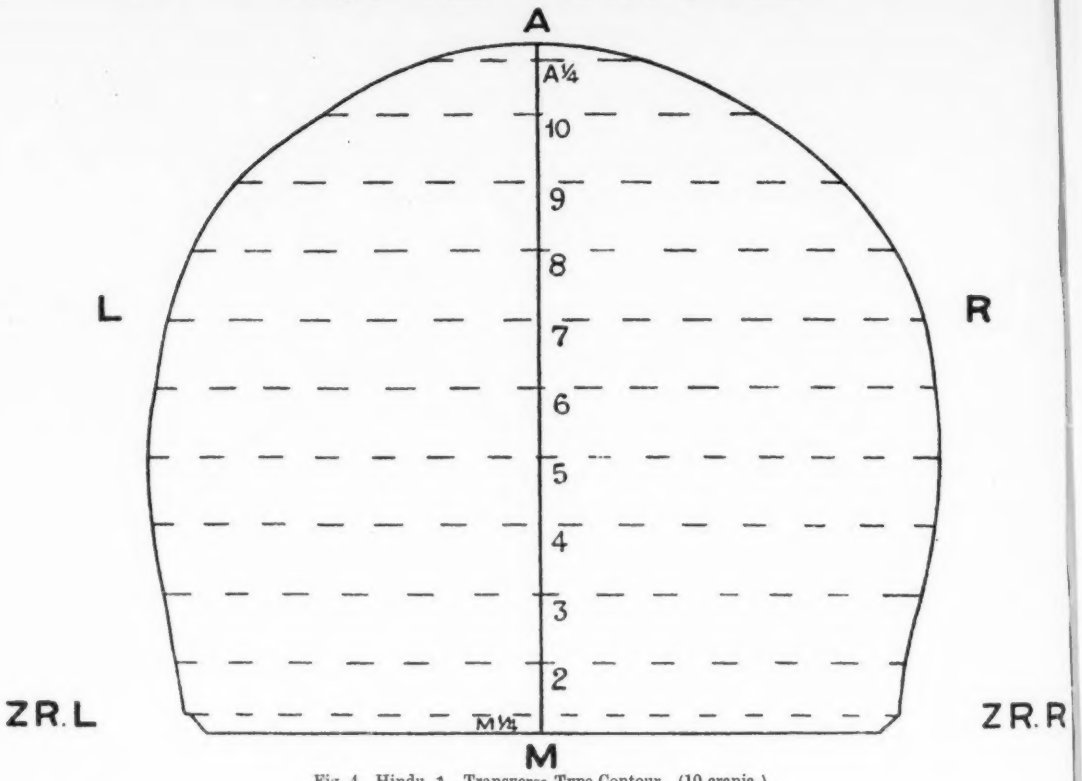


Fig. 4. Hindu ♂. Transverse Type Contour. (10 crania.)

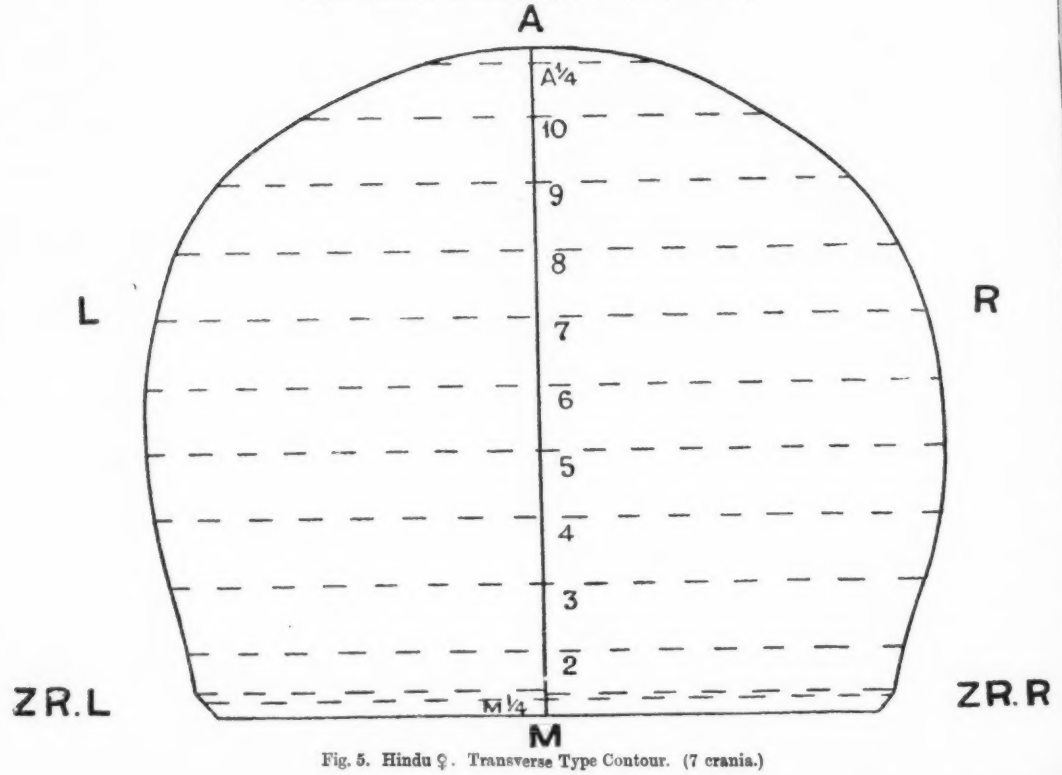


Fig. 5. Hindu ♀. Transverse Type Contour. (7 crania.)

the square root of the number of crania on which the type is based. The following widths are found:

Race	Number of Crania	Total width of type zone at vertex (mm.)	Total width of type zone at auricular points (mm.)
Nepalese ♂...	46	2·1	1·2
Tibetan A ♂...	35	2·4	1·4
Hindu ♂...	10	4·4	2·5
Hindu ♀...	7	5·3	3·0
Nepalese ♀...	6	5·7	3·3

Actually the type zones should be rather wider than this, for the English are less variable than the Nepalese or Tibetans, but the figures given may be supposed good approximations. They show how unreliable the types based on ten or fewer crania are. The type zones are not shown on the figures of the transverse contours as they would obscure the outlines, but they were marked on the tracings when comparisons were being made.

No single section of the human cranium is by itself a reliable guide to racial relationships, for totally unrelated races may have similar contours. The types are supplementary and subsidiary to the mean direct measurements. Besides furnishing a picture of the mean skull which may be a guide to qualitative racial differences, as in the case of the sagittal crest of the Eskimo, they enable us to determine many detailed measurements which cannot be taken directly. From a comparison of direct measurements, it has been possible to distinguish a group of allied Asiatic races which are such that the greatest differences between them are for the breadths of the calvariae, while the heights and lengths do not differ appreciably. But the only breadth measurements taken were the greatest parietal and forehead breadths, so there is no indication of the position of the greatest breadth, or of the distances apart of the temporal bones and auricular points; and there is no direct measure of the curvature of the vault of the calvaria. These deficiencies in direct measurement are compensated for by the detailed information which is provided by the transverse and horizontal contours. Unfortunately we have no contours of the Malayan, Chinese and Maravar types.

A comparison of the type contours confirms at once the general conclusions which direct measurements have suggested. The greatest differences are between the breadths of the figures; the heights are, with the possible exception of the Hindu, all much alike. The section through the auricular points is almost invariably in front of the vertical plane through the greatest parietal breadth and the mean B exceeds the greatest width of the type by quantities which usually range from 3·5 mm. to 6·0 mm. and appear to have no sexual or racial significances*. The greatest horizontal diameter of the type contour invariably falls between the 4th

* Of fifteen type contours compared with the corresponding direct measurements in this way, there was only one which differed in breadth by less than 3·5 mm. or more than 6·0 mm.; viz. the Moriori ♀ in which case the difference was 7·1 mm.

and 6th parallels and most usually between the 4th and 5th. The arrangement of the Asiatic races, excluding the Tibetan B, in order of that maximum length is: Burmese A—Burmese B (hybrids)—Burmese C (Karens)—Tibetan A—Nepalese —Hindu. This order is substantially the same as that suggested by direct measurements, the position of the hybrid Burmese being uncertain; but in another way these contours do not confirm entirely the relations previously found. In general the arcs, left and right, between the 4th and 6th parallels of any particular one of the six types fall entirely within the type zones of the adjacent races in the above sequence. But there is one exception to this rule: the Nepalese type zone falls well within the Tibetan type zone for the whole length below the 7th parallel. The Tibetan outline is farther removed from the Nepalese and closer to the Burmese than we should have anticipated. The divergence is particularly marked in the case of the lower parallels of the contours. If a tracing of the Tibetan A type is superposed on to the Burmese A it will be seen that the inter-auricular lengths and the outlines below the 3rd parallels are precisely the same for the two, while the other races show the same gradation in size for these characters—from Burmese A to Hindu—as for the greatest breadth. Now a very distinctive feature of the type contour of the Tibetans of the B type is the great breadth between the temporal bones and auricular points. The inter-auricular length is significantly greater than that of any other Asiatic race with which we are able to make comparison, but the greatest breadth is less than the Burmese A. It has already been shown that there is evidence which renders it probable that there has been some admixture between the Tibetan A and B types (see p. 40) and a comparison of the transverse vertical contours confirms that relationship. The evidence from the totality of direct measurements is sufficiently conclusive and we cannot conceive that there is any such close affinity between the Tibetans and Burmese of the A types as the transverse contours might have suggested.

The following table gives the auricular heights, as read off from the transverse

TABLE XXXIII.

Measurements of the Transverse Type Section.

Race	Sex	Number of Skulls	Inter-auricular Length	Auricular Height	Index
Tibetan B ...	♂	15	123·2	115·9	93·3
Tibetan A ...	,,	35	118·4	113·1	95·5
Burmese A ...	,,	44	118·2	116·5	98·6
Nepalese ...	,,	46	113·6	113·2	99·6
Burmese C ...	,,	8	114·6	115·6	100·9
Burmese B ...	,,	7	114·6	115·7	101·0
Hindu ...	,,	10	106·8	109·6	102·6
Burmese A ...	♀	38	112·8	110·0	97·5
Hindu ...	,,	7	104·6	106·1	101·4
Burmese B ...	,,	17	109·2	111·0	101·6
Burmese C ...	,,	18	106·6	108·9	102·2
Nepalese ...	,,	6	107·0	109·6	102·4

contours, together with the inter-auricular lengths and the resulting indices for the Asiatic races. In general there is a closer parity between the two lengths than in the case of European and African peoples for which the width always exceeds the height. But the Tibetan B does not conform to the type of the races which are in geographical proximity to it and the Tibetan A is also differentiated from its neighbours.

The order of arrangement of the races in the above table is obviously one to which we can attribute no significance. There appear to be no sexual differences between the male and female transverse contours of the same race except those of size; the female section is a slightly reduced copy of the male. The Hindu female section, though less in inter-auricular length and height, actually exceeds the male in the length of all parallels above the second, but that fact is of no great significance since the two series of mean measurements are based on so few as ten and seven crania respectively. Greater stress may be laid on the significance of the differences between the Hindu auricular height and the corresponding measurements of the other races.

No special comparison need be made between the vaults of the various types. The flattening of each is roughly proportional to its greatest parietal breadth and there are no outstanding differences. A comparison of the measures of curvature of the walls of the skull is of more interest. We may obtain such a measure by joining the apex (A) to the right auricular point and measuring the length of the greatest subtense from that line to the right half of the outlines*. The measurements obtained in this way and the resulting indices are given in Table XXXIV (p. 82). The figures show the same sequence from Burmese A to Nepalese that was found from direct measurements, but the Hindu indices are very discordant. It must be remembered that in the case of that race the mean direct measurements were deduced from many more crania than the few for which contours were drawn. It may be that the latter do not represent a random sample drawn from the Hindu population previously dealt with, and it is probable that the female contour deviates farther from the pure type than the male. In the case of the female Nepalese figure, however, we appear to have quite a good representation of the true racial type section.

There is no very close correspondence between the two Tibetan types when measured as in Table XXXIV. No significance need be attached to the positions of the male and female Hindu types relative to the other races.

To sum up the conclusions derived from a comparison of Transverse Vertical type contours: on the whole they confirm the relations between the races observed when comparing direct measurements. Excluding the Tibetan B, the heights of the mean skulls are very similar, but profound differences are found between the breadths, the Burmese A being greatest and the Hindu least. The Tibetan B type is most clearly differentiated from all the others and at the same time there is

* In the case of nineteen contours examined, the point of greatest subtense fell either between the 7th and 8th, or very slightly above the 8th parallel.

TABLE XXXIV.

Measurements of the Curvature of the Transverse Type Section.

Race	Sex	Apex to Right Auricular Point (AA')	Subtense (y)	Index ($100\,y/AA'$)
Burmese A ...	♂	129·7	40·4	31·1
Burmese B ...	„	128·8	39·5	30·7
Burmese C ...	„	129·2	39·6	30·7
Hindu ...	„	121·7	35·9	29·5
Tibetan A ...	„	127·6	37·5	29·4
Nepalese ...	„	126·4	35·8	28·3
Tibetan B ...	„	130·7	36·4	27·9
Hindu ...	♀	118·5	37·8	31·9
Burmese A ...	„	125·6	39·5	31·4
Burmese B ...	„	123·4	38·0	30·8
Burmese C ...	„	122·3	37·5	30·7
Nepalese ...	„	121·8	33·2	27·3

a suggestion that the Tibetan A type has been modified by admixture with the Tibetan B, though such a relation is the very reverse of that which direct measurements suggested. Judging from the meagre material at present available, there is apparently no sexual difference whatever between the male and female transverse type contours except one of absolute size.

(*b*) *The Glabella Horizontal Section.* The horizontal type contour is prepared in the way described in *Biometrika*, Vol. XIV. p. 234, but additional parallels 2½ R. and 2½ L. are now given for the first time; they define more exactly the outline immediately behind the temporal lines. The mean measurements from which the types (Figs. 6 to 10) were constructed are given in Table XXXV; the right and left of that table have reference to the individual contours and must be interchanged to correspond with the diagrams which show the *norma verticalis*.

The horizontal type zone shown by Benington for 100 English crania has an approximate width of 1·2 mm. all round. The corresponding widths for our types will be:

Race	Number of Crania	Approximate width of type zone
Nepalese ♂ ...	46	1·8
Tibetan A ♂ ...	35	2·0
Hindu ♂ ...	10	3·8
Hindu ♀ ...	7	4·5
Nepalese ♀ ...	6	4·9

The type zones should have widths rather greater than these since the Asiatic standard deviations are greater than the English. The lines defining the type zones were marked on the tracings used when comparing the various types, but are not shown on Figs. 6 to 10.

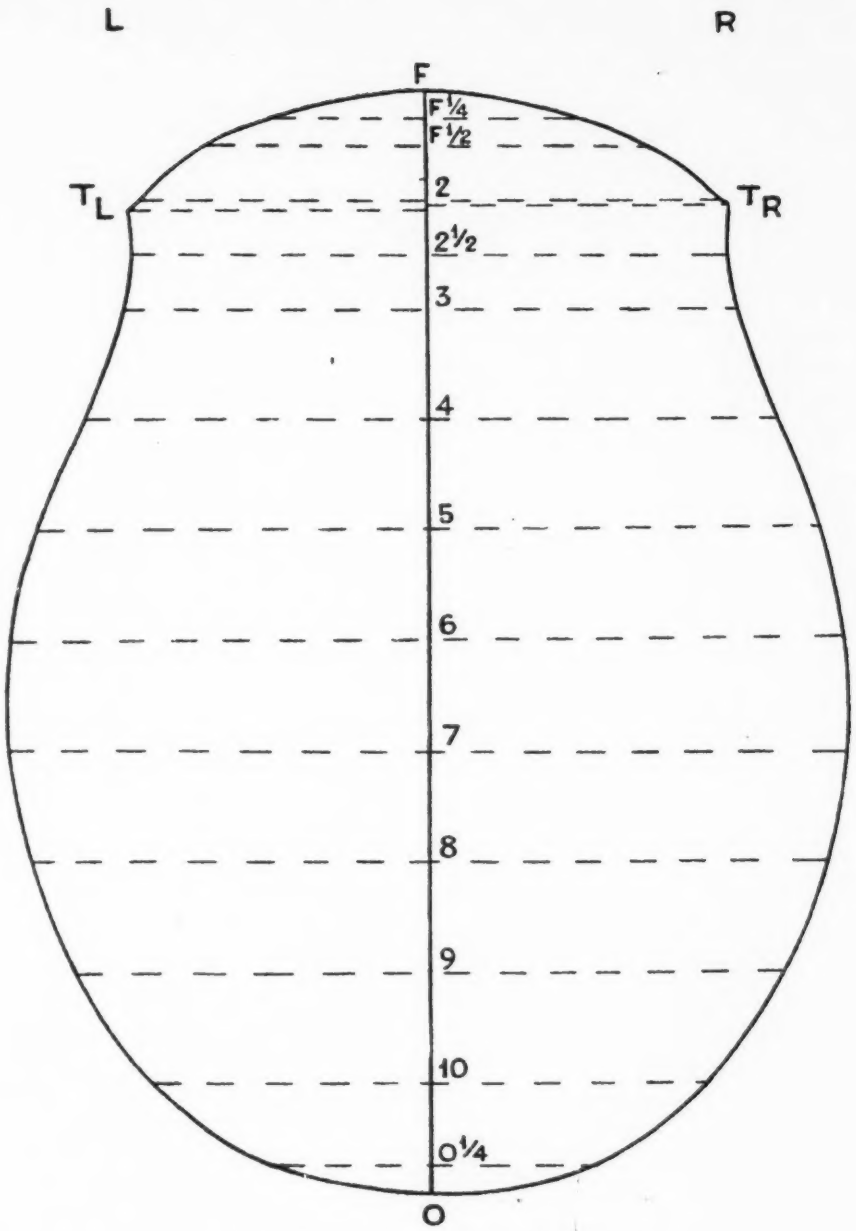


Fig. 6. Nepalese ♂. Horizontal Type Contour. (46 crania.)

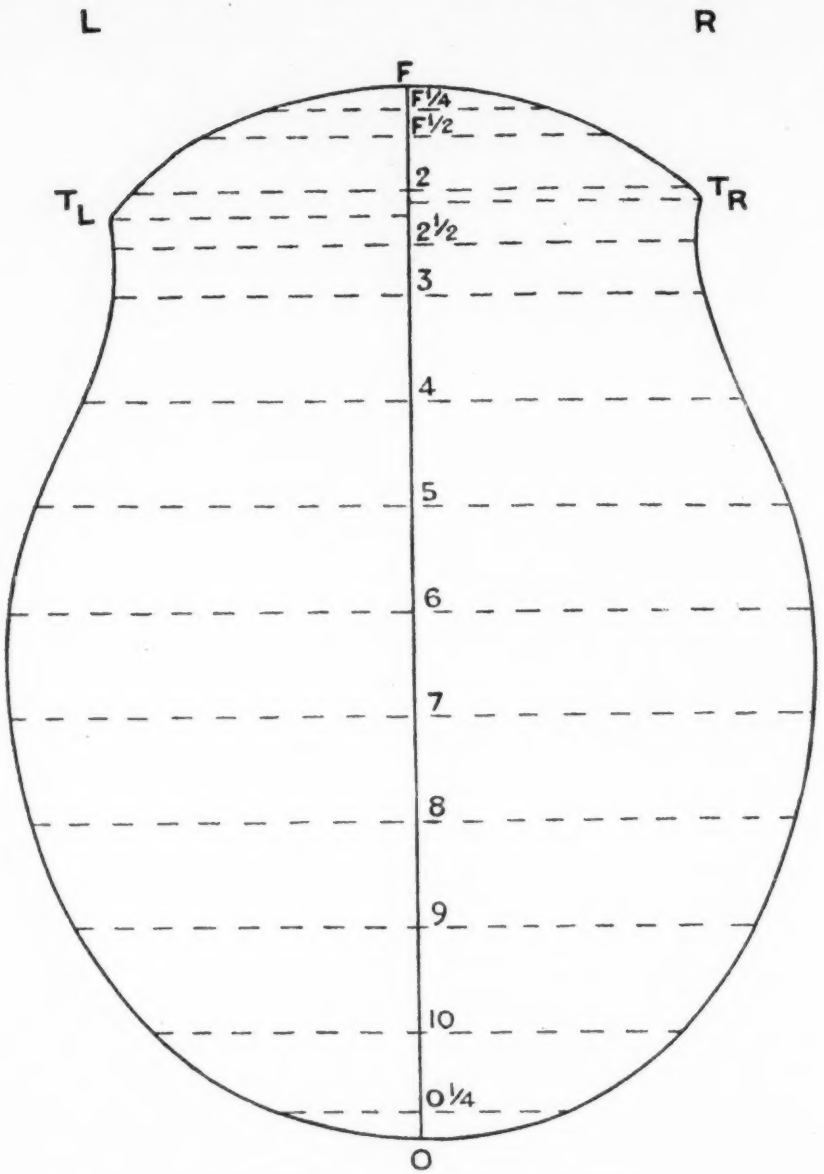


Fig. 7. Nepalese ♀. Horizontal Type Contour. (6 crania.)

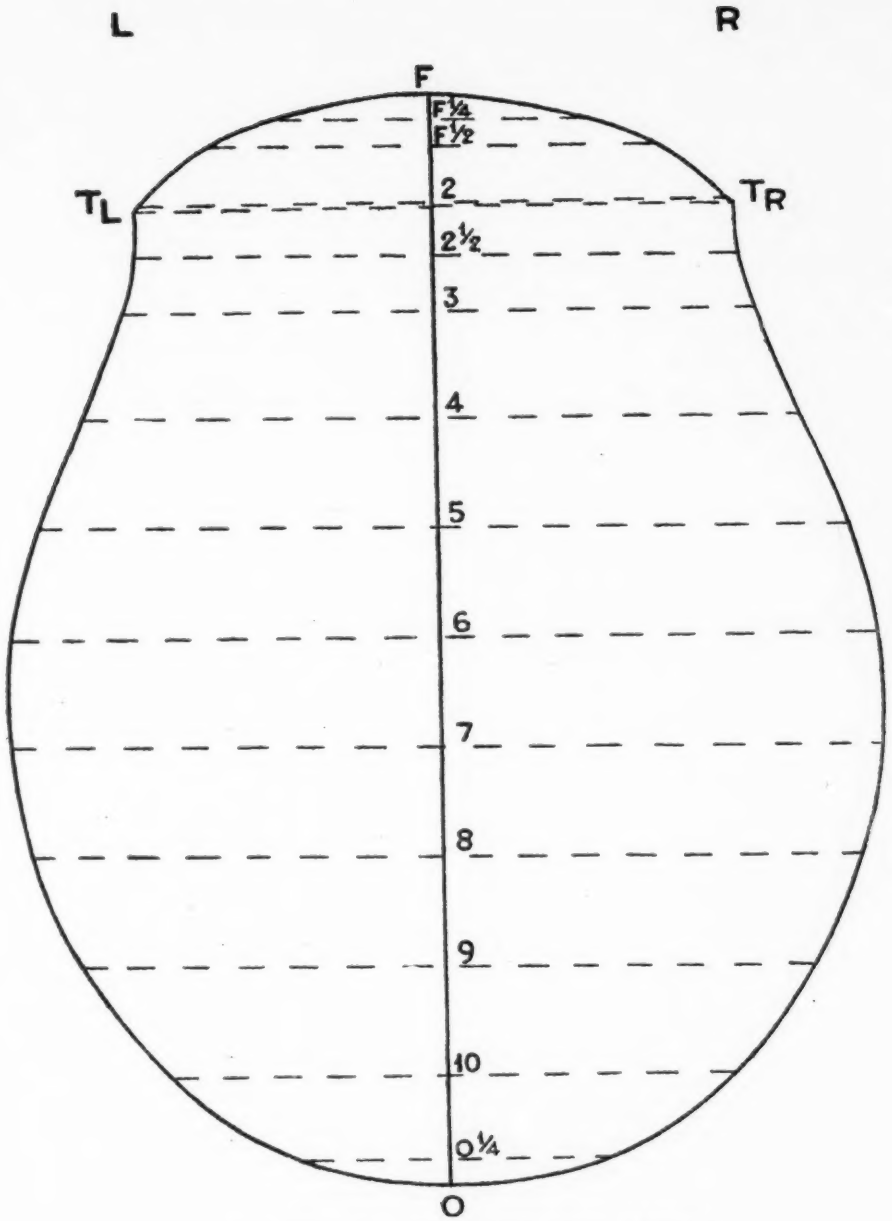


Fig. 8. Tibetan A ♂. Horizontal Type Contour. (35 crania.)

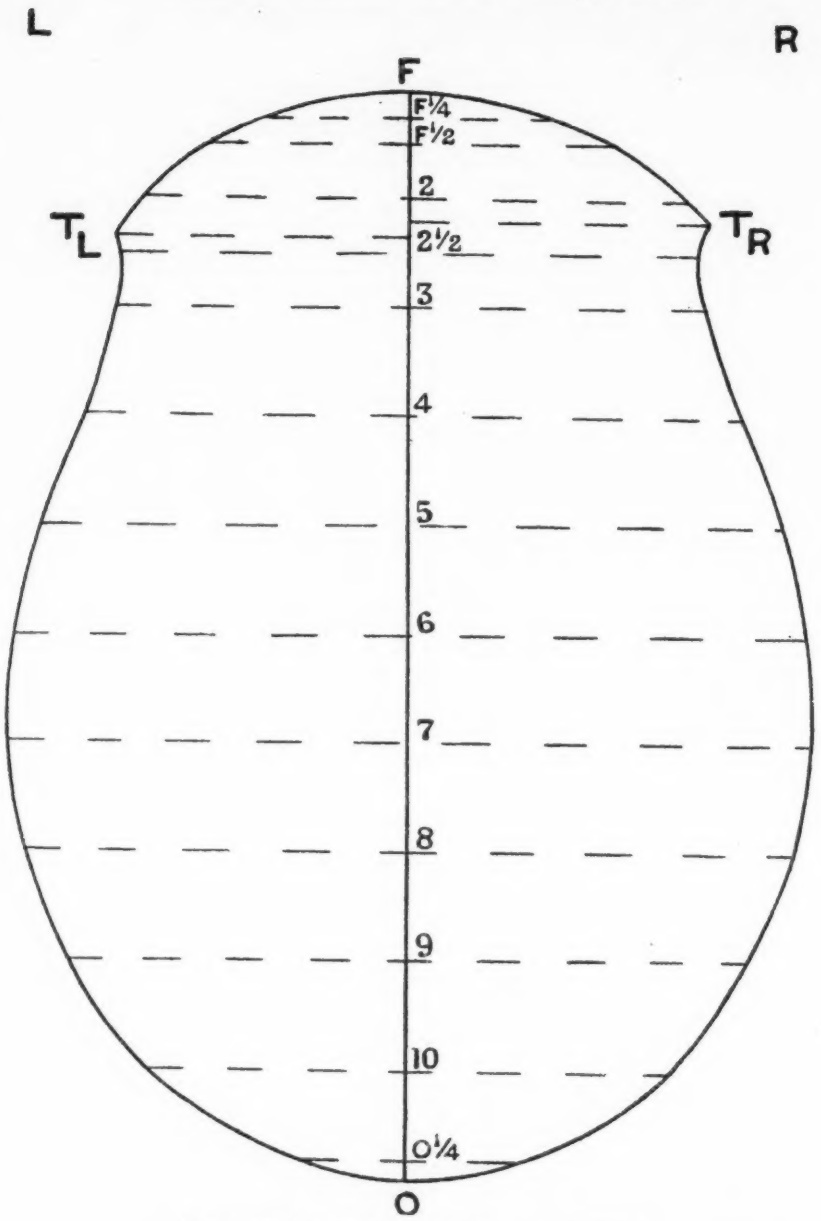


Fig. 9. Hindu ♂. Horizontal Type Contour. (10 crania.)

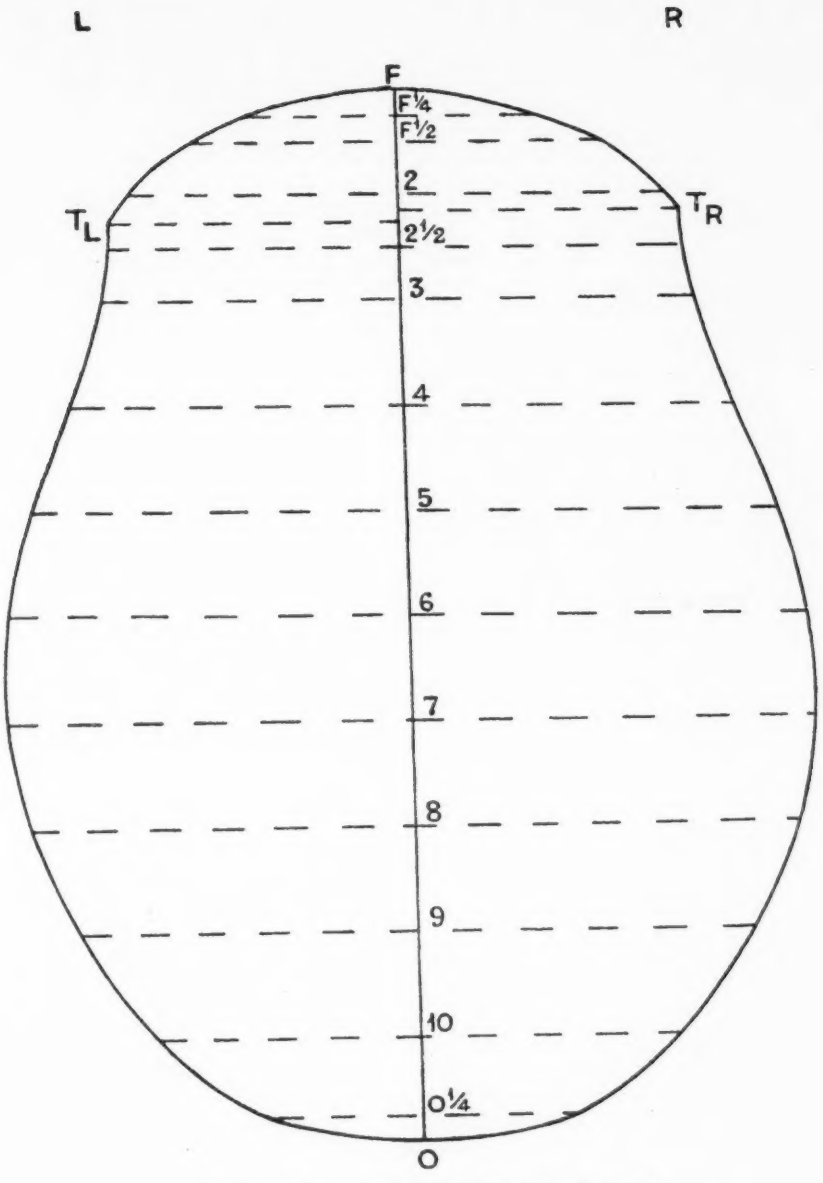


Fig. 10. Hindu ♀. Horizontal Type Contour. (7 crania.)

TABLE XXXV.

Mean Values of Horizontal Contours.

Race	Sex	No.	*FO*	*F* ¼*R*	*F* ¼*L*	*F* ½*R*	*F* ½*L*	2*R*	2*L*	2 ½*R*	2 ½*L*	3*R*	3*L*	4*R*	4*L*	5*R*	5*L*
Tibetan A	♂	35	174·4	23·9	23·5	36·0	36·0	46·5	47·2	47·4	48·6	49·6	51·1	56·0	57·6	63·8	65·5
Nepalese	♂	46	176·3	22·9	22·6	34·3	34·4	45·5	46·5	47·0	47·7	48·5	49·3	54·6	55·2	62·8	62·3
Hindu	♂	10	173·5	21·7	22·7	31·9	32·8	42·2	44·2	45·9	45·9	46·3	47·0	50·9	52·7	58·5	59·5
Dravidian	♂	2	177·1	22·7	19·4	36·4	31·4	47·4	44·0	48·8	46·4	50·1	47·4	56·5	52·7	63·9	58·0
Tibetan A	♀	3	166·0	28·2	19·3	37·1	33·5	45·2	42·9	46·8	44·4	49·5	47·5	58·6	54·7	63·7	63·8
Nepalese	♀	6	167·9	21·4	21·5	33·5	32·3	44·1	44·7	47·1	45·6	47·1	46·5	52·2	52·5	60·2	59·6
Hindu	♀	7	167·6	23·6	21·6	33·1	32·4	43·1	42·6	46·4	45·0	47·6	46·6	52·9	52·6	59·8	59·1
Dravidian	♀	1	162·8	14·1	19·6	32·0	34·9	44·7	44·0	44·8	44·0	44·6	46·8	51·4	55·1	59·6	61·9
Tibetan A Young (i) ...	♂	1	161·7	28·5	18·8	36·1	32·7	44·1	45·8	48·2	49·8	50·8	53·6	57·3	60·7	64·2	67·9
Tibetan A Young (ii) ...	♂	1	141·9	23·0	21·3	31·7	33·6	41·7	42·3	44·8	47·0	49·3	49·5	56·3	54·2	64·7	62·8

Race	Sex	No.	6*R*	6*L*	7*R*	7*L*	8*R*	8*L*	9*R*	9*L*	10*R*	10*L*	*O* ¼*R*	*O* ¼*L*	*TR*		*TL*	
															x	*y*	*x*	*y*
Tibetan A	♂	35	68·2	69·8	68·8	70·2	65·4	66·6	57·9	59·1	44·7	46·0	25·1	26·4	18·2	47·7	18·1	48·1
Nepalese	♂	46	66·7	65·7	67·0	66·1	63·7	63·3	56·6	56·4	44·4	43·9	26·4	26·4	19·3	47·8	18·4	48·2
Hindu	♂	10	62·4	63·4	63·3	64·2	61·0	61·7	54·1	54·6	41·5	41·7	22·3	21·9	23·2	46·5	20·8	47·1
Dravidian	♂	2	68·4	61·7	69·4	61·1	67·0	57·4	59·8	50·8	47·9	39·0	27·4	18·7	19·7	48·9	20·2	46·0
Tibetan A	♀	3	70·2	67·8	68·7	66·2	66·3	63·2	58·4	57·3	45·6	45·8	26·1	26·6	14·1	45·1	13·7	43·0
Nepalese	♀	6	64·6	63·0	64·6	63·3	61·3	60·1	54·8	53·5	42·7	41·5	23·8	24·6	20·8	47·5	18·6	46·3
Hindu	♀	7	63·8	63·3	64·4	64·1	60·9	61·3	53·6	54·0	41·9	41·7	25·7	25·9	21·1	46·1	19·8	44·7
Dravidian	♀	1	63·3	65·9	63·3	63·6	59·0	59·8	52·9	52·7	42·5	40·1	27·2	23·9	16·4	44·8	14·8	44·5
Tibetan A Young (i) ...	♂	1	66·5	71·7	65·2	71·9	58·6	65·5	50·0	58·2	39·6	49·0	22·2	32·1	19·3	46·6	20·8	48·8
Tibetan A Young (ii) ...	♂	1	68·1	64·9	67·2	64·2	64·2	59·1	58·7	51·9	47·3	36·5	33·0	13·5	16·2	43·3	17·3	45·2

The lengths of the horizontal type contours of the Asiatic races we are considering differ less than the auricular heights; in the Burmese A the *FO* length is least (170·9 mm.) and in the Nepalese greatest (176·3 mm.), provided the alien Tibetan B type with the extreme length for an Asiatic race of 183·2 mm. be excluded. On examining twenty-one male and female racial type contours it was found that in twenty cases the greatest parallel, adding the lengths left and right of the axis *FO*, was the 7th, and in one case only—that of the Eskimo—the 6th was greater than the 7th; while for all twenty-one contours the greatest breadth was between the 6th and 7th parallels. The contour of the cast of a Cro-Magnon skull* has the 6th as the greatest parallel and the maximum breadth is between the 5th and 6th, but it is unsafe to draw any inferences from a single specimen. This uniformity among modern races of man is a point well worthy of notice; a similar agreement between types which differed widely in other particulars was noted in the case of the transverse contour (see pp. 79, 80). If our attention is confined to male types only we find that the greatest breadths of the contours arrange the races in the order: Burmese A—Tibetan B—Tibetan A—Burmese B—Burmese C—Nep 'ese—Hindu. Again we are led to believe that the interposition of the Tibetan A type between the Burmese types is possibly to be attributed to the fact that the former has been modified by some admixture with the Tibetan B which is itself widely different from the Burmese though adjacent to that type in the sequence. The effect of this supposed hybridity is more clearly seen if the frontal parts of the horizontal contours are compared. A "Temporal Index" has been suggested as a measure of frontal development; it is formed by expressing the total length of ordinate 3 as a percentage of the length of the section (*FO*). In the following table the races are arranged in the order of the lengths of the 3rd ordinate.

TABLE XXXVI.

The Temporal Index of Asiatic Races.

Race	Sex	Number of Skulls	Ordinate 3	Length of Section (*FO*)	Temporal Index
Tibetan B ...	♂	15	104·5	183·2	57·0
Tibetan A ...	,,	35	100·7	174·4	59·0
Nepalese ...	,,	46	97·8	176·3	55·5
Burmese A...	,,	44	97·5	170·9	57·1
Burmese C ...	,,	8	97·1	174·4	55·7
Burmese B ...	,,	7	94·1	173·7	54·2
Hindu ...	,,	10	93·3	173·5	53·8
Burmese A...	♀	39	95·6	164·1	58·3
Hindu ...	,,	7	94·2	167·6	56·2
Burmese B ...	,,	17	93·9	161·6	58·1
Nepalese ...	,,	6	93·6	167·9	55·7
Burmese C ...	,,	18	90·1	168·2	53·6

* Benington, *loc. cit.* p. 153.

Evidently the orders in which the Temporal Indices and the lengths of Ordinate 3 arrange the races are not ones to which any significance can be given, but the values suggest again that the Tibetan A type inclines towards the Tibetan B. There is no clear indication of sexual differences in these or in any other measurements of the horizontal section but the temporal index tends apparently to be greater for female than male sections. As in the case of the transverse types, the Nepalese ♂ and ♀ contours are in excellent accordance, but the two Hindu contours are not so similar in form as it would be reasonable to expect them to be if they were based on longer series of crania. There is clearly little correlation between the greatest widths of the type contours and the widths of the frontal portions, the differences between the latter being so small that they are in the majority of cases less than the widths of the corresponding type zones as far back as the 3rd parallel. It may be remembered that the least forehead breadth (B') was very approximately the same for all the races dealt with. Between the 4th and the more posterior parallels the differences become more marked. Both Tibetan types exceed the Burmese A in the length of the 3rd parallel to a significant extent but they narrow down rapidly on nearing the temporal lines until the three are practically identical. A peculiarity of the Tibetan B skull is the flattening of the vertical portion of the frontal bone so that the points marking the temporal lines on the horizontal type contour fall in front of the 2nd parallel while for all other racial contours that have been constructed they fall behind that line. The degrees of frontal flattening may be readily compared by constructing an index as in the following table:

TABLE XXXVII.

Index of Frontal Flattening for Asiatic Races.

Race	Sex	Number of Skulls	$\frac{1}{2}[TR(x)+TL(x)]$	Length of Section (FO)	Index $\frac{100\times\frac{1}{2}[TR(x)+TL(x)]}{FO}$
Tibetan B ...	♂	15	17·5	183·2	9·6
Burmese C ...	„	8	17·65	174·4	10·1
Burmese B ...	„	7	17·9	173·7	10·3
Tibetan A ...	„	35	18·15	174·4	10·4
Burmese A ...	„	44	18·05	170·9	10·6
Nepalese ...	„	46	18·85	176·3	10·7
Hindu ...	„	10	22·0	173·5	12·7
Burmese A ...	♀	39	17·25	164·1	10·5
Burmese B ...	„	16	17·15	161·6	10·6
Nepalese ...	„	6	19·7	167·9	11·7
Burmese C ...	„	18	20·5	168·2	12·2
Hindu ...	„	7	20·45	167·6	12·2

The temporal lines of the Tibetan B are characteristically more forward than are those of the other races, and the Hindu have the deepest forehead when the depth is measured as a proportion of the length of the skull. If an index be constructed expressing the percentages that the lengths $\frac{1}{2}[TR(x)+TL(x)]$ are of

the inter-temporal widths of the sections it will again be found that the Tibetan B type has the least value and the Hindu the greatest, so we may conclude that the former is characterised by a flat and the latter by a rounded forehead. It has been shown that there is little correlation between the widths of the frontal portions of the types and their greatest widths. There is a closer correspondence between the magnitudes of the occipital parallels and the greatest widths: the 7th, 8th, 9th and 10th parallels all arrange the races in very similar orders with the Burmese A greatest and the Hindu least.

A comparison of horizontal contours has confirmed the relationships between the races suggested by direct measurements and transverse contours. Excluding the alien Tibetan B type, the races considered show no significant differences in length but very marked differences between the breadths of the calvariae, the Burmese A having the greatest breadth and the Hindu the least. The magnitudes of the greatest—the 7th—or any of the posterior parallels, would arrange the races in almost identical orders, but on approaching the frontal parts of the contours the lengths of the parallels of the various races tend to equality. The Hindu type is apparently distinguished from the others by having a more rounded frontal bone. The Tibetan A section immediately behind the temporal lines is markedly different from the Nepalese and in its great width it approaches the Tibetan B type, though we can hardly suppose that admixture with the latter has modified the skull form of the prevailing Tibetan population.

(*c*) *The Sagittal or Median Section.* The type sagittal section is prepared from the mean measurements of the individual contours in the way described in *Biometrika*, Vol. XIV. pp. 239 and 240. Table XXXVIII gives the measurements actually used. The abbreviations used in the table and diagrams (Figs. 11 to 15) are:

N = Nasion. *G* = Glabella. *Bas.* or *B* = Basion. β = Bregma. *V* = Vertex. *A* or *Alv.* = Alveolar point. *Sub-Orb.* = Left infra-orbital point. *Aur.* = Right auricular point. *I* = Inion. *Op.* = Opisthion. *Sp.* = Sphenoidal point, i.e. the point of intersection of the median plane and the suture between the sphenoid and basi-occipital bones. *P* = Point of intersection of the palatine sutures. *P′* = Extremity of *spina nasalis posterior*. *NS* = Extremity of anterior nasal spine. *L* = Tip of nasal bone. *L′* = Point at which *NL* first meets the outline of the nasal bone.

The type zone of 100 ♂ English sections provided by Benington* has a total width of 1·6 mm. for the greater part of its length, tapering down to 0·8 mm. at the gamma and 0·9 mm. at the nasion. Unfortunately it is not shown for any part of the outline below the $N\gamma$ line. The approximate widths† of the type zones for the Asiatic types will be:

* *Biometrika*, Vol. VIII. p. 147.

† The standard deviations of the Asiatic races are known to be greater than the English, so the widths shown are rather less than they ought to be.

Race	Number of Crania	Total width of type zone in mm. at		
		Vault	Gamma	Nasion
Nepalese ♂ ...	46	2·4	1·2	1·3
Tibetan A ♂ ...	35	2·7	1·4	1·5
Hindu ♂ ...	10	5·1	2·5	2·8
Hindu ♀ ...	7	6·0	3·0	3·4
Nepalese ♀ ...	6	6·5	3·3	3·7

As in the case of the transverse and horizontal types, the "points" of the sagittal contours occupy very much the same relative positions for all modern races of man. Thus the closest ordinate to β is always the 4th and the bregma falls usually between ordinates 4 and 5*. Similar uniformities are found with regard to λ and the points of maximum subtense, but the position of the vertex appears to have some racial significance. For all the diagrams as yet available the vertex falls between the 5th and 6th ordinates, but it is nearer to the 5th than the 6th for the Hindu ♂, Nepalese ♂ and ♀, Tibetan A ♂, Tibetan B ♂, Burmese A ♂ and ♀, Burmese C ♀, the Guanche and the Cro-Magnon figures; while the vertex is nearer to the 6th ordinate than the 5th in the case of the Burmese B and C ♂, Hindu ♀, Burmese B ♀, Moriori ♂ and ♀, Congo (1860), Congo (1880), Congo (Batetelu), Egyptian, Eskimo and English types. The contours of all the Asiatic races that fall into the latter category are based on small numbers of skulls, so we may conclude that the vertex of the skulls of Oriental races tends to be relatively more forward than the vertex of other races of man.

A detailed comparison of the various sagittal type contours need hardly be made; it would do little more than confirm the relationships which were observed between the direct measurements. The differences between the Hindu, Nepalese, Tibetan and Burmese sagittal sections are evidently smaller and less significant than those already found between the transverse and horizontal sections. Estimating the significance of the difference by superposing the two zones and noting for what regions one falls well outside the other, we conclude that the lengths of the contours are not appreciably different, while the Hindu with least height is just differentiated from the Burmese A with the greatest, but the same characters do not distinguish the Nepalese from the Burmese A. The Hindu, Nepalese and Tibetan A arcs are practicaliy coincident when the $N\gamma$ axes are superposed, but the three Burmese types—and particularly the A—differ from the others in having shorter occipital lengths below the $N\gamma$ line. The opisthions of the Burmese contours fall behind the 7th ordinate, but in the case of the more Western types the points are in front of that line. Now the chord from λ to opisthion (S_3') was measured directly and found to be the same for the Burmese A as for the Nepalese and Tibetan A types and the occipital index also showed no significant differences; the position

* The β of the individual contour of the cast of a Cro-Magnon skull (*Biometrika*, Vol. VIII. p. 149) is rather nearer to the 3rd than to the 4th ordinate.

TABLE XXXVIII. *Means of Contour Measurements used in plotting Sagittal Type Contours* *.

Race	No.	$N\gamma$	Ordinates above $N\gamma$													
			$0=N$	$N\frac{1}{4}$	1	2	3	4	5	6	7	8	9	$\gamma\frac{1}{4}$	$\gamma\frac{1}{8}$	$\gamma=10$
Hindu ♂	10	170·5	30·0	41·3	58·0	70·6	78·4	83·1	84·7	83·4	79·5	71·1	53·7	29·4	22·9	6·8
Nepalese ♂	46	174·0	27·2	44·1	62·3	74·2	81·5	86·4	88·7	88·5	83·8	73·7	54·0	25·6	16·8	2·0
Tibetan A ♂	35	172·6	26·9	42·2	60·5	72·6	80·0	85·1	87·5	87·4	83·5	74·2	55·5	28·4	19·9	5·4[k]
Dravidian ♂	2	175·8	34·6	46·9	62·6	73·9	80·2	85·5	85·7	84·6	79·0	68·9	47·6	17·4	10·4	−6·3/†
Tibetan A ♂ ‡	1	159·2	42·6	50·8	64·7	74·7	81·2	85·8	87·3	87·4	83·2	74·0	52·3	31·0	19·5	2·8
Hindu ♀	7	164·8	37·0	45·6	60·9	71·1	77·4	81·5	82·7	82·7	78·8	70·0	52·6	31·8	25·6	10·7
Nepalese ♀	6	165·7	36·2	46·6	62·0	73·1	80·5	85·3	87·0	86·9	83·1	73·5	54·1	29·0	20·1	4·7
Tibetan A ♀	3	164·7	30·1	44·8	60·0	71·3	79·1	84·5	87·8	87·5	83·8	75·6	60·5	36·2	25·5	0·0
Dravidian ♀	1	161·6	24·5	40·2	54·0	64·9	70·0	72·9	74·3	74·6	71·8	62·9	43·8	19·7	14·8	0·0

Race	No.	Ordinates below $N\gamma$								Vertex		β		Glabella	
		$0=N$	$N\frac{1}{4}$	1	2	8	9	$\gamma\frac{1}{2}$	$\gamma\frac{1}{4}$	x from N	y	x from N	y	x from N	y
Hindu ♂	10	58·1[a]	55·9[b]	49·0[b]	50·6[c]	46·0	35·0	26·7	18·5	87·0	85·1	71·9	84·1	2·5	10·7
Nepalese ♂	46	60·1[e]	58·3[f]	51·6	51·1[g]	46·0	35·0	25·9	16·2	93·0	89·4	67·5	86·2	2·5	10·9
Tibetan A ♂	35	66·1[k]	63·0[k]	54·3	51·9[l]	45·1	34·0	26·2	18·0	91·3	88·3	70·1	85·3	2·3	11·9
Dravidian ♂	2	58·5	55·8	48·9	46·5	46·8	40·1	32·2	23·7	90·9	86·2	70·4	85·4	3·3	9·7
Tibetan A ♂	1	60·4	56·6	46·1	43·9	40·8	25·6	18·2	12·9	88·1	87·9	58·1	85·4	2·8	12·6
Hindu ♀	7	58·5	55·0	48·4	47·4	41·5	30·0	22·3	16·0	92·1	83·0	65·0	81·3	2·6	14·7
Nepalese ♀	6	58·2	54·5	46·4	45·7	40·9	28·4	20·4	13·9	85·7	87·4	63·7	85·0	1·8	11·1
Tibetan A ♀	3	60·2	56·1	48·0	49·4[n]	46·2	33·5	24·8	15·0	90·7	88·5	64·9	84·2	1·8	11·1
Dravidian ♀	1	55·6	53·8	47·0	47·1	43·7	35·6	27·4	18·3	91·2	74·4	64·6	73·1	1·7	11·3

Race	No.	Occipital Pt.		λ		Sub-Orb.		Aur.		Op.		Inion		Basion	
		x from N	y	x from γ	y	x from N	y	x from γ	y	x from γ	y	x from γ	y	from γ	from N
Hindu ♂	10	0·5	3·8	4·9	29·2	8·2	26·3	87·0	25·4	51·7	51·5	12·2	32·3	98·1	95·7
Nepalese ♂	46	0·2	0·9	6·3	31·5	8·6	26·7	90·5	25·9	52·9	51·9	13·8	32·7	100·7	96·8
Tibetan A ♂	35	0·4	2·9	5·7	31·5	7·1	27·4	87·7	26·8	51·7	50·6	10·2	28·9	98·9	96·1
Dravidian ♂	2	0·3	−5·3	4·4	16·9	10·6	26·3	95·0	26·8	61·3	52·3	12·0	35·4	105·3	90·2
Tibetan A ♂	1	0·2	4·5	6·4	36·0	7·1	24·2	85·7	24·2	48·9	43·9	17·1	26·6	90·5	85·7
Hindu ♀	7	0·6	7·8	5·5	33·6	7·9	24·9	85·8	24·4	50·7	48·0	9·6	25·1	94·2	92·7
Nepalese ♀	6	0·1	3·9	6·0	33·0	8·2	24·3	84·8	24·0	49·9	45·7	11·8	24·4	93·8	90·6
Tibetan A ♀	3	0·1	−0·8	3·6	33·2	7·8	26·2	83·7	26·0	49·4	52·2	11·4	28·8	94·3	91·3
Dravidian ♀	1	0·1	−3·0	4·7	21·4	6·9	26·5	85·9	25·4	47·7	47·8	15·9	35·4	96·6	89·7

Race	No.	Alv. Pt.		Nose					Palate				Frontal		Occipital	
		from N	from Bas.	NL	NL'	$\angle LN\gamma$	Max. Sub.		P'		P		Max. Sub. to $N\beta$		Max. Sub. to λ Op.	
							x from N	y	x from N	y	x from Alv.	y	x from N	y	x from λ	y
Hindu ♂	10	64·0[b]	92·1[b]	19·3[c]	17·0[c]	121°·8[c]	7·4[c]	2·2[c]	40·1[d]	50·5[d]	34·0[b]	15·1[b]	46·2	25·0	51·1	25·7
Nepalese ♂	46	68·0[h]	93·9[h]	21·1[i]	17·0[i]	116°·1[i]	8·1[i]	2·2[i]	42·5[g]	49·6[g]	34·8[h]	17·1[h]	49·7	25·5	49·7	26·6
Tibetan A ♂	35	69·2	92·3	23·1[k]	19·6[k]	111°·8[k]	9·2[k]	1·5[k]	43·7[l]	49·7[l]	33·8	17·5	49·3	25·9	48·9	26·3
Dravidian ♂	2	65·3	90·3	18·6	17·8	112°·6	8·6	2·8	41·3	45·2	37·5	17·3	48·8	27·1	41·3	27·9
Tibetan A ♂	1	62·1	81·3	22·0	13·3	101°·9	7·9	1·0	36·8	41·9	27·9	18·0	46·9	25·6	39·0	25·2
Hindu ♀	7	61·5[e]	86·9[e]	20·4[m]	20·4[m]	120°·6[m]	6·8[m]	2·5[m]	37·5[e]	45·9[e]	32·5[e]	13·9[e]	43·5	26·4	43·0	25·5
Nepalese ♀	6	62·8	88·8	17·6[m]	15·7[m]	109°·4[m]	6·0[m]	2·1[m]	41·4	43·6	32·4	16·9	45·7	25·3	42·3	24·6
Tibetan A ♀	3	65·8	86·1	19·2[n]	17·2[n]	116°·5[n]	8·5[n]	2·0[n]	41·2[n]	49·1[n]	30·6	18·5	43·0	24·6	48·8	24·0
Dravidian ♀	1	—	—	—	—	—	—	—	38·7	46·7	—	—	40·4	25·8	45·0	23·1

TABLE XXXVIII—(*continued*).

Race	No.	Max. Sub. to $N\lambda$		Max. Sub. to GI		Sp.		Sub. from ½ Bas. Sp. Chord		N. S.		Max. Sub. of Alv. N. S. arc	
		x from N	y	x from G	y	x from N	y	x from Bas.	y	x from N	y	x from N	y
Hindu ♂	10	85·3	70·4	91·1	99·0	63·0	31·8	12·7	0·7	6·9[b]	47·2[b]	5·0[b]	51·9[b]
Nepalese ♂	46	84·5	72·5	92·6	103·7	64·1	31·2	12·9	0·6	5·5[j]	49·0[j]	3·7[h]	53·4[h]
Tibetan A ♂	35	88·0	71·1	94·7	100·1	63·7	30·1	12·8	1·3	3·6	48·6	2·4	56·9
Dravidian ♂	2	81·1	78·2	88·0	103·3	61·6	28·5	12·0	0·6	4·4	44·7	2·6	47·3
Tibetan A ♂	1	74·9	69·8	85·9	98·7	60·2	27·7	8·9	0·7	0·2	42·8	0·2	42·8
Hindu ♀	7	77·3	66·3	93·0	91·7	59·7	31·6	12·4	1·0	5·2[e]	44·6[e]	3·5[e]	49·9[e]
Nepalese ♀	6	83·2	70·3	89·7	97·8	61·6	28·1	11·8	0·6	2·4	43·7	1·7	48·9
Tibetan A ♀	3	87·1	69·4	85·1	100·9	60·2	29·5	12·2	0·3	3·7	45·8	2·7	47·2
Dravidian ♀	1	76·1	63·9	88·1	92·9	56·0	28·8	13·5	0·8	7·7	47·7	—	—

* The sagittal contour of the young Tibetan A skull 1 *i.i.i.i.i.i.* numbered (ii) in Tables XXXII and XXXV could not be drawn.

† The negative sign here means that the Ordinate 10 of the type contour meets the outline below the $N\gamma$ line.

‡ Young 1 *g.g.g.g.g.g.* of Appendix and Museum numbering.

[a] Mean of 8 contours
[b] „ 9 „
[c] „ 6 „
[d] „ 4 „
[e] „ 40 „
[f] Mean of 44 contours
[g] „ 42 „
[h] „ 41 „
[i] „ 31 „
[j] „ 45 „
[k] Mean of 34 contours
[l] „ 33 „
[m] „ 3 „
[n] „ 2 „

of the point γ is in fact the only difference of any moment between the occipital portions of the six sections. Before investigating this point further it will be well to compare other parts of the contours. The very considerable differences, found by direct measurement, between the facial heights of the races can be clearly observed when the type figures are superposed, the alveolar points of the Hindu and Burmese A being 8 mm. apart and the anterior nasal spines are widely removed. If it were supposed that the Hindu section might be converted into the Burmese by merely increasing all the facial lengths, and in particular the distance of the sub-orbital point below the $N\gamma$ line, while all other regions and points, and in particular the auricular point, remained unaltered, then it is clear that the Burmese γ would be nearer to λ than the Hindu. But actually the $\gamma\lambda$ length is greater for the Burmese than for the Hindu, so we must suppose the position of the auricular point would also have to be moved in such a transformation. To test whether that is actually a true representation of the difference we should compare the observed distances of the auricular points of the types from some independent base line such as $N\lambda$ and not from the $N\gamma$ line. Such a comparison is made in Table XXXIX on p. 100, which gives the measurements of the male contours only.

The distance of the sub-orbital points from the $N\lambda$ line increases steadily as the races are passed from Hindu to Burmese and the distance of the auricular points from the same line increases in the same way but to a much greater extent. The auricular point then is far from being a constant one relative to circumference

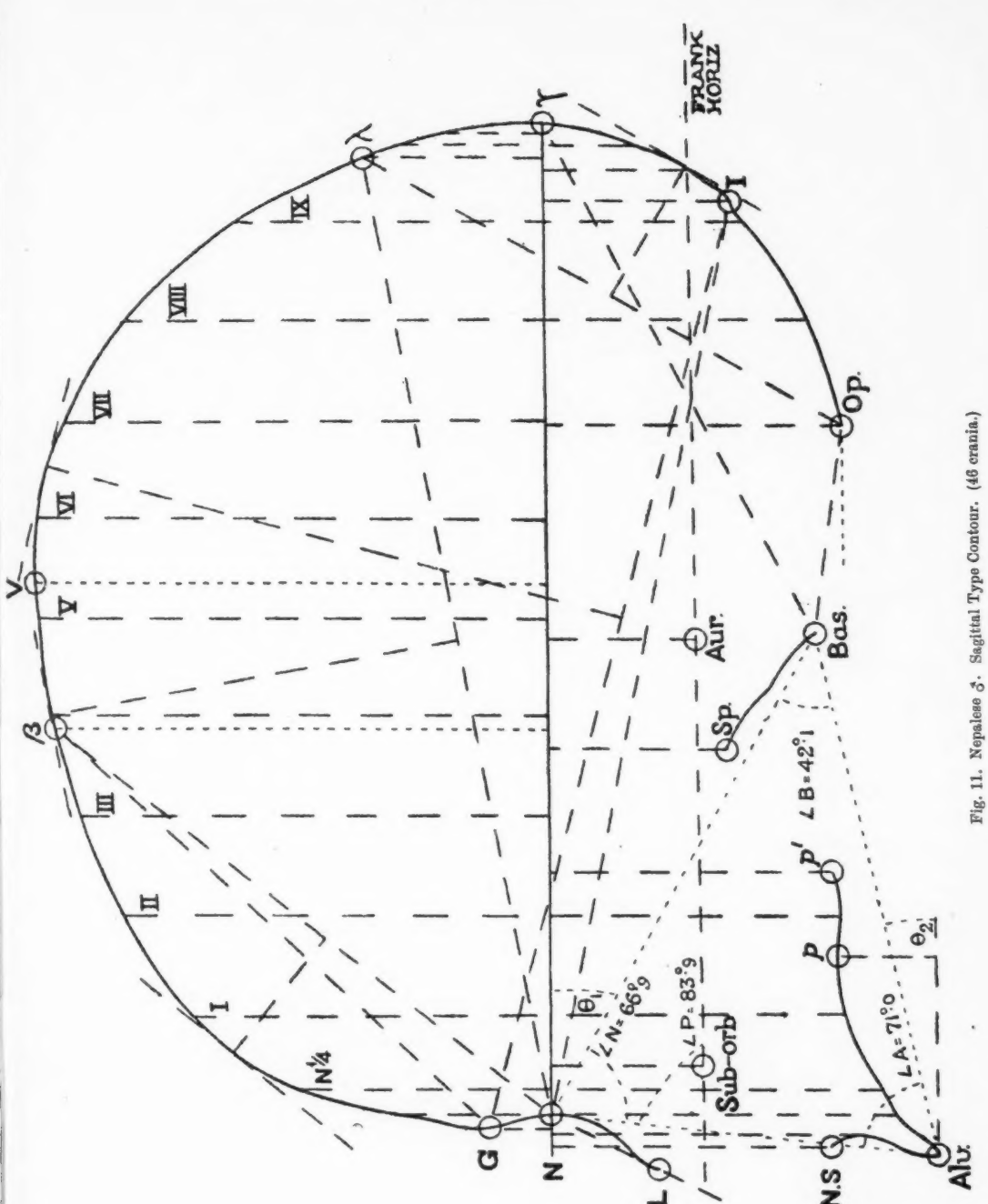
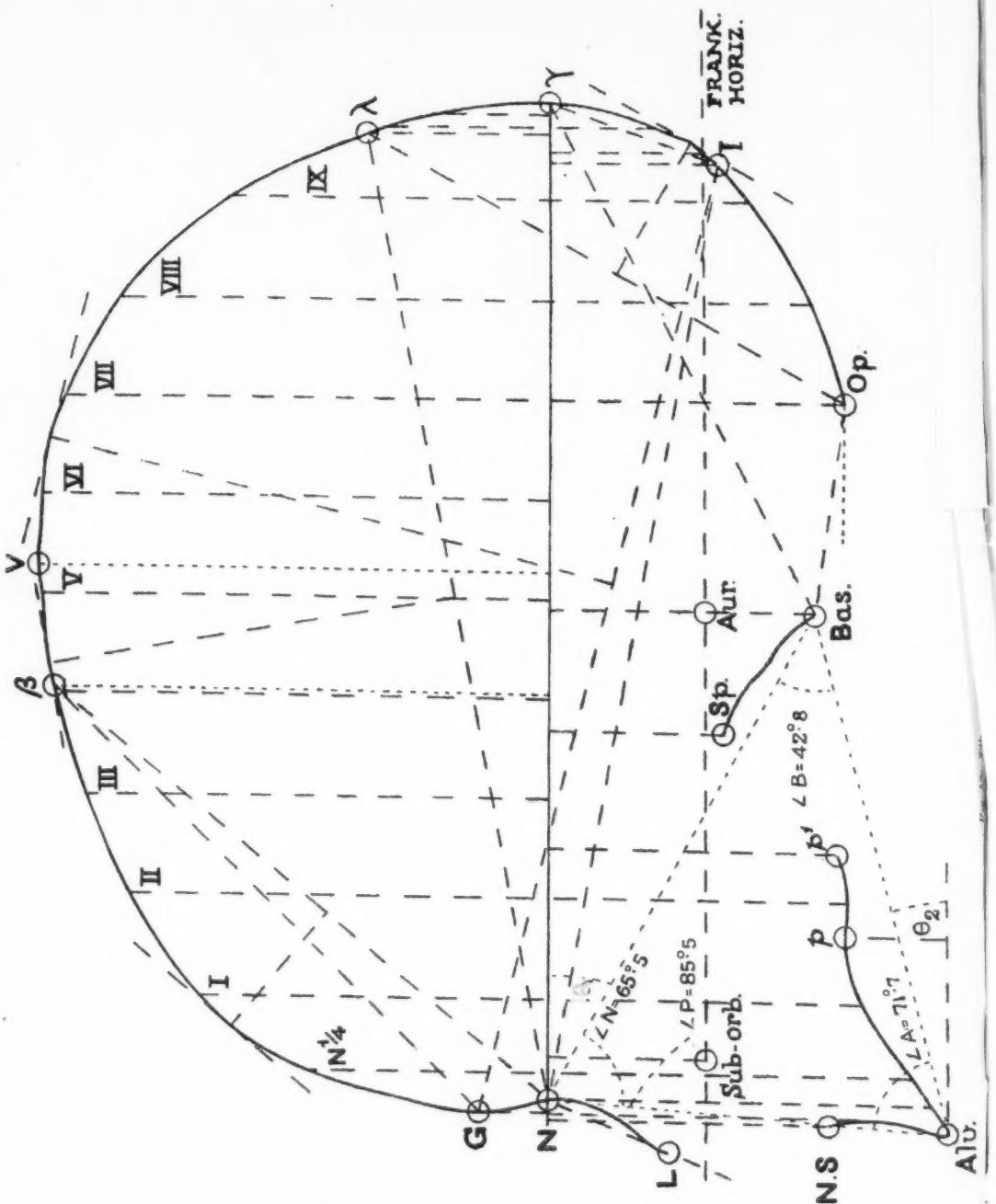


Fig. 11. Nepalese ♂. Sagittal Type Contour. (46 crania.)


FRANK. HORIZ.
λ
γ
I
IX
VIII
VII
VI
V
β
III
II
I
N¼
G
N
L
N.S
Alv.
Op.
Bas.
Aur.
Sp.
Sub-orb.
∠N=65°5
∠P=85°5
∠B=42°8
∠A=71°7
p'
p
θ2'

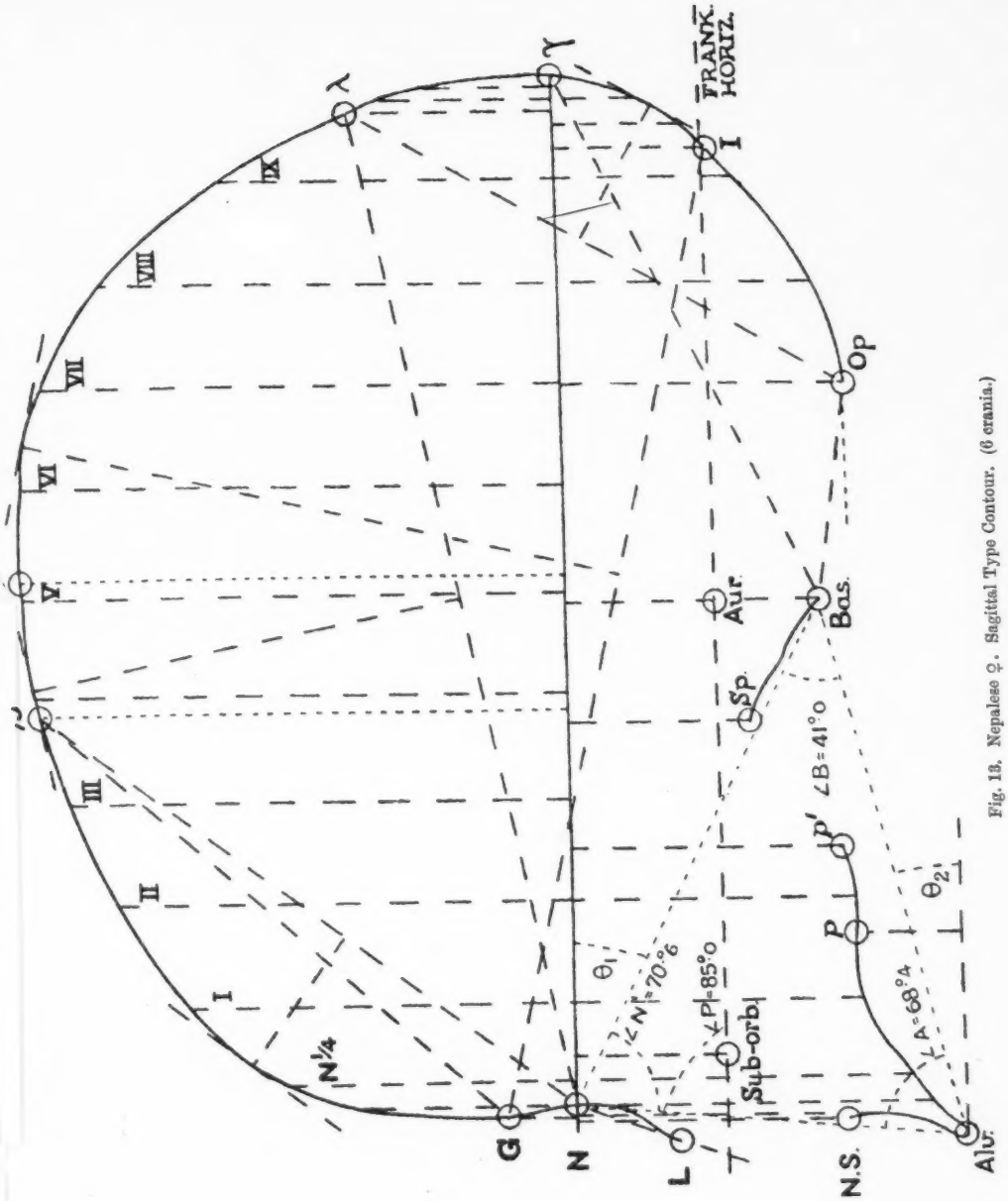


Fig. 13. Nepalese ♀. Sagittal Type Contour. (6 crania.)

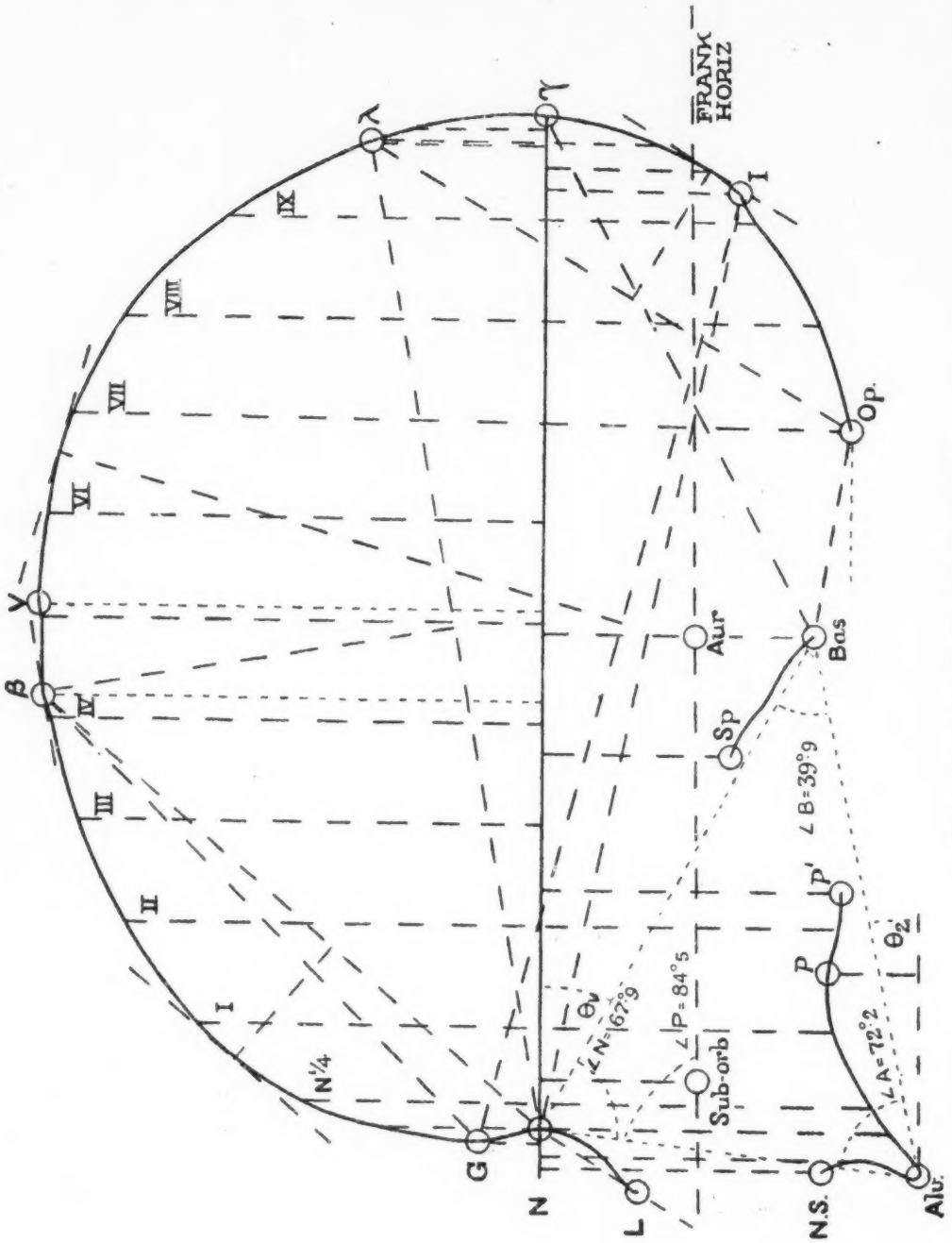


Fig. 14. Hindu ♂. Sagittal Type Contour. (10 crania.)

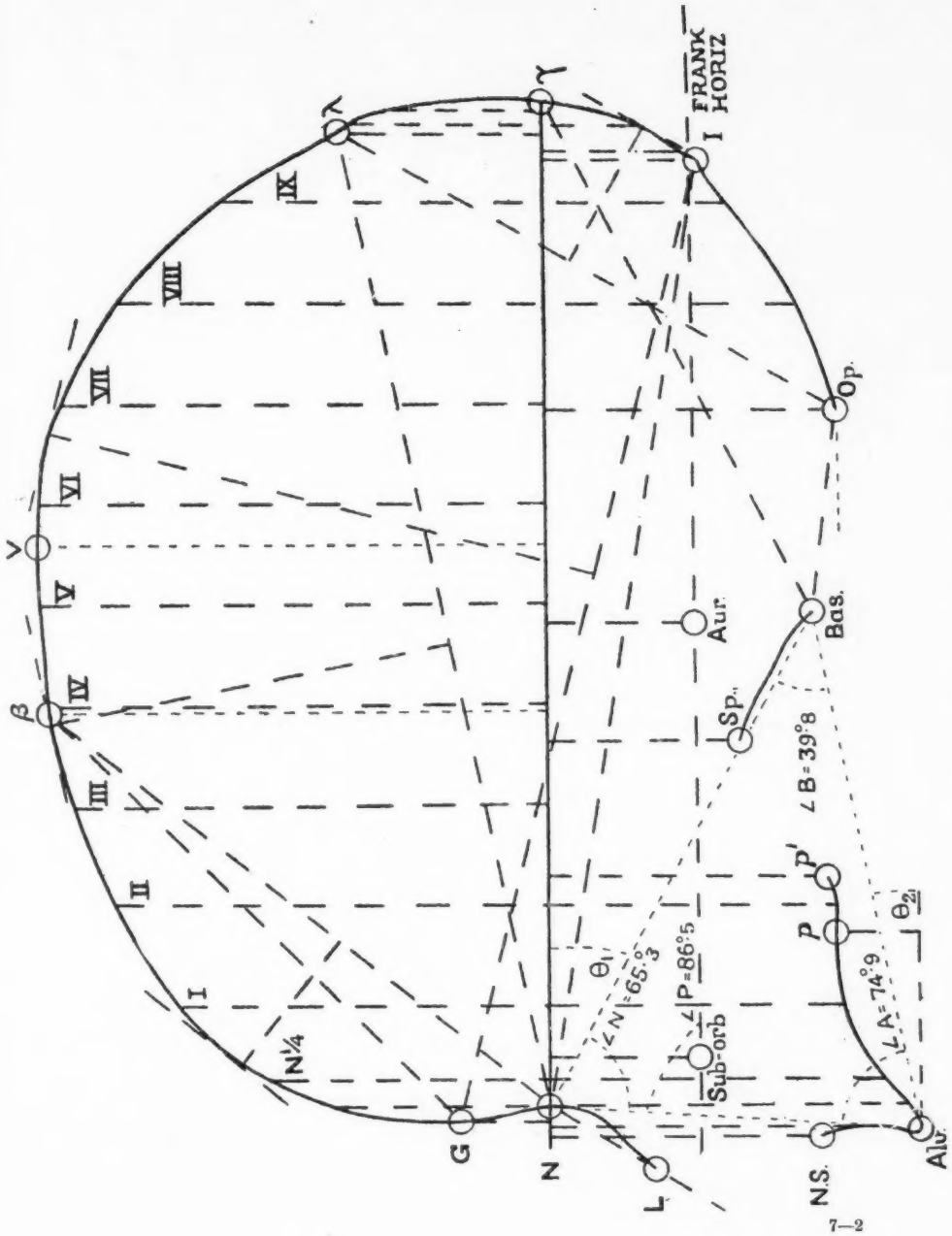


Fig. 15. Hindu ♀. Sagittal Type Contour. (7 crania.)

Fig. 14. Hindu ♂. Sagittal Type Contour. (10 crania.)

of the calvaria and we can at once understand why the γ is lower for the Burmese than for the other races; that it is so is shown by the angles $\lambda N\gamma$ in the last

TABLE XXXIX.

Measurements of the Sagittal Type Contour.

Race	Number of Crania	Distance of Aur. Point from $N\lambda$	Distance of Sub-Orb. Point from $N\lambda$	Max. Sub. from $N\lambda - y$	*OH* Direct Measurement	$\angle \lambda N\gamma$
Hindu ...	10	39·7	27·5	70·4	111·4	9°·8
Nepalese ...	46	41·0	27·9	72·5	115·1	10°·7
Tibetan A ...	35	42·2	28·3	71·1	114·4	10°·5
Burmese A ...	44	44·9	29·2	72·9	117·7	11°·6
Burmese B ...	7	45·2	29·6	71·1	116·7	11°·5
Burmese C ...	8	46·0	30·3	70·7	116·9	11°·4
Tibetan B ...	15	42·8	30·2	71·8	115·5	8°·8

column of the table. The differences between the *OH* measurements appear to be attributable almost entirely to the lowering of the auricular point relative to the vault of the cranium and not to any differences between the arcs from nasion to opisthion. The tilt of the calvaria and not the shape of the sagittal section is the feature that distinguishes the Burmese from the Tibetan and Nepalese types.

The greater length of calvaria and height of face and the more massive occiput of the Tibetan B contour clearly differentiates it from the Tibetan A. The sagittal section of the latter occupies an intermediate position between the Nepalese and Burmese A and it has no characteristics which would have suggested that admixture with an alien type had modified the outline in any way. From the evidence of direct measurements and the three type contours we cannot suppose that any admixture between the Tibetan A and B types has modified the skull form of the former although it is highly probable that the B type has been slightly modified as the result of some degree of crossing with the A. But still it is hard to explain satisfactorily why the transverse and horizontal contours of the two races should bear the observed relation to each other.

From the meagre material available we may conclude that there are no well-marked sexual differences between the horizontal and transverse type sections of the same race except those of absolute size. The sagittal sections, however, seem to possess some sexual characteristics. Nearly all are associated with the shape of the frontal region of the calvaria and they are well known to craniologists although previously they do not appear to have been exactly measured. In columns 1 and 2 of Table XL on p. 102 the ratio that the ordinate $N\frac{1}{4}$ above the $N\gamma$ line is of the length of the section ($N\gamma$) will be found expressed as a percentage, and that index is smaller for all the ♂ sections than for the ♀. The differences would appear to be quite significant in the case of every race considered except the Burmese A. The ♀ $N\frac{1}{4}$ ordinates of the other five races are actually greater than the ♂. If all the other ordinates above the $N\gamma$ line are expressed in terms of the corresponding lengths of the sections, then it will be found that the sexual difference between the indices in

the case of any one race is greatest for the index $100\,N\frac{1}{4}/N\gamma$. The ♀ values of all the races shown in Table XL, except the Burmese A, exceed the ♂ in the cases $100\,I/N\gamma$, $100\,II/N\gamma$, $100\,III/N\gamma$ and $100\,IV/N\gamma$, but the differences between the indices involving more posterior ordinates are evidently insignificant. The sexual difference in the curvature of the frontal bones may be supposed to be due to the lesser development of the frontal sinuses of the female skull.

Indices formed by dividing the y's of the frontal subtense (see Table XXXVIII, p. 93) by the $N\beta$ lengths are shown in columns 8 and 9 of Table XL and the sexual difference is again well marked, though least for the Burmese A. A similar index for the parietal portions of the sections—from β to λ—shows smaller divergences, but they are still all of the same sign: the curvatures of the ♂ and ♀ occipital arcs from λ to opisthion do not differ.

The female frontal bone has not only a greater curvature than the male, but it is also thrown more forward as in the infantile skull. This may be seen by comparing (see Table XLI, p. 102) the bregmatic angles ($\angle\,\beta N\gamma$) or the "physiognomic angles of flatness*." The latter is defined to be the angle subtended at the nasion by the point of maximum frontal subtense and the gamma; it is found by adding the bregmatic angle to the angle between the line joining the point of maximum subtense to the nasion and the $N\beta$ line. The sexual difference is more due to "frontal bone flatness" than to the rotation of the frontal bone, but both are characteristic. The least differences are found again between the ♂ and ♀ Burmese A types.

10. MEASUREMENTS OF THE MANDIBLES IN THE HODGSON COLLECTION.

In the Hodgson Collection there are nineteen Nepalese ♂, nine Tibetan A ♂ and one Tibetan A ♀ adult mandibles and one of a young male Tibetan. Full measurements of these were taken in the way described in Appendix I to "A First Study of the Tibetan Skull" (*Biometrika*, Vol. XIV. p. 253) and for an account of the definitions of "points," measurements and methods of measurement employed the reader is referred to that paper† which provides mean measurements of 16 ♂ Tibetan A and 12 ♂ Tibetan B mandibles in the Museum of the Royal College of Surgeons. The means of the former series were pooled with the nine British Museum specimens to give the Tibetan A values in Table XLII, p. 103. The material is so scanty that a detailed comparison of the racial means would serve no useful purpose. The Nepalese and Tibetan A types appear to be remarkably similar so that the majority of their mandibular measurements differ by less than 1 mm., while the alien Tibetan B type is clearly distinguished from the other two by having greater lengths and heights. The Tibetan A slightly exceed the Nepalese in width characters but not in length. "Remarks" relating to the mandibles will be found in the Appendix to this paper. One Tibetan specimen (1 *a.a.a.a.a.*) has a condylar height ($c_y h$) exceeding its coronoidal height ($c_r h$), which is quite a rare condition, and there are three cases for which the intradental height ($d_i h$) is greater than the height of the lowest point of the incisura (ih).

* See *Biometrika*, Vol. XI. p. 110.

† The young Tibetan mandible had no second molars, and accordingly it was supposed to be in the "standard basal position" when the first left molar was held (see *Biometrika*, Vol. XIV. pp. 253, 254).

TABLE XL.

Sexual Differences between Measurements of the Sagittal Contour.

Race	100 $N\frac{1}{4}/N\gamma$			Frontal Arc							Parietal Arc						
	♂ Index	♀ Index	♂ − ♀ Index	♂ $N\beta$	♀ $N\beta$	♂ Sub.	♀ Sub.	♂ 100 Sub./$N\beta$	♀ 100 Sub./$N\beta$	♂ − ♀ 100 Sub./$N\beta$	♂ $\beta\lambda$	♀ $\beta\lambda$	♂ Sub.	♀ Sub.	♂ 100 Sub./$\beta\lambda$	♀ 100 Sub./$\beta\lambda$	♂ − ♀ 100 Sub./$\beta\lambda$
Hindu ...	24·2	27·7	−3·5	110·5	103·9	25·0	26·4	22·6	25·4	−2·8	109·0	105·5	21·2	21·4	19·4	20·3	−0·9
Nepalese .	25·3	28·1	−2·8	109·2	105·5	25·5	25·3	23·4	24·0	−0·6	114·0	109·7	24·1	24·0	21·1	21·9	−0·8
Burmese A	26·3	26·5	−0·2	112·6	106·7	26·1	25·2	23·2	23·6	−0·4	110·0	106·8	24·6	24·3	22·4	22·8	−0·4
Burmese B	21·1	25·2	−4·1	111·3	106·0	24·0	23·9	21·6	22·5	−0·9	110·3	105·0	24·7	23·8	22·4	22·7	−0·3
Burmese C	20·9	24·5	−3·6	113·1	108·2	25·0	24·8	22·1	22·9	−0·8	108·7	106·4	23·1	22·9	21·3	21·5	−0·2
Moriori .	16·9	18·6	−1·7	112·2	107·5	21·7	22·7	19·3	21·1	−1·8	111·7	107·1	22·0	21·5	19·7	20·1	−0·4

TABLE XLI.

The Physiognomic Angle of Flatness.

Race	Angle of Frontal Bone Flatness			Bregmatic Angle			Physiognomic Angle of Flatness		
	♂	♀	♂ − ♀	♂	♀	♂ − ♀	♂	♀	♂ − ♀
Hindu ...	28°·4	31°·3	−2°·9	49°·5	51°·4	−1°·9	77°·9	82°·7	−4°·8
Nepalese ...	27°·2	29°·0	−1°·8	51°·9	53°·2	−1°·3	79°·1	82°·2	−3°·1
Burmese A ...	26°·7	27°·8	−1°·1	51°·8	51°·6	+0°·2	78°·5	79°·4	−0°·9
Burmese B ...	24°·2	26°·3	−2°·1	50°·7	51°·8	−1°·1	74°·9	78°·1	−3°·2
Burmese C ...	25°·2	26°·9	−1°·7	49°·6	50°·9	−1°·3	74°·8	77°·8	−3°·0
Moriori ...	22°·3	23°·4	−1°·1	49°·3	49°·8	−0°·5	71°·6	73°·2	−1°·6

TABLE XLII.

Mandibular Mean Measurements.

Race		Lengths with Callipers												
		w_1	w_2	h_1	zz	$c_r c_r$	rb	rb'	G_2'	$c_y c_r$	$g_o g_o$	$g_n g_o$ (l)	$g_n g_o$ (r)	$c_y l$
Nepalese ♂	Mean	115·0	96·2	32·0	46·2	93·4	36·7	31·9	45·4	33·8	92·2	83·2	83·8	18·9
	No.	19	19	18	19	19	19	19	18	19	19	19	19	19
Tibetan A ♂	Mean	117·0	96·2	30·6	45·7	93·8	37·2	32·1	46·7	34·2	92·8	83·8	83·8	18·8
	No.	25	25	25	25	25	25	25	25	25	24	24	24	25
Tibetan B ♂	Mean	122·0	99·4	35·9	47·0	93·9	37·3	34·2	48·6	38·1	96·5	88·7	88·0	19·9
	No.	12	12	12	12	12	12	12	12	12	11	11	12	12
Tibetan A ♀	(1)	119·0	102·0	30·9	45·9	97·6	37·2	35·7	44·7	31·8	96·8?	87·0?	86·6?	20·2
Tibetan A Young ♂ ...	(1)	100·4	78·9	23·8	40·8	81·2	31·6	28·0	35·6	26·6	75·4	64·8	65·6	15·2

Race		Lengths with Callipers							Tape	Lengths and Heights on Board				
		$c_y b$	$m_2 p_1$	$p_a d_t$	$p_a g_n$	$p_a d_f$	$g_n d_f$	$p_a p_b$	$g_o p_a g_o$	ih	ih'	$c_r h$	$c_y h$	$d_t h$
Nepalese ♂	Mean	7·9	29·5	25·2	9·5	22·0	29·6	3·1	192·3	42·2	13·8	61·3	51·5	35·2
	No.	19	19	18	19	18	18	18	19	19	19	19	19	18
Tibetan A ♂	Mean	8·1	29·2	25·4	8·2	23·0	28·3	3·6	190·4	41·8	15·0	60·8	50 8	35·0
	No.	25	24	25	25	24	25	25	24	25	25	25	25	24
Tibetan B ♂	Mean	8·2	29·2	31·2	7·8	28·1	33·8	4·0	201·0	43·6	16·6	65·2	54·9	40·1
	No.	12	12	12	12	12	12	7	12	12	12	12	12	12
Tibetan A ♀	(1)	11·2	30·2	25·6	8·2	23·1	27·9	2·2	206?	49·8	13·1	68·1	58·9	31·5
Tibetan A Young ♂ ...	(1)	7·7	—	17·0	8·1	14·9	21·7	1·7	152	31·3	12·1	47·5	38·1	24·0

TABLE XLII—(*continued*).

Race		Lengths and Heights on Board					Indices								
		m_2h	p_1h	c_pl	rl	ml	c_rh/ml	c_rc_r/ml	g_og_o/c_pl	rb'/rl	c_yb/c_yl	g_og_o/c_rc_r	c_yh/c_rh	ih'/c_rc_y	d_th/c_rh
Nepalese ♂	Mean	24·5	28·7	74·3	58·6	104·5	58·8	90·2	124·6	55·6	41·8	98·9	84·0	41·3	57·6
	No.	19	18	19	19	19	19	19	19	19	19	19	19	19	18
Tibetan A ♂	Mean	24·0	28·9	74·6	58·4	105·2	57·8	89·2	124·9	55·3	43·3	98·9	83·8	44·0	57·9
	No.	25	25	25	25	25	25	25	24	25	25	24	25	25	24
Tibetan B ♂	Mean	27·6	33·9	78·9	61·0	109·6	59·7	86·0	121·7	56·3	41·3	102·9	84·0	43·8	61·7
	No.	12	12	12	12	12	12	12	11	12	12	11	12	12	12
Tibetan A ♀	(1)	25·3	28·3	78·0	61·1	101·1	67·4	96·5	124·1 ?	58·4	55·4	99·2 ?	86·5	41·2	46·3
Tibetan A Young ♂ ...	(1)	—	—	56·7	43·7	80·2	59·2	101·2	133·0	64·1	50·7	92·9	80·2	45·5	50·5

Race		Angles								
		$M\angle$	$R\angle$	$G\angle$	$C\angle$	$C'\angle$	$L\angle$	$L'\angle$	$F\angle$	$S\angle$
Nepalese ♂	Mean	124°·8	72°·4	66°·9	67°·3	66°·4	93°·0	92°·1	92°·0	26°·6
	No.	19	19	14	18	18	16	16	17	16
Tibetan A ♂	Mean	125°·3	70°·5	67°·1	64°·8	62°·9	95°·0	93°·5	90°·6	32°·1
	No.	25	25	24	25	24	24	24	25	24
Tibetan B ♂	Mean	123°·0	73°·1	65°·9	66°·7	63°·1	101°·1	99°·8	95°·5	38°·0
	No.	12	12	11	12	12	7	7	7	7
Tibetan A ♀	(1)	113°·8	83°·2	67°·7?	70°·7	67°·9	81°·0	81°·0	89°·0	13°·1
Tibetan A Young ♂ ...	(1)	125°·9	74°·3	70°·4	77°·9	79°·8	102°·2	103°·0?	96°·7?	22°·4

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APPENDIX.

Index letter	Sex	C	Lengths F	L′	L	B	B′	H
1 *d.d.d.d.d.* ...	♀	1245	170	172	170·5	124·4	87·6	127·5
1 *g.g.g.g.g.* ...	,,	1220	163	165	163·5	123·7	89·1	127·1
1 *i.i.i.i.i.* ...	,,	1240	164·5	171·5	165	125·1	90·0	130·0
1 *l.l.l.l.*	♀	—	163?	167?	163?	127·0	79·4	—
1 *n.*	♂	—	161?	162?	159?	128·5	81·2	—
1 *c.e.e.*	♂	1480	174·5	180	177	138·1	93·6	127·2
1 *n.n.n.*	,,	1480	179·5	182·5	180	138·3	94·3	136·2
1 *p.p.p.*	,,	1275	172	178	176	128·4	85·2	121·0
1 *q.q.q.*	,,	1540	187	186	186	143·0	98·9	137·9
1 *r.r.r.*	,,	1460	173	176	172	139·9	91·0	130·1
1 *q.q.q.q.* ...	,,	1275	167·5	172	168	130·3	86·2	128·5
1 *r.r.r.r.* ...	,,	—	173·5	—	174	—	90·9	—
1 *a.*	,,	1440	175	181	179	138·2	89·1	132·6
1 *b.*	,,	1270	167	171	169	128·1	93·1	125·0
1 *c.*	,,	1620	182·5	185	181	138·4	93·0	136·4
1 *d.*	,,	1480	183	185	183·5	136·2	93·8	138·1
1 *a.a.a.a.a.* ...	,,	1685	177	179·5	177	141·9	90·0	138·2
1 *z.z.z.z.* ...	,,	1335	156	162	159	142·7	95·0	123·4
1 *m.m.m.m.m.* ...	,,	1420	168	171	169	144·5	92·2	132·1
1 *n.n.n.n.n.* ...	,,	1355	166	166·5	166	139·1	90·7	126·1
1 *o.o.o.o.o.* ...	,,	1530?	185	—	186·5	—	—	—
1 *p.p.p.p.p.* ...	,,	1380	170·8	175	172	139·9	91·2	133·7
1 *j.j.j.j.j.* ...	,,	1670	182	184·5	182·5	146·5	89·8	128·2
1 *k.k.k.k.k.* ...	,,	1600	181·5	185·5	184	134·6	98·8	139·0
1 *u.u.u.u.u.* ...	,,	1630	185·5	190·5	188	137·0	87·2	139·6
1 *j.j.j.j.*	♀	1580	162	166·5	163	143·9	81·4	132·0
1 *q.q.q.q.q.* ...	,,	1225	165	167·5	164	131·2	84·9	129·0
1 *t.t.t.t.t.* ...	,,	1365	174	175	172	127·8	82·3	130·1
1 *g.g.g.g.g.g.* ...	♂	1490	164	163·5	162	138·2	91·1	122·1
1 *i.i.i.i.i.i.* ...	,,	—	143	145	143	130·1	84·1	—
1 *o.*	♂	1360	181	174·5	180	129·8	90·5	133·7
1 *m.m.m.* ...	,,	1400	174·5	181	179	135·0	93·1	128·3
1 *p.*	♂	1300	166·5	—	165	132·0	82·9	—
1 *n.n.n.n.* ...	♂	1455	181·5	185·5	183	129·0	91·9	129·0
1 *o.o.o.o.* ...	,,	1335	177	180	177	126·1	91·4	127·8
1 *p.p.p.p.* ...	♀	1140	163	165·5	163	125·0	84·2	121·8

K.

	Lengths					Arcs								Face		
	B'	*H*	*H'*	*OH*	*LB*	*Q*	*Q'*	*S*	S_1	S_2	S_3	S_4'	*U*	*PH*	*G'H*	*GE*
·4	87·6	127·5	127·1	110·7	90·2	287	288	356	125	113	118	96·7	476	17·3	65·7	92·
·7	89·1	127·1	128·0	113·2	89·8	288	287	350·5	120·5	120	110	91·0	457	16·5	56·8	—
·1	90·0	130·0	129·3	111·4	97·0	288·5	289	348	123	119	106	88·4	470	18·4	66·1	92·
·0	79·4	—	—	107·5?	—	275·5?	274?	345·5	123·5	121	101	82·3	455?	13·6	49·7?	70·
·5	81·2	—	—	114·4?	—	276·5?	280?	—	110	119·5	—	—	443	15·0	48·2	75·
·1	93·6	127·2	123·5	117·8	100·0	288	290	349	120	115·5	113·5	96·6	503·5	19·2	70·7	98·
·3	94·3	136·2	137·2	118·9	98·5	308	309·5	367	141	121	105	89·8	502	18·8	70·0	94·
·4	85·2	121·0	121·1	100·4	95·1	270·5	271	344·5	122	123	99·5	84·1	481	13·7	64·9	91·
·0	98·9	137·9	134·1	121·2	101·0	312	311·5	374·5	120·5	139·5	114·5	92·9	530	23·2	76·6	110·
·9	91·0	130·1	128·1	115·9	99·2	306·5	308·5	349·5	119·5	119	111·0	93·7	500·5	18·2	67·1	88·
·3	86·2	128·5	127·2	112·5	87·8	291	290	350·5	120	123	107·5	89·2	469	17·2	61·2	91·
	90·9	—	—	—	98·8	—	—	356	130·5	115·5	110	94·8	—	14·6	67·3	99·
·2	89·1	132·6	130·6	115·2	95·6	307·5	305	355	121·5	126·5	107	96·4	508	20·3	71·0	99·
·1	93·1	125·0	125·1	110·7	100·9	290·5	290·5	334·5	113·5	116	105	88·0	481	15·9	54·1	101·
·4	93·0	136·4	135·0	116·3	98·1	306·5	304·5	388·5	129	131	128·5	103·9	504	16·1	71·1	86·
·2	93·8	138·1	135·9	116·7	96·2	302	300·5	379	131	116·5	131·5	103·7	503	13·8	65·2	108·
·9	90·0	138·2	139·3	124·5	97·2	324	324	383·5	138·5	131·5	113·5	97·2	512	16·9	74·6	110·
·7	95·0	123·4	121·9	111·4	94·0	301·5	300	335	120·5	115·5	99	87·0	476	12·8	60·1	98·
·5	92·2	132·1	131·9	115·4	101·2	305·5	304	349	123·5	120·5	105	89·2	498·5	17·7	70·7	102·
·1	90·7	126·1	126·4	111·5	91·1	296	294	340	116·5	116	110·5	92·1	488	—	—	106·
	—	—	—	—	103·3	—	—	381·5	132·5	133	116	96·9	—	15·8	63·1	—
·9	91·2	133·7	131·3	111·4	102·1	295	296	347·5	122·5	118·5	106·5	92·3	500·5	16·0	68·9	108·
·5	89·8	128·2	126·2	114·4	93·1	312·5	315	372	121	137	114	95·9	529·5	21·2	73·2	99·
·6	98·8	139·0	139·1	125·3	99·6	320·5	319	388	138	141·5	108·5	94·1	511	19·2	68·8	92·
·0	87·2	139·6	137·7	119·5	105·1	306·5	308·5	384	126·5	137·5	120	100·2	519	17·6	69·2	97·
·9	81·4	132·0	129·8	114·2	93·8	310·5	311	354	117·5	129·5	107	96·2	484	21·6	68·1	94·
·2	84·9	129·0	127·2	111·1	90·1	298·5?	301?	351·5	120	119·5	112	95·3	480?	18·1	63·2	92·
·8	82·3	130·1	128·1	109·9	93·0	288	288	358	118	125	115	97·2	485	16·9	62·9	92·
·2	91·1	122·1	121·3	113·7	85·3	308	312	344·5	119	118	107·5	89·9	469	17·6	61·0	86·
·1	84·1	—	—	100·3	—	274·5?	278?	324?	107?	116?	91?	83·0	429?	11·5	41·8?	74·
·8	90·5	133·7	134·2	114·6	105·4	295	297	377	128	131	118	93·3	495	20·7	70·1	93·
·0	93·1	128·3	126·4	108·6	94·8	289·5	288	360	115·5	126·5	118	99·1	499	19·0	67·1	87·
·0	82·9	—	—	—	—	—	—	354	127·5	124	102·5	84·9	455	—	—	—
·0	91·9	129·0	129·8	110·2	92·9	287	288	377	125·5	135	116·5	94·0	501	—	—	87·
·1	91·4	127·8	127·0	113·7	91·1	296	296·5	368·5	130	134	104·5	83·9	494·5	21·2	68·9	101·
·0	84·2	121·8	120·8	102·5	85·0?	270	271·5	327·5	118	110·5	99	81·1	468	—	—	91·

Face				Nose								Orbits			
PH	*G'H*	*GB*	*J*	*NH R*	*NH L*	*NB*	*DS*	*DC*	*DA*	*SS*	*SC*	O_1R	O_1L	O_2R	O_2L
17·3	65·7	92·7	117·1	48·9	49·1	27·8	7·5	20·0	26·8	2·4	7·9	38·8	37·4	32·3	32·
16·5	56·8	—	—	40·2	41·1	23·0	8·5	20·1	27·5	2·5	9·8	—	36·5	—	30·
18·4	66·1	92·5	—	47·6	50·8	24·1	12·5	20·6	36·6	5·0	9·7	40·1	40·4	29·8	29·
13·6	49·7?	70·9	94·0?	35·8	37·0	19·5	—	—	—	—	—	33·3	33·5	32·8	33·
15·0	48·2	75·0	—	34·1	33·8	19·0	8·1	16·9	25·6	1·8	7·8	32·9	32·8	29·1	29·
19·2	70·7	98·1	128·3	53·1	51·0	26·4	10·6	22·9	33·0	3·7	10·2	44·3	43·1	36·6	37·
18·8	70·0	94·0	125·6	50·8	50·2	22·5	13·1	22·3	37·2	4·6	12·8	41·8	41·1	32·6	32·
13·7	64·9	91·8	122·2	51·1	51·0	26·1	11·9	20·3	31·9	3·1	7·2	43·1	41·2	30·2	30·
23·2	76·6	110·5	139·0	55·4	53·7	29·0	10·4	31·2	38·6	2·7	9·9	44·4	43·0	35·8	34·
18·2	67·1	88·6	133·9	49·0	48·9	23·6	11·2	19·3	31·5	3·5	10·9	43·1	41·2	33·4	34·
17·2	61·2	91·9	118·0	46·1	46·1	24·9	8·5	21·9	28·3	2·5	8·1	36·6	36·9	32·6	32·
14·6	67·3	99·0	—	51·4	51·8	25·9	8·0	20·2	29·1	2·7	10·4	40·3	39·7	32·6	34·
20·3	71·0	99·8	133·0	50·8	50·1	28·0	11·6	24·9	35·7	1·3	6·1	42·5	41·1	34·1	35·
15·9	54·1	101·1	133·6	42·8	42·5	28·6	9·9	23·9	33·6	1·8	9·3	42·0	41·7	30·7	31·
16·1	71·1	86·4	122·8	53·3	53·4	19·2	11·8	17·1	28·2	3·1	5·8	38·0	37·2	35·5	36·
13·8	65·2	108·7	132·8	51·6	51·8	25·8	8·3	23·6	29·9	1·3	6·1	43·0	41·1	30·3	32·
16·9	74·6	110·5	139·2	57·4	56·3	26·0	8·8	20·9	29·1	2·1	7·0	40·9	40·1	32·3	31·
12·8	60·1	98·0	129·2	47·9	47·3	26·3	10·4	21·1	29·3	1·9	8·9	42·3	42·3	31·8	31·
17·7	70·7	102·2	138·2	53·4	53·8	28·6	11·6	22·6	34·8	4·3	10·2	40·3	40·3	34·4	34·
—	—	109·0	132·7	48·9	50·0	30·1	8·5	26·3	33·4	1·9	8·8	44·4	44·1	32·1	33·
15·8	63·1	—	—	—	47·0	25·5	—	—	—	—	—	41·9	42·9	28·6	29·
16·0	68·9	108·7	141·8	53·1	52·8	26·0	9·7	23·0	31·5	3·1	9·8	45·2	43·1	34·1	34·
21·2	73·2	99·3	133·4	52·2	52·8	26·0	9·4	26·9	31·0	2·2	10·0	41·1	39·4	34·6	35·
19·2	68·8	92·1	130·0	52·7	52·2	27·2	10·9	24·4	31·8	1·7	4·7	44·2	42·3	36·2	35·
17·6	69·2	97·9	125·3	52·5	51·8	22·8	12·0	17·8	31·3	3·8	7·1	46·7	46·1	34·9	36·
21·6	68·1	94·2	—	48·4	49·1	25·6	7·4	19·7?	26·9	2·1	6·2	42·0	42·0	—	35·
18·1	63·2	92·9	—	47·9	48·8	26·1	6·9	17·2	24·9	2·2	7·7	41·1	40·1	33·8	33·
16·9	62·9	92·2	118·3	45·8	46·4	27·6	9·9	17·0	28·0	1·5	5·7	41·5	41·6	32·8	31·
17·6	61·0	86·0	111·1	42·9	43·0	22·3	6·8	18·4	22·2	1·6	6·8	38·3	37·6	34·7	34·
11·5	41·8?	74·5	98·8	31·1?	32·1?	19·3	—	—	—	—	—	34·2	34·7	31·9	30·
20·7	70·1	93·8	—	48·9	50·1	25·3	14·3	22·8	36·8	4·7	10·1	44·1	43·3	32·0	32·
19·0	67·1	87·2	125·1	50·1	49·1	23·9	10·5	21·2	30·3	3·0	8·1	43·9	42·6	32·1	33·
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	87·6	119·2	44·3	44·9	25·2	11·4	20·0	31·6	3·1	8·2	40·2	39·9	32·1	33·
21·2	68·9	101·3	—	48·0	47·8	23·9	10·7	20·0	32·7	2·6	8·2	43·1	41·0	31·1	31·
—	—	91·0	120·8	—	—	23·9	—	18·7	—	—	—	39·8	38·0	32·5	33·

I. NEPALESE (*continued*).

Orbits			Palate and Profile						Foramen							
O_2R	O_2L	O_1'	G_1	G_1'	G_2	GL	EB	EH	fml	fmb	$Oc.I$	B/L'	H/L'	B/L	H/L	B/H
32·3	32·9	37·4	48·6	45·1	38·4	86·2	39·5	12·2	31·8	25·1	58·6	72·3	74·1	73·0	74·8	97·6
—	30·3	—	44·9	41·2	34·0	87·4	34·1	8·9	34·4	26·9	59·4	75·0	75·8	75·8	78·8	97·3
9·8	29·9	37·8	43·9	45·2	37·1	89·0	36·8	11·7	35·8	—	60·1	72·0	77·0	75·7	77·7	96·2
2·8	33·8	33·0	35·2	32·2	—	—	—	—	—	—	58·2	76·0?	—	77·9?	—	—
29·1	29·0	31·2	34·9	33·7	—	—	—	—	—	—	—	79·3?	—	80·8?	—	—

II. TIBETAN A.

30·6	37·3	42·0	52·2	48·1	43·6	99·4	42·8	12·3	36·2	31·7	62·2	76·7	70·7	78·0	71·9	108·6
2·6	32·1	39·2	49·1	45·2	40·8	91·0	40·4	14·1	35·0	28·8	62·8	75·8	74·6	76·8	75·7	101·5
30·2	30·9	38·4	48·5	45·9	36·6	93·3	37·2	12·3	37·7	30·1	61·4	72·1	68·0	73·0	68·8	106·1
35·8	34·8	40·1	54·0	54·0	43·3	101·2	43·1	15·0	37·1	29·9	57·9	76·9	74·1	76·9	74·1	103·7
33·4	34·0	40·6	47·6	46·3	40·1	94·8	39·9	13·4	37·4	31·2	61·2	79·5	73·9	81·3	75·6	107·5
32·6	32·1	34·1	44·7	41·2	35·8	83·9	35·2	10·7	36·9	27·2	59·6	75·8	74·7	77·6	76·5	101·4
32·6	34·1	37·9	45·2	42·9	41·5	94·7	41·7	10·0	38·4	30·8	63·8	—	—	—	—	—
34·1	35·8	38·3	50·9	47·7	41·0	98·2	40·4	15·1	41·6	34·3	72·0	76·4	73·3	77·2	73·0	104·2
30·7	31·0	38·6	55·1	50·2	38·1	104·2	37·9	11·2	33·5	28·9	60·5	74·9	73·1	76·3	74·4	102·5
35·5	36·8	35·9	47·2	43·6	35·8	87·8	36·0	12·3	33·9	30·9	57·7	74·8	73·8	76·5	75·4	101·5
30·3	32·6	41·2	46·0	42·9	44·2	88·4	44·6	13·9	34·6	25·2	56·3	73·6	74·6	74·2	75·3	98·6
2·3	31·8	38·8	49·2	44·8	44·6	96·3	44·9	10·3	35·9	30·6	62·9	79·1	77·0	80·2	78·1	102·7
31·8	31·2	40·9	—	—	42·6	93·9	42·6	—	30·9	27·6	66·7	88·1	76·2	89·7	77·6	115·6
34·4	34·2	39·8	48·1	44·9	44·4	97·3	45·0	15·7	33·2	28·4	62·0	84·5	77·3	85·5	78·2	109·4
32·1	33·9	41·7	58·0	52·7	44·2	—	44·0	15·9	38·8	30·4	60·0	83·5	75·7	83·8	76·0	110·3
28·6	29·5	39·8	48·1	45·8	38·7	96·7	38·1	12·1	34·1	25·4	60·2	—	—	—	—	—
34·1	34·5	41·7	48·7	45·9	46·6	97·0	46·2	14·0	35·7	30·8	64·5	79·9	76·4	81·3	77·7	104·6
34·6	35·7	37·5	47·1	43·0	40·0	91·7	40·0	9·1	39·2	28·3	60·9	79·4	69·5	80·3	70·2	114·2
36·2	35·6	41·3	45·2	41·4	37·8	86·2	37·4	12·9	40·7	32·1	64·6	72·6	74·9	73·2	75·5	96·8
34·9	36·3	43·1	—	—	39·1	92·6	39·1	9·9	36·1	28·9	60·2	71·9	73·3	72·9	74·3	98·1
—	35·1	40·4	48·2	45·0	43·4	87·1	42·9	18·3	39·0	34·8	71·5	86·4	79·3	88·3	81·0	109·0
33·8	33·1	39·7	43·5	40·4	39·2	86·6	39·2	12·9	31·0	25·8	62·1	78·3	77·0	80·0	78·7	101·7
32·8	31·6	38·2	—	—	38·7	94·6	38·8	13·3	35·9	30·6	61·4	73·0	74·3	74·3	75·6	98·2
34·7	34·9	37·1	37·2	34·4	—	81·8	—	—	34·2	27·2	60·3	84·5	74·7	85·3	75·4	113·2
31·9	30·8	32·3	29·8	27·7	—	—	—	—	—	27·8	75·6	89·7	—	91·0	—	—

III. BENGALEE.

2·0	32·0	41·9	—	—	39·1	106·1	38·9	16·6	33·8	32·1	56·4	74·4	76·2	72·1	74·3	97·1
2·1	33·0	41·9	48·8	43·9	38·8	90·9	39·0	13·9	38·1	31·6	60·7	74·6	70·9	75·4	71·7	105·2
—	—	—	—	—	—	—	—	—	—	—	59·5	—	—	80·0	—	—

IV. URAON (Dravidian).

2·1	33·1	37·6	—	—	—	—	—	—	36·2	28·7	57·6	69·5	69·5	70·5	70·5	100·0
1·1	31·1	40·8	50·0	46·2	42·9	89·3	42·9	13·1	35·7	28·1	57·2	70·1	71·0	71·2	72·2	99·3
2·5	33·1	36·1	—	—	—	—	—	—	37·0	28·7	58·6	75·5	73·6	76·6	74·7	102·6

TABLE OF ABSOLUTE AND INDICIAL MEASU

Indices

	H/L	B/H	H'/L	B/H'	$\frac{(B-H')}{L}$	$G'H/GB$	$\frac{NB/NH}{R}$	$\frac{NB/NH}{L}$	$\frac{O_2/O_1}{R}$	$\frac{O_2/O_1}{L}$	O_2/O_1'	G_2/G_1	EH/EB	fmb/fml	DS/DC
	74·8	97·6	74·5	97·9	−1·6	70·9	56·9	56·6	73·1	88·0	86·4	79·0	30·9	78·9	37·5
	78·8	97·3	78·4	96·8	−2·5	—	57·2	47·4	—	83·0	—	75·7	26·1	78·2	60·7
	77·7	96·2	78·3	96·6	−2·6	71·5	50·6	56·0	74·3	74·0	78·8	75·9	31·8	—	42·3
?	—	—	—	—	—	70·1?	54·5	52·7	98·5	100·9	99·4	—	—	—	—
?	—	—	—	—	—	64·2	55·7	56·2	88·4	88·4	93·3	—	—	—	47·9
	71·9	108·6	69·8	111·8	8·2	72·1	49·7	51·8	82·6	86·5	87·1	83·5	28·7	87·6	46·3
	75·7	101·5	76·2	100·8	0·6	74·5	44·3	44·8	78·0	78·1	83·2	83·1	34·9	82·3	58·7
	68·8	106·1	68·8	106·0	4·1	70·7	51·1	51·2	70·1	75·0	78·6	75·5	33·1	79·8	58·6
	74·1	103·7	72·1	106·6	4·8	69·3	52·3	54·0	80·6	80·9	89·3	80·2	34·8	80·6	33·3
	75·6	107·5	74·5	109·2	6·9	75·7	48·2	48·3	77·5	82·5	82·3	84·2	33·6	83·4	58·0
	76·5	101·4	75·7	102·4	1·8	66·6	54·0	54·0	88·9	87·0	95·6	80·1	30·4	73·7	38·8
	—	—	—	—	—	68·0	50·4	50·0	80·9	85·9	86·0	91·8	24·0	80·2	39·6
	73·0	104·2	73·0	105·8	4·2	71·1	55·1	55·9	80·2	87·1	89·0	80·6	37·4	82·5	46·6
	74·4	102·5	74·5	102·4	1·8	53·5	66·8	67·3	73·1	74·3	79·5	69·1	29·6	86·3	41·4
	75·4	101·5	74·6	102·5	1·9	82·3	36·0	36·0	93·4	98·9	98·9	75·8	34·2	91·2	69·0
	75·3	98·6	74·1	100·2	0·2	60·0	50·0	49·8	70·5	79·3	73·5	96·1	31·2	72·8	35·2
	78·1	102·7	78·7	101·9	1·5	67·5	45·3	46·2	9·0	79·3	83·2	90·7	22·9	85·2	42·1
	77·6	115·6	76·7	117·1	13·1	61·3	54·9	55·6	75·2	73·8	77·8	—	—	89·3	49·3
	78·2	109·4	78·0	110·5	7·5	69·2	53·6	53·2	85·4	84·9	86·4	92·3	34·9	85·5	51·3
	76·0	110·3	76·1	110·0	7·7	—	61·6	60·2	72·3	76·9	77·0	76·2	36·1	88·4	32·3
	—	—	—	—	—	—	—	54·3	68·3	68·8	71·9	80·5	31·8	74·5	—
	77·7	104·6	76·3	106·5	5·0	63·4	48·9	49·2	75·4	80·0	81·8	95·7	30·3	86·3	42·2
	70·2	114·2	69·2	116·1	11·1	73·7	49·8	49·2	84·2	90·6	92·2	84·9	22·8	72·2	34·9
	75·5	96·8	75·6	96·8	−2·4	74·7	51·6	52·1	81·9	84·2	87·7	83·6	34·5	78·9	44·7
	74·3	98·1	73·2	99·5	−0·4	70·7	43·4	44·0	84·1	78·7	81·0	92·3	25·3	76·8	67·4
	81·0	109·0	79·6	110·9	8·7	72·3	52·9	52·1	—	83·6	—	90·0	42·7	89·2	37·6?
	78·7	101·7	77·5	103·1	2·4	68·0	54·5	53·5	82·2	82·5	85·1	90·1	32·9	83·2	40·1
	75·6	98·2	74·5	99·8	−0·2	68·2	60·3	59·5	79·0	76·0	85·9	—	34·3	85·2	58·2
	75·4	113·2	74·9	113·9	10·4	70·9	52·0	51·9	90·6	92·8	93·5	—	—	79·5	37·0
	—	—	—	—	—	56·1?	62·1?	60·1?	93·3	88·8	98·8	—	—	—	—
	74·3	97·1	74·6	96·7	−2·4	74·7	51·7	50·5	72·6	73·9	76·4	—	42·7	94·9	62·7
	71·7	105·2	70·6	106·8	4·8	76·9	47·7	48·7	73·1	77·5	76·6	79·5	35·6	82·9	49·1
	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
	70·5	100·0	70·9	99·4	−0·4	—	56·9	56·1	79·9	83·0	85·4	—	—	79·3	57·0
	72·2	99·3	71·8	99·3	−0·5	68·0	48·8	50·0	72·2	75·9	76·2	85·8	30·5	78·7	53·5
	74·7	102·6	74·1	103·5	2·6	—	—	—	81·7	87·1	90·0	—	—	77·6	—

* For an explanation of the

REMENTS OF THE INDIVIDUAL CRANIA IN THE HODGSON COLLECTIO

	Angles						
SS/SC	*N∠*	*A∠*	*B∠*	θ_1	θ_2	*P∠*	Remarks
30·4	64°·8	71°·8	43°·4	30°·2	13°·2	85°·0	Lowlander. Nepalese Terai. P.B. downwards with rounded and double
51·5	68°·9	73°·9	37°·2	21°·1	16°·1	84°·9	Lowlander. Nepalese Terai. Young, about 12 years; P.B. flat with blu
25·5	60°·1	78°·1	41°·8	35°·0	6°·8	90°·0	Lowlander. Nepalese Terai. Aging; P.B. downwards with blunt edge; S
—	—	—	—	—	—	92°·7?	Murmi. Valley of Nepal. Young, about 7 years; inner palate bridge L epipteric bone L.; 2 ossicles lambdoid L.; basilar portion of occipita
23·1	—	—	—	—	—	90°·3?	Newar? Found on banks of Bagmatti, Valley of Nepal. Young, age?; squama temporalis articulating with frontal bone; wormians lambd
36·3	68°·9	69°·9	41°·2	29°·3	11°·9	81°·8	Khampa. East Tibet. P.B. downwards with blunt edge; SR; J equal;
35·9	62°·8	74°·2	43°·0	26°·5	16°·5	90°·7	From Sakya, Province of Utsang, South Tibet. Mandible; lower 3rd edge; SR; JR; epipteric bones R. and L.; ossicle of λ and wormians
43·1	68°·1	71°·5	40°·4	31°·9	8°·5	80°·0	Bhotia from Ukyaba. Tibet. Mandible; adult but no upper 3rd molars with rounded edge; SR; JR; traces of horizontal sutures across mal
27·3	67°·9	67°·9	42°·0	24°·2	14°·0	81°·9	Inu-Bhotia. Tibet. Sagittal suture closing; P.B. flat with blunt edge; S
32·1	63°·8	76°·0	40°·2	30°·1	10°·1	86°·1	Sokpa. Koko-Nor district, Tibet. Mandible; tympanic perforation R. lambdoid R. and L.
30·9	65°·6	72°·8	41°·6	28°·8	12°·8	85°·6	Dirge-Bhotia. Khams, East Tibet. Young adult; P.B. downwards wit
26·0	65°·1	73°·2	41°·7	—	—	—	Dirge-Bhotia. Khams, East Tibet. Single pre-condyle; P.B. downward bone L. missing.
21·3	70°·3	66°·7	43°·0	26°·7	16°·3	83°·0	Probably Cachar-Bhotia. Symbhunath, Central Nepal (temporary resi blunt edge; SR; J equal; epipteric bones R. and L.; ossicle of λ; la
19·4	78°·0	71°·5	30°·5	27°·1	3°·4	74°·9	Probably Cachar-Bhotia. Symbhunath, Central Nepal (temporary resid L.; P.B. flat with blunt edge; SR; JR; marked sub-nasal prognathi
53·4	60°·0	75°·8	44°·2	30°·3	13°·9	89°·7	Probably Cachar-Bhotia. Symbhunath, Central Nepal (temporary resi sharp edge; SR; JR; several epipteric bones R. and L.; single ossic
21·3	62°·9	76°·5	40°·6	29°·0	11°·6	88°·1	Probably Cachar-Bhotia. Symbhunath, Central Nepal (temporary resi edge; SR; JR; complete tripartite interparietal.
30·0	66°·6	68°·2	45°·2	26°·8	18°·4	86°·6	Cachar-Bhotia. Temporarily resident in Central Nepal. Coronal suture slight occipital asymmetry, bulging to right; small wormians lambd
21·3	71°·0	71°·9	37°·1	22°·9	14°·2	86°·1	Sharpa-Bhotia. Temporarily resident in Central Nepal. P.B. flat with r
42·2	66°·1	72°·3	41°·6	24°·0	17°·6	89°·9	Sunwar. West Nepal. Mandible; no upper 3rd molars or lower 3rd edge; SR; JR; 2 ossicles lambdoid L.
21·6	—	—	—	—	—	—	Sunwar. West Nepal. Mandible; 1 incisor lost; P.B. downwards wit asterions R. and L.
—	65°·7	77°·9	36°·4	—	—	—	Sunwar. West Nepal. P.B. upwards with sharp edge; S equal; JR; w
74·0	65°·8	74°·0	40°·2	23°·1	11°·0	85°·0	Sunwar. West Nepal. Tympanic perforation R.; P.B. downwards with
22·0	65°·3	68°·0	46°·7	25°·8	20°·9	88°·9	Gurung. West Nepal. Mandible; P.B. flat with blunt edge; S equal; JI bone L.; projecting squama temporalis with multiple edge.
36·2	64°·3	69°·8	45°·9	19°·2	19°·3	95°·0	Gurung. West Nepal. P.B. downwards with sharp edge; SR; JL; slig
53·5	59°·8	80°·1	40°·1	29°·6	10°·5	90°·6	Magar. West Nepal. Aging; coronal and sagittal sutures closing; no 3 with sharp edge; SR; JR; occipital asymmetry, bulging to L.
33·9	62°·6	73°·4	44°·0	30°·6	13°·4	86°·8	Sunwar. West Nepal. Sutures closing; small 3rd molars; P.B. flat wit
28·6	65°·7	72°·4	41°·9	26°·8?	15°·1?	87°·5?	Magar. West Nepal. P.B. flat with sharp edge; S equal; JR; very pr
26·3	71°·3	69°·3	39°·4	27°·1	12°·3	81°·6	Magar. West Nepal. Mandible; no lower 3rd molars or upper 3rd mol bone L.
23·5	65°·2	72°·0	42°·8	23°·5	19°·3	91°·3	Gurung. West Nepal. Young, about 11 years; 2 small pre-condyles; t SL; J equal; process of squama temporalis articulating with fronta
—	—	—	—	—	—	96°·0	Gurung. West Nepal. Young, about 6 years; tympanic perforation lambdoid R. and L.; basilar portion of occipital bone and nasal bo
46·5	71°·0	70°·4	38°·6	28°·0	9°·6	81°·0	Banks of Ganges. P.B. downwards with rounded edge; SR; JR; mass
37·0	65°·7	72°·2	42°·1	29°·5	12°·6	84°·8	Bengal Islamite. A Fakeer. No 3rd molar R.; inner palate bridge R. R. and L.
—	—	—	—	—	—	—	Bengalee? Young, 7–10 years; S equal; J equal; single ossicle lambd
37·8	—	—	—	—	—	—	Uraon. Chota Nagpur. Inner palate bridge L.; tympanic perforation lambdoid R. and L.
31·7	66°·0	69°·2	44°·8	29°·4	15°·4	84°·6	Uraon. Chota Nagpur. P.B. upwards with blunt edge; SL; J equal; R. and L.
—	—	—	—	—	—	—	Uraon. Chota Nagpur. Aging; all teeth lost and alveoli absorbed; s wards with sharp edge; SR; JR; few wormians lambdoid R. and

abbreviations used see Section 3 of text. The tribal division and locality from which the specimen was procured are given firs

OLLECTION (LENGTHS IN MILLIMETRES).

REMARKS*
nded and double edge; SR; JL; epipteric bone R.; wormians lambdoid R. and L. P.B. flat with blunt edge; S equal; J equal. vith blunt edge; SR; small epipteric bones R. and L.
palate bridge L.; tympanic perforation R. and L.; P.B. flat with blunt edge; ortion of occipital bone missing. al. Young, age?; tympanic perforation L.; P.B. flat with blunt edge; process of wormians lambdoid R. and L.; basilar portion of occipital bone missing.
ge; SR; J equal; single ossicle lambdoid R. dible; lower 3rd molar L. growing horizontally backwards; P.B. flat with blunt f λ and wormians lambdoid R. and L. upper 3rd molars; large lower 1st premolar L. shaped like canine; P.B. downwards utures across malar bones R. and L. vith blunt edge; SL; JR; metopic; wormians lambdoid R. and L. e perforation R. and L.; P.B. downwards with blunt edge; SR; J equal; wormians
. downwards with rounded edge; S equal; J equal; single ossicle in lambdoid L. ; P.B. downwards with blunt edge; occipital asymmetry, bulging to right; temporal
l (temporary residence). Sutures beginning to close; teeth perfect; P.B. flat with .; ossicle of λ; large mastoid foramen R. (6 mm. ×6 mm.) but foramen L. closed. (temporary residence). Sutures closing; 1 molar lost; tympanic perforation R. and b-nasal prognathism and very hollow maxillae; possible post-mortem deformation. l (temporary residence). Mandible; slight tympanic perforation L.; P.B. flat with d L.; single ossicles lambdoid R. and L. l (temporary residence). Mandible; tympanic perforation L.; P.B. flat with sharp
l. Coronal suture closing; mandible; P.B. downwards with sharp edge; SR; JR; wormians lambdoid R. and L. P.B. flat with rounded edge; SR; JL; epipteric bones R. and L.; flattened occiput. rs or lower 3rd molar L.; small upper canine L.; P.B. downwards with rounded
downwards with rounded edge; SR; JR; wormians lambdoid R. and L. and at
; S equal; JR; wormians λ R. and L. and ossicle asterion L.; sawn sagittally. downwards with rounded edge; SL; J equal; epipteric bone R. edge; S equal; JR; tympanic perforation R.; traces of suture half-way across malar ple edge. lge; SR; JL; slight occipital asymmetry, bulging to L. ures closing; no 3rd molars; 1 molar lost; tympanic perforation R. and L.; P.B. flat ulging to L.
iars; P.B. flat with blunt edge; SR; J equal. ual; JR; very projecting anterior nasal spine. or upper 3rd molar R.; P.B. downwards with rounded edge; SR; J equal; epipteric
ll pre-condyles; tympanic perforation R. and L.; P.B. downwards with blunt edge; lating with frontal bone; epipteric bone L.; wormians lambdoid R. and L. anic perforation R. and L.; P.B. downwards with blunt edge; SL; JR; wormians one and nasal bones missing.
ge; SR; JR; massive inionic protuberance. palate bridge R.; P.B. downwards with blunt edge; SR; JR; wormians lambdoid
gle ossicle lambdoid L.; part of occipital bone and facial bones missing.
anic perforation L.; P.B. flat with blunt edge; SL; JR; epipteric bone L.; wormians
ge; SL; J equal; epipteric bone L.; large ossicle of lambda and wormians lambdoid
veoli absorbed; single pre-condyle R.; tympanic perforation R. and L.; P.B. down- ambdoid R. and L.

red are given first. Unless otherwise stated the skull is adult and has no mandible.

Index letter	Sex	C	Lengths F	L′	L	B	B′	H
1 *e.*	♂	1595	185	188	185	136·0	90·9	133·7
1 *f* (48.1.12.436)	,,	1355?	176	179	176	130·7	94·2	138·1
1 *f′* (45.1.12.436)	,,	1340	171	175	173	124·3	97·1	126·8
1 *g.*	,,	1395	183	187·5	182	129·8	91·4	124·3
1 *h.*	,,	1510	185	188	187	131·8	103·2	138·0
1 *k.*	,,	1220	162	166	164	132·3	88·0	130·0
1 *m.*	,,	1580	183	182·5	182	138·6	91·6	138·0
1 *s.s.s.s*	,,	1285	169	173·5	171	128·2	88·1	132·0
1 *t.t.t.t.*	,,	1440	184	186	182·5	127·8	85·4	130·5
1 *u.u.u.u.* ...	,,	1510	182	182	181	134·3	97·9	140·9
1 *v.v.v.v.* ...	,,	1520	187·5	191	188	128·9	93·5	141·6
1 *w.w.w.w.* ...	,,	1580	175·5	178	176	137·0	89·2	136·0
1 *x.x.x.x.* ...	,,	1515	166·5	166·5	165	147·1	95·7	125·1
1 *f.f.f.f.* ...	,,	1195	167·5	171·5	169	122·8	86·9	127·5
1 *g.g.g.g.* ...	,,	1625	187·5	185·5	184	133·9	93·1	139·2
1 *i.i.i.i.*	,,	1420	166·5	172	168	134·5	96·2	137·2
1 *k.k.k.k.* ...	,,	1380	176	179	176	128·1	93·1	139·2
1 *a.a.a.a.a.a.* ...	,,	1560	177	176	176·5	135·9	97·1	135·3
1 *b.b.b.b.b.b.* ...	,,	1445	174·5	177	176	130·0	93·2	142·8
1 *c.c.c.c.c.c.* ...	,,	1680	180	183·5	182·5	134·1	93·7	146·0
1 *d.d.d.d.d.d.* ...	,,	1210	169	173·5	172	118·1	84·9	128·0
1 *e.e.e.e.e.e.* ...	,,	1500	175	175	173	126·2	88·2	140·9
1 *f.f.f.f.f.f.* ...	,,	1210	168	174	171	127·1	81·2	126·1
1 *v.v.v.v.v.* ...	,,	1635	186	190·5	189	134·0	93·8	137·0
1 *w.w.w.w.w.* ...	,,	1590	192	192·5	191	138·6	92·0	136·0
1 *x.x.x.x.x.* ...	,,	1345	163·5	167	165	134·2	80·7	133·0
1 *y.y.y.y.y.* ...	,,	1520	177	179·5	178	135·6	91·8	149·0
1 *z.z.z.z.z.* ...	,,	1545	171	176·5	169	143·2	91·2	132·6
1 *w.w.w.* ...	,,	1520	177	178	177	144·8	93·1	128·0
1 *x.x.x.*	,,	1425	176	175·5	176·5	130·2	90·8	132·0
1 *y.y.y.*	,,	1525	180·5	184	182·5	140·8	90·2	134·5
1 *z.z.z.*	,,	1540	179·5	182·5	180·5	133·9	90·2	141·1
1 *a.a.a.a.* ...	,,	1490	186	192	190·5	127·7	80·9	134·2
1 *b.b.b.b.* ...	,,	1265	164	169	166·5	136·0	89·0	128·0
1 *c.c.c.c.*	,,	—	—	—	—	—	86·4	—
1 *d.d.d.d.* ...	,,	1155	161	161·5	160·5	135·0	93·9	126·4
1 *e.e.e.e.*	,,	1505	178	179·5	178	140·3	92·4	135·0
1 *u.u u.*	,,	1425	180·5	187	185	133·0	93·0	130·0
1 *c.c.*	,,	1380	168	170·5	168·5	136·9	87·1	134·0
1 *b.b.b.b.b.* ...	♂?	1275	164	171·5	168·5	132·0	95·0	128·9
1 *e.e.e.e.e.* ...	,,	1315	176	178·5	176	127·2	72·8	128·0
1 *f.f.f.f.f.* ...	,,	1340	170·5	176·5	174	129·1	91·6	130·4
1 *h.h.h.h.h.* ...	,,	1470	180	184	178·5	131·1	92·8	137·5
1 *k.k.k.k.k.* ...	,,	1325	176	178·5	176	125·2	92·0	127·6
1 *k.k.k.*	♂	1405?	176·5	178	175·5	130·2	90·1	138·0
1 *s.s.s.*	,,	1320	184	188	184·5	124·9	88·2	132·2
1 *o.o.o.*	,,	1505	183·5	190	183	135·2	93·1	138·0
1 *i.i.i.*	,,	1550	175·5	179·5	175·5	136·2	95·2	138·1
1 *h.h.h.h.* ...	♀	1180	163·5	165·5	162·5	124·2	87·9	121·2
1 *m.m.m.m.* ...	,,	1240	164·5	167	163·5	136·0	83·9	120·2
1 *i.*	,,	1370	179	181·5	179·5	131·1	96·0	131·1

Lengths						Arcs								Face						
	B'	H	H'	OH	LB	Q	Q'	S	S_1	S_2	S_3	S_3'	U	PH	$G'H$	GB	J	$\frac{NH}{R}$	$\frac{NH}{L}$	NB
·0	90·9	133·7	133·8	118·2	103·0	308	307·5	372	132	122	118	96·0	514	19·6	72·2	101·0	130·1	52·1	52·0	28·7
·7	94·2	138·1	136·3	119·9	97·1	312·5	314	367·5	129·5	129	109	92·4	491·5	19·4	63·7	93·4	—	43·8	44·6	22·9
·3	97·1	126·8	125·8	109·5	95·2	286·5	288	356	122·5	126	107·5	88·9	485	14·6	65·6	90·7	119·0	51·4	51·2	22·8
·8	91·4	124·3	125·2	116·4	95·0	300·5	301	380·5	131	134·5	115	89·3	507	21·7	68·2	98·8	126·9	48·3	48·8	28·3
·8	103·2	138·0	135·1	117·4	105·0	309·5	309·5	368	114·5	137·5	116	96·8	517	18·6	67·0	93·9	129·3	49·7	51·0	25·8
·3	88·0	130·0	130·0	110·8	91·9	289·5	288	341·5	117	120	104·5	92·2	471	17·0	61·9	97·4	123·2	46·1	46·3	25·9
·6	91·6	138·0	135·5	117·4	104·9	309	309	376	126	128	122	100·2	515	20·9	72·2	102·1	—	52·0	52·0	27·1
·2	88·1	132·0	134·1	109·6	98·6	285·5	285·5	350	116	131	103	90·0	478	18·8	66·8	—	125·2	47·0	48·2	23·4
·8	85·4	130·5	132·0	116·4	101·8	300	300	375	134	130	111	91·2	506	18·4	60·3	92·1	127·2	49·4	50·1	23·2
·3	97·9	140·9	140·2	117·2	100·0	305	307	375	125	134	116	93·1	510	19·9	68·5	98·2	130·0	48·3	51·2	25·6
·9	93·5	141·6	137·1	115·6	97·2	298	300	392	120·5	132	139·5	109·8	509·5	—	—	91·7	124·1	45·5	43·9	26·4
·0	89·2	136·0	133·0	116·3	93·0	303·5	303	369	122	135	112	94·9	502	18·3	70·3	100·1	131·1	51·9	50·5	24·3
·1	95·7	125·1	124·6	115·8	93·0	312	313	352	119	115	118	100·1	495	18·0	63·7	96·0	127·0	47·5	47·5	25·7
·8	86·9	127·5	126·8	111·1	94·8	287	287·5	352	115·5	127·5	109	92·9	467	15·5	61·1	100·3	123·2	46·2	45·1	23·1
·9	93·1	139·2	137·2	118·7	99·8	309·5	309·5	385·5	134	131·5	120	96·2	519·5	20·1	72·7	101·0	131·9	52·0	52·2	27·1
·5	96·2	137·2	136·9	116·6	97·0	302	302	350	125·5	114·5	110	95·7	484	16·1	65·9	88·6	125·8	51·8	51·1	27·3
·1	93·1	139·2	138·4	111·8	101·3	295	295	358·5	124·5	125	109	92·1	491	18·0	71·5	105·2	132·2	54·3	55·0	26·3
·9	97·1	135·3	133·8	116·4	98·1	303·5	302·5	363·5	119	128·5	116	98·2	505	20·3	67·2	103·6	135·3	50·0	49·8	28·2
·0	93·2	142·8	141·6	115·7	100·6	301	301	362·5	125·5	126	111	93·0	491·5	17·8	65·7	100·0	127·9	50·5	49·1	24·8
·1	93·7	146·0	143·7	120·3	97·2	309	307	390	138	138·5	113·5	94·1	511	16·9	64·3	102·1	130·6	47·2	47·2	26·3
·1	84·9	128·0	127·0	106·2	90·0	271·5	273·5	349	113	124	112	91·8	470·5	18·8	65·8	97·5	120·1	47·1	46·7	25·8
·2	88·2	140·9	140·1	118·1	97·6	306·5	305	366	128	116	122	102·2	490·5	15·8	63·0	94·1	128·0	48·5	49·1	24·2
·1	81·2	126·1	125·8	104·9	96·1	278	276	341	115	126	100	85·9	477	16·7	65·5	94·1	119·6	48·8	49·1	24·2
·0	93·8	137·0	134·9	119·2	102·8	309·5	306·5	392·5	131	148	113·5	94·7	520·5	24·4	72·1	103·9	134·1	48·5	48·0	28·2
·6	92·0	136·0	135·7	117·5	100·0	307	307·5	384	122·5	124	137·5	104·9	526·5	22·4	69·6	97·2	137·0	52·2	51·8	28·0
·2	80·7	133·0	133·8	116·3	97·2	302·5	302	345·5	127·5	111	107	89·7	468	18·6	67·0	93·8	118·3	47·8	47·8	22·8
·6	91·8	149·0	144·2	115·2	—	298·5	298	—	128·5	130	—	—	492	19·6	68·2	93·3	120·2	52·0	51·0	24·1
·2	91·2	132·6	130·8	117·4	96·3	310·5	310	362	128	122	112	97·3	499·5	17·8	67·7	105·9	130·0	51·9	51·7	29·0
·8	93·1	128·0	126·3	110·8	96·0	304	306	355	124	118	113	93·3	517·5	18·0	67·1	92·3	132·3	51·3	50·6	24·4
·2	90·8	132·0	132·1	111·5	97·4	295	291	364·5	114	129	121·5	95·7	498·5	14·2	67·2	89·1	130·5	54·6	53·3	22·7
·8	90·2	134·5	134·0	114·7	104·0	306·5	303·5	361·5	124	124·5	113	93·3	517	13·7	67·0	102·0	133·0	53·4	53·8	27·9
·9	90·2	141·1	139·9	118·2	100·1	303·5	302	367	116·5	135·5	115	99·8	502	20·7	71·4	102·2	140·7	51·7	52·2	27·6
·7	80·9	134·2	131·8	116·5	100·3	300	299	388	130	137	121	97·2	512	18·8	70·4	104·7	129·8	52·3	50·9	28·1
·0	89·0	128·0	126·0	109·3	90·3	291	291	350	123·5	121	105·5	89·9	480·5	15·8	61·8	99·7	123·0	46·4	46·3	24·9
	86·4	—	119·9	106·5	89·0	—	—	—	—	—	104	86·2	—	16·6	67·1	93·6	118·5	49·4	49·0	23·9
·0	93·9	126·4	128·1	112·5	101·8	298	297·5	335·5	114	113	108·5	95·1	468	23·5	71·8	100·3	124·8	46·8	48·7	23·9
·3	92·4	135·0	132·9	117·4	99·2	309	309	374	126	119·5	128·5	106·2	508	15·4	69·2	97·0	125·8	54·6	54·2	23·3
·0	93·0	130·0	129·2	115·3	102·3	300·5	303	366·5	128	119	119·5	94·2	504	17·4	68·3	98·1	127·3	49·2	48·8	27·9
·9	87·1	134·0	133·8	111·3	96·1	297	297	353	122·5	119·5	111	91·9	488	20·1	71·1	96·2	129·3	51·1	52·2	27·2
·0	95·0	128·9	128·8	115·4	98·9	300	300·5	344	121·5	122	100·5	87·2	479	—	—	100·0	132·0	50·3	50·2	28·1
·2	72·8	128·0	127·3	113·6	96·1	290	291·5	356·5	127	120	109·5	91·0	489	—	—	93·9	121·9	51·0	50·9	26·3
·1	91·6	130·4	127·2	111·5	98·2	294	293·5	352·5	117·5	130	105	88·9	487	21·4	74·1	98·2	119·1	51·9	52·9	23·1
·1	92·8	137·5	136·8	118·5	93·6	309	312	384	135	131	118	96·8	508	—	—	92·0	—	46·2	46·1	23·8
·2	92·0	127·6	129·2	111·5	100·3	295	293	354·5	127	121·5	106	91·1	494	13·9	61·2	97·9	—	46·6	46·9	26·1
·2	90·1	138·0	—	129·8	92·9	330	333	379	—	—	106·5	89·0	495	22·3	67·4	92·9	118·9	47·0	47·1	24·5
·9	88·2	132·2	132·7	116·3	103·1	296	293·5	363·5	121·5	124·5	117·5	95·9	501·5	—	—	95·9	128·9	50·9	49·9	28·2
·2	93·1	138·0	136·0	119·7	100·0	307	309·9	385	128	135	122	99·0	518	23·3	78·1	103·0	131·2	53·0	53·0	25·4
·2	95·2	138·1	136·7	120·5	94·4	330	—	376	126·5	136·5	113	96·3	497	19·1	65·9	100·1	123·6	47·1	47·4	25·9
·2	87·9	121·2	119·6	106·5	90·0	280·5	282	337	112	123	102	83·9	462	17·8	57·5	84·1	110·3	39·4	39·9	24·1
·0	83·9	120·2	119·9	108·3	90·1	289	290	339·5	117·5	123	99	84·6	479	18·2	62·1	97·2	123·2	42·9	43·9	25·6
·1	96·0	131·1	129·9	118·3	98·2	302	306	363	123·5	127·5	112	93·1	502·5	20·1	66·8	98·5	124·4	47·4	48·1	24·9

Nose							Orbits					Palate and Profile						Foramen	
NH L	*NB*	*DS*	*DC*	*DA*	*SS*	*SC*	O_1R	O_1L	O_2R	O_2L	O_1'	G_1	G_1'	G_2	*GL*	*EB*	*EH*	*fml*	*fmb*
52·0	28·7	13·6	22·0	35·6	4·4	10·5	41·5	42·2	32·6	31·9	39·7	57·9	54·0	42·8	104·5	42·6	16·1	36·6	29·7
44·6	22·9	11·4	20·1	33·6	3·0	5·3	43·8	43·1	32·1	32·1	39·6	46·0	41·6	43·9	92·8	42·9	9·9	37·5	28·6
51·2	22·8	11·6	19·9	31·9	3·7	6·6	39·4	39·8	31·6	31·1	39·0	50·2	47·4	38·0	92·6	37·9	11·5	33·0	30·2
48·8	28·3	12·1	21·9	32·6	3·3	11·0	41·1	41·1	32·1	32·4	38·8	48·8	45·3	37·8	94·1	38·6	10·9	32·3	27·2
51·0	25·8	11·6	24·7	36·7	4·2	11·1	42·7	41·0	32·2	32·2	42·6	50·6	46·6	44·2	98·2	43·6	14·6	36·8	30·5
46·3	25·9	7·7	24·9	31·0	2·8	8·1	39·3	40·1	31·2	30·9	37·9	46·2	42·6	40·0	91·0	40·0	10·2	35·0	27·1
52·0	27·1	10·0	22·8	31·4	2·8	9·0	42·4	41·5	38·0	29·2	41·8	—	—	44·4	97·6	44·7	14·0	34·0	28·4
48·2	23·4	11·4	19·4	31·7	4·2	8·7	42·8	43·2	32·7	32·0	39·6	46·5	43·9	40·0	92·9	40·8	8·1	34·2	32·2
50·1	23·2	14·0	17·9	36·0	5·7	10·9	42·7	41·9	33·1	33·7	40·7	44·3	41·1	—	89·0	—	—	33·9	30·1
51·2	25·6	11·6	22·5	32·1	4·1	9·3	44·2	45·4	35·0	34·5	42·6	50·3	47·1	43·2	91·2	42·6	12·3	39·1	31·3
43·9	26·4	12·0	20·1	32·8	4·9	9·1	41·8	40·8	29·7	29·6	40·7	—	—	38·2	—	38·2	9·6	36·8	29·3
50·5	24·3	6·6	21·2	26·9	1·0	3·7	41·5	41·0	34·9	35·0	39·9	47·0	43·1	43·4	85·5	43·0	14·3	38·3	29·4
47·5	25·7	10·4	24·0	30·6	2·2	5·2	42·0	41·3	34·0	34·4	39·8	45·7	43·8	39·6	93·9	40·0	7·9	34·9	31·7
45·1	23·1	11·3	22·0	33·4	2·5	9·6	40·6	38·9	28·0	28·8	37·4	51·0	47·2	—	97·2	—	—	35·1	27·6
52·2	27·1	8·8	21·9	30·4	4·0	5·4	41·9	41·2	33·8	34·1	40·6	54·7	51·9	40·4	103·6	40·2	15·8	33·1	27·8
51·1	27·3	12·7	23·9	38·3	3·7	10·7	42·2	41·9	36·3	36·7	38·1	42·2	39·6	40·0	83·1	40·2	12·1	34·6	29·1
55·0	26·3	10·8	26·9	35·7	1·8	18·1	41·6	40·9	35·0	34·4	37·8	49·7	47·7	44·0	97·2	43·4	16·3	37·1	29·8
49·8	28·2	9·9	25·3	32·4	3·1	10·7	41·3	39·1	34·8	35·6	38·5	45·3	43·7	44·7	95·2	44·9	12·0	37·4	32·1
49·1	24·8	9·8	20·0	29·2	1·6	5·2	42·5	41·2	33·9	33·0	40·1	49·3	45·8	43·8	94·3	43·7	12·8	39·2	34·0
47·2	26·3	8·5	19·9	28·2	2·3	6·1	44·3	45·3	35·0	35·1	43·0	46·8	41·9	40·9	90·9	40·9	11·1	37·7	30·8
46·7	25·8	8·6	19·7	29·0	2·6	7·9	40·6	39·1	33·3	33·3	40·0	49·2	45·8	42·7	89·0	42·9	14·1	36·3	29·2
49·1	24·2	9·5	18·7	25·9	2·4	8·4	41·8	41·2	32·7	32·7	39·3	46·6	43·1	41·1	88·3	42·2	8·9	39·0	31·1
49·1	24·2	12·1	23·2	33·0	2·7	5·8	40·5	38·6	30·9	32·1	36·3	43·6	40·5	38·1	88·1	38·6	9·2	31·8	28·0
48·0	28·2	7·7	22·8	29·5	1·1	8·6	42·3	42·1	31·7	33·8	40·9	59·2	52·6	44·9	112·0	44·1	14·0	32·9	31·3
51·8	28·0	7·7	19·2	28·3	2·1	5·1	44·0	44·2	34·9	34·9	41·8	48·2	45·1	43·8	95·6	44·1	14·1	40·5	31·6
47·8	22·8	11·3	19·5	32·9	4·1	7·4	37·2	36·2	29·7	30·8	35·5	47·1	47·1	40·8	91·6	41·0	12·5	32·1	28·4
51·0	24·1	15·6	19·9	38·0	5·9	9·0	43·6	41·5	32·8	32·6	38·7	49·1	45·0	34·5	—	34·1	9·3	—	—
51·7	29·0	8·0	20·0	26·2	3·1	7·0	42·1	42·3	33·8	34·0	39·8	44·5	41·1	41·5	92·2	41·0	16·5	36·6	30·1
50·6	24·4	7·7	21·0	29·8	1·3	5·2	42·1	42·2	34·0	33·6	40·7	46·8	44·1	44·4	94·8	44·7	11·7	39·8	32·3
53·3	22·7	9·7	18·2	24·1	2·3	6·9	41·0	40·8	37·2	37·7	39·4	43·9	40·9	38·7	85·3	38·7	12·6	37·6	28·2
53·8	27·9	10·3	22·1	30·1	2·1	8·8	45·1	43·1	34·8	35·7	40·3	47·2	42·9	43·4	90·4	43·0	11·1	36·7	31·8
52·2	27·6	6·8	22·1	28·1	1·4	7·0	45·2	43·1	33·9	34·9	43·3	52·8	49·7	47·6	109·0	47·5	14·8	35·6	32·7
50·9	28·1	8·5	20·7	29·6	3·4	9·7	40·1	39·1	32·2	32·7	38·3	50·1	46·5	42·8	96·4	42·1	13·9	36·7	29·4
46·3	24·9	9·3	24·0	32·7	2·1	8·2	38·7	38·9	32·1	31·5	36·6	42·8	40·9	37·9	86·3	37·1	12·3	32·5	25·6
49·0	23·9	7·7	20·1	26·5	3·1	10·4	38·4	36·9	35·0	35·0	37·2	47·7	43·8	41·1	89·6	40·8	12·3	34·4	25·7
48·7	23·9	5·3	21·2	25·0	1·2	6·3	41·7	41·1	30·9	31·6	40·1	50·9	48·3	39·9	103·1	39·6	14·6	29·2	26·0
54·2	23·3	9·8	20·0	28·2	2·8	7·2	40·1	37·9	31·5	31·2	39·1	52·1	48·1	45·1	97·3	44·8	8·0	37·7	29·3
48·8	27·9	12·0	20·3	35·7	5·0	10·7	42·0	41·4	29·6	29·8	40·9	54·2	50·7	41·9	97·6	42·0	17·0	34·6	26·1
52·2	27·2	7·8	16·8	22·5	2·6	7·0	40·3	40·7	31·9	32·5	39·7	50·4	47·7	42·1	96·0	42·3	11·9	34·2	27·1
50·2	28·1	10·6	18·6	29·9	4·5	8·9	42·6	41·8	35·0	34·9	40·8	—	—	41·2	—	44·9	15·2	34·5	32·9
50·9	26·3	9·9	17·6	28·3	2·8	6·2	39·0	38·4	31·1	31·5	37·6	47·0	44·3	40·3	—	40·9	14·4	33·8	28·8
52·9	23·1	11·9	19·9	34·6	4·6	7·5	41·8	41·1	33·6	33·9	40·0	50·0	46·6	37·3	92·0	37·9	12·1	38·1	28·1
46·1	23·8	8·8	20·9	29·2	0·4	3·4	40·3	41·3	32·1	30·6	38·6	—	—	36·2	—	37·0	9·8	36·1	28·6
46·9	26·1	12·3	22·1	36·0	4·0	6·6	44·2	43·8	32·8	32·4	41·1	—	—	41·6	100·0	41·2	11·1	38·3	—
47·1	24·5	9·3	20·1	31·3	1·6	9·1	41·1	40·4	32·8	32·2	38·1	47·8	45·3	36·8	95·2	36·6	11·4	35·9	30·6
49·9	28·2	12·9	22·8	36·5	2·9	9·2	42·7	42·7	32·9	32·7	39·8	—	—	—	—	—	—	36·4	28·1
53·0	25·4	9·3	22·5	30·2	2·0	7·2	45·1	43·2	36·1	36·7	42·4	51·1	48·0	41·3	103·2	42·0	14·3	39·8	34·8
47·4	25·9	—	21·4	—	—	9·1	41·0	39·7	33·4	33·8	39·2	47·7	43·2	37·2	93·2	36·3	13·0	38·1	28·7
39·9	24·1	11·3	19·1	35·9	3·9	7·3	38·7	38·4	27·8	29·0	36·3	46·9	43·0	35·7	93·5	35·0	9·4	31·7	26·7
43·9	25·6	7·8	22·8	29·2	1·5	8·1	40·2	41·2	34·8	33·6	38·6	45·0	41·9	—	89·0	—	—	34·9	30·7
48·1	24·9	12·4	23·0	30·1	2·7	10·0	44·1	42·8	34·8	32·8	41·4	48·1	43·9	40·6?	92·6	40·6?	—	38·0	28·8

	Palate and Profile							Foramen		Indices											
	O_1'	G_1	G_1'	G_2	GL	EB	EH	fml	fmb	$Oc.I$	B/L'	H/L'	B/L	H/L	B/H	H'/L	B/H'	$\frac{(B-H')}{L}$	$G'H/GB$	$\frac{NB/NH}{R}$	NI
	39·7	57·9	54·0	42·8	104·5	42·6	16·1	36·6	29·7	58·1	72·3	71·1	73·5	72·3	101·7	72·3	101·6	1·2	71·5	55·1	
	39·6	46·0	41·6	43·9	92·8	42·9	9·9	37·5	28·6	61·7	73·0	77·2	74·2	78·5	94·6	76·1	95·9	−3·2	68·2	52·3	
	39·0	50·2	47·4	38·0	92·6	37·9	11·5	33·0	30·2	59·4	71·0	72·5	71·8	73·3	98·0	72·7	98·8	−0·9	72·3	44·4	
	38·8	48·8	45·3	37·8	94·1	38·6	10·9	32·3	27·2	55·6	69·2	66·3	71·3	68·3	104·4	68·8	103·7	2·5	69·0	58·6	
	42·6	50·6	46·6	44·2	98·2	43·6	14·6	36·8	30·5	60·2	70·1	73·4	70·5	73·8	95·5	72·2	97·6	−1·8	71·4	51·9	
	37·9	46·2	42·6	40·0	91·0	40·0	10·2	35·0	27·1	67·5	79·7	78·3	80·7	79·3	101·8	79·3	101·8	1·4	63·6	56·2	
	41·8	—	—	44·4	97·6	44·7	14·0	34·0	28·4	58·8	75·9	75·6	76·2	75·8	100·4	74·5	102·2	1·7	70·7	52·1	
	39·6	46·5	43·9	40·0	92·9	40·8	8·1	34·2	32·2	65·8	73·9	76·1	75·0	77·2	97·1	78·4	95·6	−3·5	—	49·8	
	40·7	44·3	41·1	—	89·0	—	—	33·9	30·1	58·8	68·7	70·2	69·6	71·5	97·9	72·3	96·8	−2·3	72·0	47·0	
	42·6	50·3	47·1	43·2	91·2	42·6	12·3	39·1	31·3	57·2	73·8	77·4	74·2	77·8	95·3	77·5	95·8	−3·3	69·8	53·0	
	40·7	—	—	38·2	—	38·2	9·6	36·8	29·3	56·2	67·5	74·1	68·6	75·3	91·0	72·9	94·0	−4·4	—	58·0	
	39·9	47·0	43·1	43·4	85·5	43·0	14·3	38·3	29·4	61·7	77·0	76·4	77·8	77·3	100·7	75·6	103·0	2·3	70·2	46·8	
	39·8	45·7	43·8	39·6	93·9	40·0	7·9	34·9	31·7	61·8	88·3	75·1	89·2	75·8	117·6	75·5	118·1	13·6	66·4	54·1	
	37·4	51·0	47·2	—	97·2	—	—	35·1	27·6	62·3	71·6	74·3	72·7	75·4	96·3	75·0	96·8	−2·4	60·9	50·0	
	40·6	54·7	51·9	40·4	103·6	40·2	15·8	33·1	27·8	57·2	72·2	75·0	72·8	75·7	96·2	74·6	97·6	−1·8	72·0	52·1	
	38·1	42·2	39·6	40·0	83·1	40·2	12·1	34·6	29·1	65·1	78·2	79·8	80·1	81·7	98·0	81·5	98·2	−1·4	74·4	52·7	
	37·8	49·7	47·7	44·0	97·2	43·4	16·3	37·1	29·8	61·4	71·6	77·8	72·8	79·1	92·0	78·6	92·6	−5·9	68·0	48·4	
	38·5	45·3	43·7	44·7	95·2	44·9	12·0	37·4	32·1	61·6	77·2	76·9	77·0	76·7	100·4	75·8	101·6	1·2	64·9	56·4	
	40·1	49·3	45·8	43·8	94·3	43·7	12·8	39·2	34·0	60·5	73·4	80·7	73·9	81·1	91·0	80·5	91·8	−6·6	65·7	49·1	
	43·0	46·8	41·9	40·9	90·9	40·9	11·1	37·7	30·8	59·6	73·1	79·6	73·5	80·0	91·8	78·7	93·3	−5·3	63·0	55·7	
	40·0	49·2	45·8	42·7	89·0	42·9	14·1	36·3	29·2	58·6	68·1	73·8	68·7	74·4	92·3	73·8	93·0	−5·2	67·5	54·8	
	39·3	46·6	43·1	41·1	88·3	42·2	8·9	39·0	31·1	60·5	72·1	80·5	72·9	81·4	89·6	81·0	90·1	−8·0	67·0	49·9	
	36·3	43·6	40·5	38·1	88·1	38·0	9·2	34·8	28·0	63·3	73·0	66·7	74·3	73·7	100·8	73·6	101·0	0·8	69·6	49·6	
	40·9	59·2	52·6	44·9	112·0	44·1	14·0	32·9	31·3	60·1	70·3	71·9	70·9	72·5	97·8	71·4	99·3	−0·5	68·4	58·2	
	41·8	48·2	45·1	43·8	95·6	44·1	14·1	40·5	31·6	54·9	72·0	70·6	72·6	71·2	101·9	71·0	102·1	1·5	71·6	53·6	
	35·5	47·1	47·1	40·8	91·6	41·0	12·5	32·1	28·4	60·5	80·4	79·6	81·3	80·6	100·9	81·1	100·3	0·2	71·4	47·7	
	38·7	49·1	45·0	34·5	—	34·1	9·3	—	—	—	75·5	83·0	76·2	83·7	91·0	81·0	94·0	−4·8	73·0	46·3	
	39·8	44·5	41·1	41·5	92·2	41·0	16·5	36·6	30·1	64·9	81·1	75·2	84·7	78·5	108·0	78·5	109·5	7·3	63·9	55·9	
	40·7	46·8	44·1	44·4	94·8	44·7	11·7	39·8	32·3	59·2	81·3	71·9	81·8	72·3	113·1	71·4	114·6	10·5	81·5	47·6	
	39·4	43·9	40·9	38·7	85·3	38·7	12·6	37·6	28·2	56·2	74·2	75·2	73·8	74·8	98·6	74·8	98·6	−1·1	75·4	41·6	
	40·3	47·2	42·9	43·4	90·4	43·0	11·1	36·7	31·8	59·2	76·5	73·1	77·2	73·7	104·7	73·5	105·1	3·7	65·7	52·2	
	43·3	52·8	49·7	47·6	109·0	47·5	14·8	35·6	32·7	64·7	73·4	77·3	74·2	78·2	94·9	77·5	95·7	−3·3	69·9	53·4	
	38·3	50·1	46·5	42·8	96·4	42·1	13·9	36·7	29·4	57·3	66·5	69·9	67·0	70·4	95·2	69·2	96·9	−2·2	67·2	53·7	
	36·6	42·8	40·9	37·9	86·3	37·1	12·3	32·5	25·6	62·2	80·5	75·7	81·7	76·8	106·3	75·7	107·9	6·0	62·0	53·7	
	37·2	47·7	43·8	41·1	89·6	40·8	12·3	34·4	25·7	59·6	—	—	—	—	—	—	—	—	71·7	48·4	
	40·1	50·9	48·3	39·9	103·1	39·6	14·6	20·2	20·0	66·3	83·6	78·3	84·1	78·8	106·8	79·8	105·4	4·3	71·6	51·1	
	39·1	52·1	48·1	45·1	97·3	44·8	8·0	37·7	29·3	59·3	78·2	75·2	78·8	75·8	103·9	74·7	105·6	4·2	71·3	42·7	
	40·9	54·2	50·7	41·9	97·6	42·0	17·0	34·6	26·1	56·3	71·1	69·5	71·9	70·3	102·3	69·8	102·9	2·1	69·6	56·7	
	39·7	50·4	47·7	42·1	96·0	42·3	11·9	34·2	27·4	59·4	80·3	78·6	81·2	79·5	102·2	79·4	102·3	1·8	73·9	53·2	
	40·8	—	—	44·2	—	44·9	15·2	34·5	32·9	64·7	77·0	75·2	78·3	76·5	102·4	76·4	102·5	1·9	—	55·9	
	37·0	47·0	44·3	40·3	—	40·9	14·4	38·8	28·8	59·8	71·3	71·7	72·3	72·7	99·4	72·3	99·9	−0·1	—	51·6	
	40·0	50·0	46·6	37·3	92·0	37·9	12·1	38·1	28·1	61·6	73·1	73·8	74·2	74·9	99·0	73·1	101·5	1·1	75·5	44·5	
	38·6	—	—	36·2	—	37·0	9·8	36·1	28·6	58·7	71·3	74·7	73·4	77·0	95·3	76·6	95·8	−3·2	—	51·5	
	41·1	—	—	41·6	100·0	41·2	11·1	38·3	—	63·3	70·1	71·5	71·1	72·5	98·1	73·4	96·9	−2·3	62·5	56·0	
	38·1	47·8	45·3	36·8	95·2	36·6	11·4	35·9	30·6	60·2	73·1	77·5	74·2	78·6	94·3	—	—	—	72·6	52·1	
	39·8	—	—	—	—	—	—	36·4	28·1	58·4	66·4	70·3	67·7	71·7	94·5	71·9	94·1	−4·2	—	55·4	
	42·4	51·1	48·0	41·3	103·2	42·0	14·3	39·8	34·8	58·0	71·2	72·6	73·9	75·4	98·0	74·3	99·4	−0·4	75·8	47·9	
	38·2	47·7	43·2	37·2	93·2	36·3	13·0	38·1	28·7	62·4	75·9	76·9	77·6	78·7	98·8	77·9	99·6	−0·3	65·8	55·0	
	36·3	46·9	43·0	35·7	93·5	35·0	9·4	31·7	26·7	58·9	75·0	73·2	76·4	74·6	102·5	73·6	103·5	2·8	68·4	61·2	
	38·6	45·0	41·9	—	89·0	—	—	34·9	30·7	62·7	81·4	72·0	83·2	73·5	113·1	73·3	113·4	9·8	63·9	59·7	
	41·4	48·1	43·9	40·6?	92·6	40·6?	—	38·0	28·8	59·8	72·2	72·2	73·0	73·0	100·0	72·4	100·9	0·8	67·8	52·5	

BLE OF ABSOLUTE AND INDICIAL MEASUREMENTS OF THE INDIVIDUAL

Indices | Angles

*B	*NB/NH* R	*NB/NH* L	O_2/O_1 R	O_2/O_1 L	O_2/O_1'	G_2/G_1	*EH/EB*	*fmb/fml*	*DS/DC*	*SS/SC*	*N*∠	*A*∠	*B*∠	θ_1	θ_2	*P*∠
	55·1	55·2	78·5	75·6	82·1	73·9	37·8	81·1	61·8	41·9	70°·7	68°·7	40°·6	27°·4	13°·2	81°·9
	52·3	51·3	73·3	74·5	81·1	95·4	23·1	76·3	56·7	56·6	66°·7	74°·3	39°·0	26°·7	13°·3	86°·6
	44·4	44·5	80·2	78·1	81·0	75·7	30·3	91·7	58·3	56·1	67°·1	72°·1	40°·8	28°·0	12°·8	84°·9
	58·6	58·0	78·1	78·8	82·7	77·5	28·2	84·2	55·3	30·0	68°·1	69°·8	42°·1	25°·2	16°·9	86°·7
	51·9	50·6	75·4	78·3	75·6	87·4	33·5	82·9	47·0	37·8	65°·1	76°·7	38°·2	26°·5	11°·5	88°·4
	56·2	55·9	79·4	77·1	82·3	86·6	25·4	77·4	30·9	34·6	69°·5	71°·1	39°·4	24°·8	14°·6	85°·7
	52·1	52·1	80·6	70·4	90·9	—	31·3	83·5	43·9	31·1	63°·7	74°·8	41°·5	21°·5	10°·0	84°·8
	49·8	48·5	76·4	74·1	82·6	86·0	19·9	94·2	58·8	48·3	65°·0	74°·5	40°·5	28°·9	11°·6	86°·1
	47·0	46·3	77·5	80·4	81·3	—	—	88·7	78·2	52·3	59°·5	80°·5	40°·0	17°·7	12°·3	92°·8
	53·0	50·0	79·2	76·0	82·2	85·9	28·9	80·1	51·6	49·4	62°·1	76°·2	41°·7	32°·9	8°·8	85°·0
	58·0	60·1	71·1	72·5	73·0	—	25·1	79·6	59·7	53·8	—	—	—	—	—	—
	46·8	48·1	84·1	85·4	87·5	92·3	33·3	76·8	31·1	27·0	61°·1	72°·9	46°·0	30°·1	15°·9	88°·8
	54·1	54·1	81·0	83·3	85·4	86·7	19·8	90·8	43·3	42·2	70°·9	69°·4	39°·7	19°·3	20°·4	89°·8
	50·0	51·2	69·0	74·0	74·9	—	—	78·6	51·4	26·0	73°·8	69°·2	37°·0	24°·3	12°·7	81°·9
	52·1	51·9	80·7	82·8	83·3	73·9	39·3	84·0	40·2	74·1	71°·9	66°·3	41°·8	28°·8	13°·0	79°·3
	52·7	53·4	86·0	87·6	95·3	94·8	30·1	84·1	53·1	34·6	57°·8	80°·3	41°·9	32°·5	9°·4	89°·7
	48·4	47·8	84·1	84·1	92·6	88·5	37·6	80·3	40·1	9·9	65°·9	72°·1	42°·0	24°·3	8°·7	80°·8
	56·4	56·6	84·3	91·0	90·4	98·7	26·7	85·8	39·1	29·0	67°·1	72°·2	40°·7	25°·1	15°·6	87°·8
	49·1	50·5	79·8	80·1	84·5	88·8	29·3	86·7	49·0	30·8	65°·1	75°·8	39°·1	33°·9	5°·2	81°·0
	55·7	55·7	79·0	77·5	81·4	87·4	27·1	81·7	42·7	37·7	64°·5	75°·7	39°·8	32°·8	7°·0	82°·7
	54·8	55·2	82·0	85·2	83·3	86·8	32·6	80·4	43·6	32·9	67°·8	69°·3	42°·9	32°·0	10°·9	80°·2
	49·9	49·3	78·2	79·4	83·2	88·2	21·1	79·7	50·8	28·6	62°·4	78°·4	39°·2	32°·8	6°·4	84°·8?
	49·6	49·3	76·3	83·2	85·1	89·7	24·2	80·5	51·2	46·6	62°·7	76°·1	41°·2	29°·7	11°·5	87°·6
	58·2	58·7	74·9	80·3	77·5	75·8	31·7	95·1	33·8	12·8	77°·9	63°·9	38°·2	24°·8	13°·4	77°·3
	53·6	54·1	79·3	79·0	83·5	90·9	32·0	78·0	40·1	41·2	65°·5	72°·9	41°·6	25°·6	16°·0	88°·9
	47·7	47·7	79·8	85·1	83·7	86·6	30·5	88·5	57·9	55·4	64°·6	74°·1	41°·3	25°·5	15°·8	89°·9
	46·3	47·3	75·2	78·6	84·8	70·3	27·3	—	78·4	65·6	—	—	—	—	—	82°·2
	55·9	56·1	80·3	80·4	84·9	93·3	40·2	82·2	40·0	44·3	65°·9	72°·2	41°·9	27°·8	14°·1	86°·3
	47·6	48·2	80·8	79·6	83·5	94·9	26·2	81·2	36·6	25·0	68°·3	70°·7	41°·0	29°·0	12°·0	82°·7
	41·6	42·6	90·7	92·4	94·4	88·2	32·6	75·6	53·3	33·3	58°·9	78°·9	42°·2	37°·2	5°·0	83°·9
	52·2	51·9	77·2	82·8	86·4	91·9	25·8	86·6	46·6	23·9	59°·1	81°·3	39°·6	25°·6	14°·0	95°·3
	53·4	52·9	75·0	81·0	78·3	90·2	31·2	91·9	30·8	20·0	70°·4	71°·2	38°·6	28°·0	10°·4	81°·6
	53·7	55·2	80·3	83·6	84·1	85·4	33·0	80·1	41·1	35·1	65°·9	72°·2	41°·9	23°·0	8°·9	91°·1
	53·7	53·8	82·9	81·0	87·7	88·6	33·2	78·8	38·8	25·6	66°·0	73°·2	40°·8	26°·1	14°·7	87°·9
	48·4	48·8	91·1	94·9	94·1	86·2	30·1	74·7	38·3	29·8	68°·1	67°·8	44°·1	24°·8	19°·3	87°·1
	51·1	49·1	74·1	76·9	77·1	78·4	36·9	89·0	25·0	19·0	70°·3	68°·8	40°·9	22°·7	18°·2	87°·0
	42·7	43·0	78·6	82·3	80·6	86·6	17·9	77·7	49·0	38·9	67°·8	71°·0	41°·2	25°·4	15°·8	86°·8?
	56·7	57·2	70·5	72·0	72·4	77·3	40·5	75·4	59·1	46·7	66°·1	74°·2	39°·7	24°·2	15°·5	89°·7
	53·2	52·1	79·2	79·8	80·4	83·5	28·1	80·1	46·4	37·1	68°·2	68°·8	43°·0	30°·7	12°·3	81°·1
	55·9	56·0	82·2	83·5	85·8	—	33·9	95·4	57·0	50·6	—	—	—	—	—	—
	51·6	51·7	79·7	82·0	82·7	85·7	35·2	74·2	56·3	45·2	—	—	—	—	—	—
	44·5	43·7	80·4	82·5	84·0	74·6	31·9	73·7	59·8	61·3	62°·7	72°·0	45°·3	36°·1	13°·7	85°·7
	51·5	51·6	79·7	74·1	83·2	—	26·5	79·2	42·1	11·8	—	—	—	—	—	—
	56·0	55·7	74·2	74·0	79·8	—	26·9	—	55·7	60·6	71°·8	72°·9	35°·3	25°·1	10°·2	83°·1
	52·1	52·0	79·8	79·7	86·1	77·0	31·1	85·2	46·3	17·6	70°·9	67°·2	41°·9	30°·0	11°·9	79°·1
	55·4	56·5	77·0	76·6	82·7	—	—	77·2	56·6	31·5	—	—	—	—	—	—
	47·9	47·9	80·0	85·0	85·1	80·8	34·0	87·4	41·3	27·8	69°·5	65°·4	45°·1	31°·7	13°·4	78°·8
	55·0	54·6	81·5	85·1	85·2	78·0	35·8	75·3	—	—	68°·2	70°·8	41°·0	30°·1	10°·9	81°·7
	61·2	60·4	71·8	75·5	76·6	76·1	26·9	84·2	59·2	53·4	74°·8	68°·7	36°·5	24°·3	12°·2	80°·9
	59·7	58·3	86·6	81·6	90·2	—	—	88·0	34·2	18·5	68°·4	71°·0	40°·6	25°·0	15°·6	86°·6
	52·5	51·8	78·9	76·6	84·1	84·4?	—	75·8	53·9	27·0	64°·7	74°·6	40°·7	23°·7	17°·0	91°·6

* For an explanation of the abbreviations used see Section 3 of text. The tribal divisio

EMENTS OF THE INDIVIDUAL CRANIA IN THE HODGSON COLLECTION (LENGTHS IN MIL

	Angles						
SS/SC	*N*∠	*A*∠	*B*∠	θ_1	θ_2	*P*∠	REMARKS*
41·9	70°·7	68°·7	40°·6	27°·4	13°·2	81°·9	Newar. Valley of Nepal. Mandible; 3 diseased teeth in upper jaw; P.B. upwards with rounded edge; suture R. and L.
56·6	66°·7	74°·3	39°·0	26°·7	13°·3	86°·6	Newar. Valley of Nepal. P.B. flat with blunt edge; SR; JR; single epipteric bone R.
56·1	67°·1	72°·1	40°·8	28°·0	12°·8	84°·9	Newar. Valley of Nepal. Mandible; adult but only 1 third molar (upper R.) in both jaws; P.B. downwa of λ and 1 ossicle lambdoid R.
30·0	68°·1	69°·8	42°·1	25°·2	16°·9	86°·7	Newar. Valley of Nepal. Sutures beginning to close; all teeth sound; mandible; P.B. upwards with
37·8	65°·1	76°·7	38°·2	26°·5	11°·5	88°·4	Newar. Valley of Nepal. P.B. upwards with rounded edge, L. aperture smaller than R.; S equal; JR; in sagittal and lambdoid suture R. and L.; asymmetrical foramen magnum.
34·6	69°·5	71°·1	39°·4	24°·8	14°·6	85°·7	Newar. Valley of Nepal. P.B. upwards with sharp edge; SR; JR; occipital asymmetry, bulging to
31·1	63°·7	74°·8	41°·5	21°·5	10°·0	84°·8	Probably Newar. Banks of Bagmatti in Valley of Nepal. No 3rd molars; P.B. flat with blunt edge;
48·3	65°·0	74°·5	40°·5	28°·9	11°·6	86°·1	Newar. Valley of Nepal. P.B. upwards with sharp edge; S equal; J equal.
52·3	59°·5	80°·5	40°·0	17°·7	12°·3	92°·8	Newar. Valley of Nepal. 3rd molar R. growing horizontally backwards; P.B. upwards with sharp edg bulging to left.
49·4	62°·1	76°·2	41°·7	32°·9	8°·8	85°·0	Newar. Valley of Nepal. Mandible; adult but only 1 3rd molar (upper R.) in both jaws; 2 diseased double on R.; S equal; JR.
53·8	—	—	—	—	—	—	Newar. Valley of Nepal. 3 teeth lost; no 3rd molar R.; P.B. flat with sharp edge; SR; JR; epipteric temporalis R. and parietal bone; single ossicles in sagittal and lambdoid L. sutures.
27·0	61°·1	72°·9	46°·0	30°·1	15°·9	88°·8	Newar. Valley of Nepal. Mandible; adult but no 3rd molar L. in either jaw; P.B. downwards with blu R. and L. and process between temporal and parietal bones; ossicle in sagittal suture at obelion
42·2	70°·9	69°·4	39°·7	19°·3	20°·4	89°·8	Newar. Valley of Nepal. Mandible; young adult; P.B. flat with blunt edge; JR; JL; tympanic perfo in occipito-mastoid sutures R. and L.
26·0	73°·8	69°·2	37°·0	24°·3	12°·7	81°·9	Murmi. Valley of Nepal. Attrition of teeth; 4 lost; P.B. flat with blunt edge; SR; JR.
74·1	71°·9	66°·3	41°·8	28°·8	13°·0	79°·3	Murmi. Valley of Nepal. Mandible; P.B. flat with blunt edge; SL; JL; wormians lambdoid suture
34·6	57°·8	80°·3	41°·9	32°·5	9°·4	89°·7	Murmi. Valley of Nepal. Mandible; no 3rd molars in lower jaw; P.B. downwards with sharp edge;
9·9	65°·9	72°·1	42°·0	24°·3	8°·7	80°·8	Murmi. Valley of Nepal. Sagittal suture closing at obelion; tympanic perforation R.; P.B. flat wit bone R.; and process of squama temporalis joining frontal bone R. and L.; ossicle of λ; single ossic ossicle of asterion L.
29·0	67°·1	72°·2	40°·7	25°·1	15°·6	87°·8	Kiranti. East Nepal. Mandible; P.B. flat with blunt edge; S equal; J equal; tympanic perforation
30·8	65°·1	75°·8	39°·1	33°·9	5°·2	81°·0	Kiranti. East Nepal. 3rd molar erupting; P.B. flat with sharp edge; S equal; JR; large epipteric b
37·7	64°·5	75°·7	39°·8	32°·8	7°·0	82°·7	Kiranti. East Nepal. P.B. upwards with blunt edge; SR; JL; large epipteric bones R. and L.; wo
32·9	67°·8	69°·3	42°·9	32°·0	10°·9	80°·2	Kiranti. East Nepal. Mandible; P.B. flat with sharp edge; SR; JR; 2 small ossicles in lambdoid L
28·6	62°·4	78°·4	39°·2	32°·8	6°·4	84°·8?	Kiranti. East Nepal. Attrition of teeth, 2 lost; no 3rd molar L.; P.B. upwards with sharp edge; S interparietal originally bipartite; wormians lambdoid L.
46·6	62°·7	76°·1	41°·2	29°·7	11°·5	87°·6	Kiranti. East Nepal. P.B. downwards with blunt edge; S equal; JR; small ossicle of λ.
12·8	77°·9	63°·9	38°·2	24°·8	13°·4	77°·3	Limbu. East Nepal. Mandible; 1 molar lost; 1 diseased molar; P.B. flat with very rounded edge prognathous; remarkably depressed roots of nasal bones.
41·2	65°·5	72°·9	41°·6	25°·6	16°·0	88°·9	Limbu. East Nepal. P.B. flat with rounded edge; SR; JR; occipital asymmetry, bulging to left; tymp bone R.; tripartite interparietal bone.
55·4	64°·6	74°·1	41°·3	25°·5	15°·8	89°·9	Limbu. East Nepal. Mandible; adult but only 1 3rd molar (upper L.); 2 premolars lost; 1 disea and L.; P.B. downwards with sharp edge; SR; JL; wormians lambdoid suture R. and L.
65·6	—	—	—	—	—	82°·2	Limbu. East Nepal. Sagittal suture closing at obelion; teeth irregularly placed; 2 teeth lost; no 3 P.B. flat with blunt edge; SL; JR; almost complete assimilation of atlas into occipital bone; ve
41·3	65°·9	72°·2	41°·9	27°·8	14°·1	80°·3	Limbu. East Nepal. P.B. downwards with sharp edge; SR; JL; small projecting spine at opisthio
25·0	68°·3	70°·7	41°·0	29°·0	12°·0	82°·7	Lepcha. Sikkim. P.B. flat with blunt edge; SR; JR; tympanic perforation R.
33·3	58°·9	78°·9	42°·2	37°·2	5°·0	83°·9	Lepcha. Sikkim. No 3rd molar R.; P.B. downwards with sharp edge; SR; JR; small ossicle of epipteric bones R. and L.
23·9	59°·1	81°·3	39°·6	25°·6	14°·0	95°·3	Lepcha. Sikkim. Mandible; adult but no 3rd molars in either jaw; P.B. flat with sharp edge; S equal
20·0	70°·4	71°·2	38°·6	28°·0	10°·4	81°·6	Lepcha. Sikkim. Mandible; no upper 3rd molar R.; P.B. flat with rounded edge; S equal; JL; co
35·1	65°·9	72°·2	41°·9	23°·0	8°·9	91°·1	Lepcha. Sikkim. No 3rd molar R.; P.B. downwards with sharp edge; SR; JL; tympanic perforati
25·6	66°·0	73°·2	40°·8	26°·1	14°·7	87°·9	Lepcha. Sikkim. Inner palatine bridge L.; P.B. flat with sharp edge and suggestion of double edg R. and L.; single ossicle lambdoid L.
29·8	68°·1	67°·8	44°·1	24°·8	19°·3	87°·1	Lepcha. Sikkim. Young adult; P.B. flat with sharp edge and traces of double edge; SR; JR.
19·0	70°·3	68°·8	40°·9	22°·7	18°·2	87°·0	Lepcha. Sikkim. P.B. upwards with blunt edge; SR; JR; occipital asymmetry, bulging to righ 2 ossicles λ R. and 1 ossicle λ L.
38·9	67°·8	71°·0	41°·2	25°·4	15°·8	86°·8?	Lepcha. Sikkim. Mandible; P.B. downwards with blunt edge; SR; JR; small epipteric bones R. a
46·7	66°·1	74°·2	39°·7	24°·2	15°·5	89°·7	Lhopa. Bhutan. Aging; all sutures closing; teeth perfect; mandible; P.B. downwards with sharp ed R. and L.; deep palate.
37·1	68°·2	68°·8	43°·0	30°·7	12°·3	81°·1	Lhopa. Bhutan. Adult but no 3rd molars; P.B. downwards with blunt edge; S equal; JR; hori and L.; epipteric bone L.; incomplete tripartite interparietal; os triangulare R. missing; wormi
50·6	—	—	—	—	—	—	Lowlander. Nepalese Terai. Mandible; only one (lower L.) 3rd molar; P.B. downwards with ro lambdoid R. and L.
45·2	—	—	—	—	—	—	Lowlander. Nepalese Terai. P.B. downwards with sharp, double edge; SR; JR; epipteric bones parietal, no os pentagonale; healed wound parietal bone L.
61·3	62°·7	72°·0	45°·3	36°·1	13°·7	85°·7	Lowlander. Nepalese Terai. No 3rd molar R.; P.B. flat with sharp edge; SR; JR.
11·8	—	—	—	—	—	—	Lowlander. Nepalese Terai. Several teeth lost; P.B. flat with blunt edge; SR; J equal; slight squama temporalis L. divided into 3 distinct parts; ossicle lambdoid L.; large mastoid foramin
60·6	71°·8	72°·9	35°·3	25°·1	10°·2	83°·1	Lowlander. Nepalese Terai. P.B. downwards with sharp edge; S equal; JR; horizontal sutures ac R. and L.
17·6	70°·9	67°·2	41°·9	30°·0	11°·9	79°·1	Kocch. Sikkimese and Bhutanese Terai. Small pre-condyle near basion; P.B. upwards with blunt
31·5	—	—	—	—	—	—	Kocch. Sikkimese and Bhutanese Terai. Aging; all sutures closing; all teeth lost except stumps edge; S equal; JR.
27·8	69°·5	65°·4	45°·1	31°·7	13°·4	78°·8	Bodo. Sikkimese and Bhutanese Terai. Tympanic perforation R. and L.; P.B. upwards with blunt wormians lambdoid R. and L.
—	68°·2	70°·8	41°·0	30°·1	10°·9	81°·7	Dimal. Sikkimese and Bhutanese Terai. No 3rd molars; P.B. flat with blunt edge; S equal; JR; e lambdoid L.; osteitis (see text).
53·4	74°·8	68°·7	36°·5	24°·3	12°·2	80°·9	Murmi. Valley of Nepal. No 3rd molar L.; P.B. upwards with blunt edge; SR; JR; epipteric bon
18·5	68°·4	71°·0	40°·6	25°·0	15°·6	86°·6	Murmi. Valley of Nepal. 4 molars lost; tympanic perforation R. and L.; P.B. downwards with sh ossicles lambdoid R. and L.
27·0	64°·7	74°·6	40°·7	23°·7	17°·0	91°·6	Newar. Valley of Nepal. Aging; 1 molar lost; tympanic perforation R. and L.; P.B. downwards w lambdoid R. and L.

breviations used see Section 3 of text. The tribal division and locality from which the specimen was procured are given first. Unless otherwise stated the

ith rounded edge; SR; J equal; wormians in lambdoid

R.
ws; P.B. downwards with blunt edge; SR; JR; ossicle

B. upwards with blunt edge; SR; J equal.
R.; S equal; JR; epipteric bones R. and L.; wormians

metry, bulging to left. Executed for forgery.
with blunt edge; S equal; J equal; epipteric bone R.

ds with sharp edge; S equal; JR; occipital asymmetry,

jaws; 2 diseased teeth; P.B. flat with sharp edge and

R; JR; epipteric bone R. and process between squama
ures.
wnwards with blunt edge; S equal; JR; epipteric bones
suture at obelion.
; tympanic perforation R. and L.; large single ossicles

; JR.
lambdoid suture R. and L.
with sharp edge; SR; JR.
R.; P.B. flat with sharp edge; S equal; JL; epipteric
e of λ; single ossicles in lambdoid suture R. and L. and

panic perforation R. and L.; ossicles of β and λ.
; large epipteric bone R.
es R. and L.; wormians lambdoid suture R. and L.
les in lambdoid L.
ith sharp edge; S equal; JL; epipteric bone R.; simple

e of λ.
ery rounded edge; S equal; JR; 2 epipteric bones R.;

lging to left; tympanic perforation R. and L.; epipteric

lars lost; 1 diseased molar; tympanic perforation R.
R. and L.
2 teeth lost; no 3rd molars; tympanic perforation R.;
occipital bone; very projecting anterior nasal spine.
spine at opisthion.

small ossicle of λ and one ossicle lambdoid R.; small

arp edge; S equal; JR; tympanic perforation R. and L.
S equal; JL; complex sutures.
ympanic perforation L.
ion of double edge; S equal; JL; large epipteric bones

ge; SR; JR.
y, bulging to right; small epipteric bones R. and L.;

ipteric bones R. and L.
rds with sharp edge; S equal; JR; wormians lambdoid

equal; JR; horizontal sutures across malar bones R.
. missing; wormians lambdoid R.
wnwards with rounded edge; S equal; JL; wormians

epipteric bones R. and L.; incomplete tripartite inter-

.
; J equal; slight occipital asymmetry, bulging to L.;
mastoid foramina.
izontal sutures across malar bones; wormians lambdoid

vards with blunt edge; S equal; JR; osteitis (see text).
st except stumps of molars; P.B. upwards with sharp

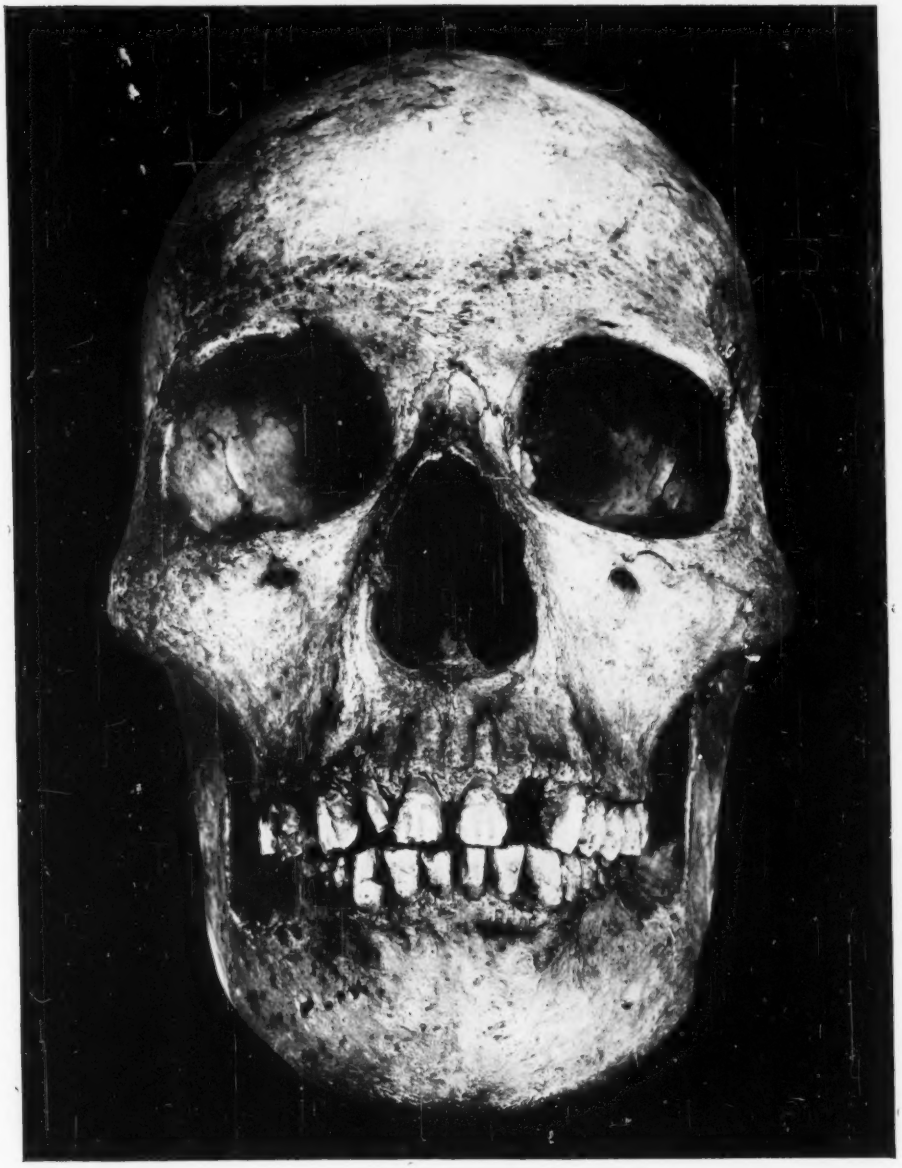
wards with blunt edge; SR; J equal; epipteric bone L.;

e; S equal; JR; epipteric bones R. and L.; single ossicle

JR; epipteric bone R.
ownwards with sharp edge; SR; JR; epipteric bone R.;

P.B. downwards with blunt edge; S equal; JR; ossicles

herwise stated the skull is adult and has no mandible.



Normal Nepalese Male (1, *e*). *Norma facialis.*

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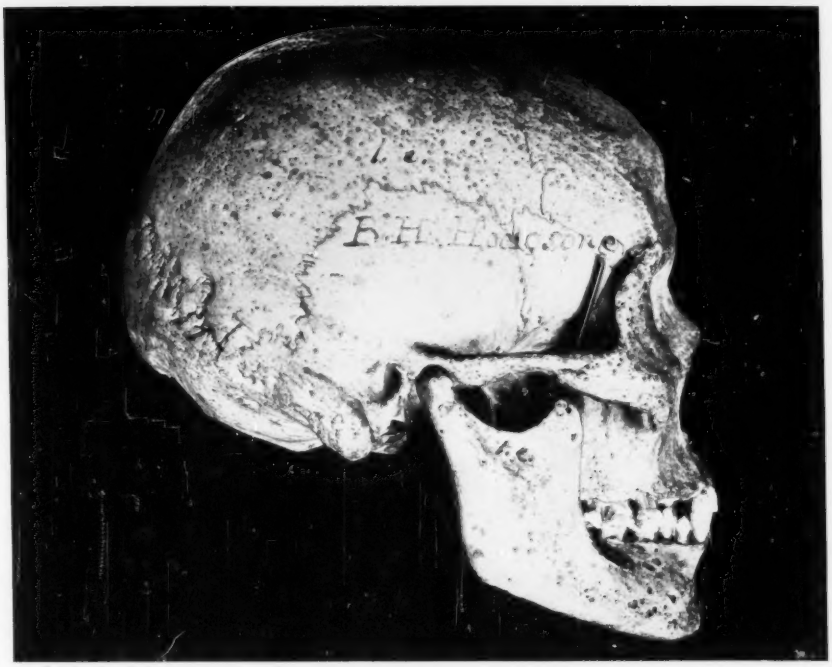
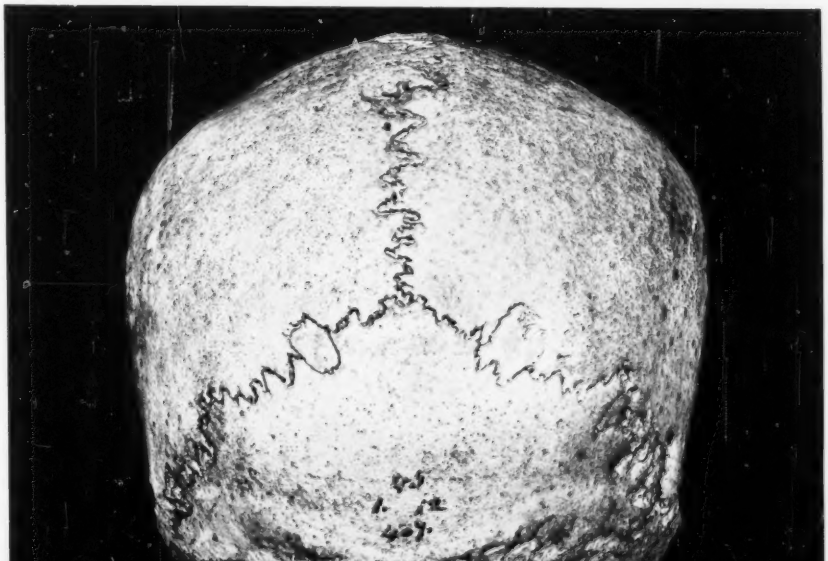


Normal Nepalese Male (1, *c*). *Norma lateralis.*



Normal Nepalese Male (1, *c*). *Norma occipitalis.*

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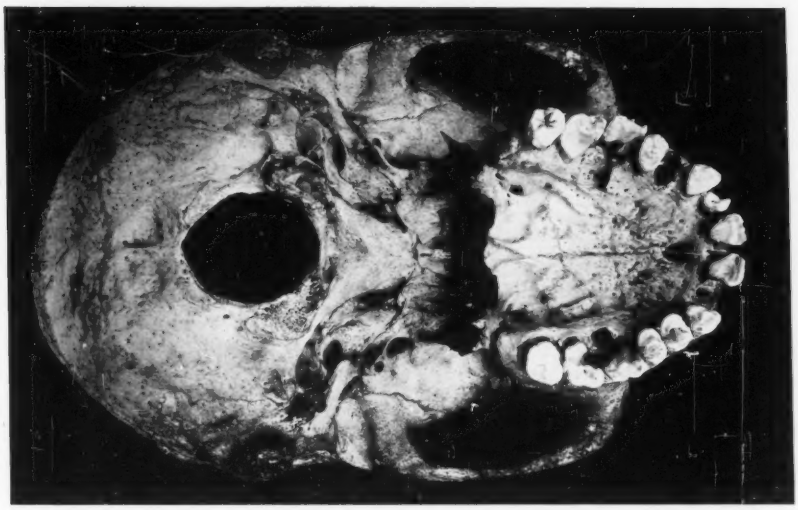
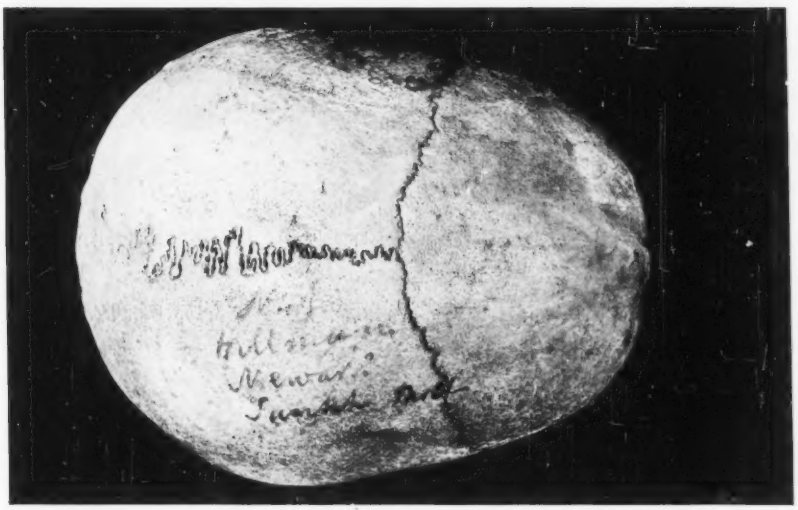


Normal Nepalese Male (1, *c*). *Norma basalis.*



Normal Nepalese Male (1, *c*). *Norma verticalis.*

Normal Nepalese Female (1, *i*). *Norma lateralis.*

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Normal Nepalese Female (1, *i*). *Norma facialis.*

Normal Nepalese Female (1, *i*). *Norma occipitalis.*

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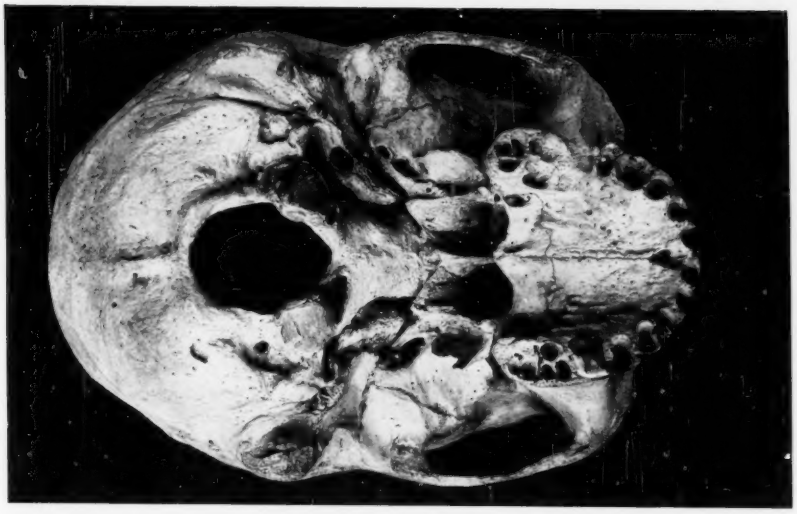
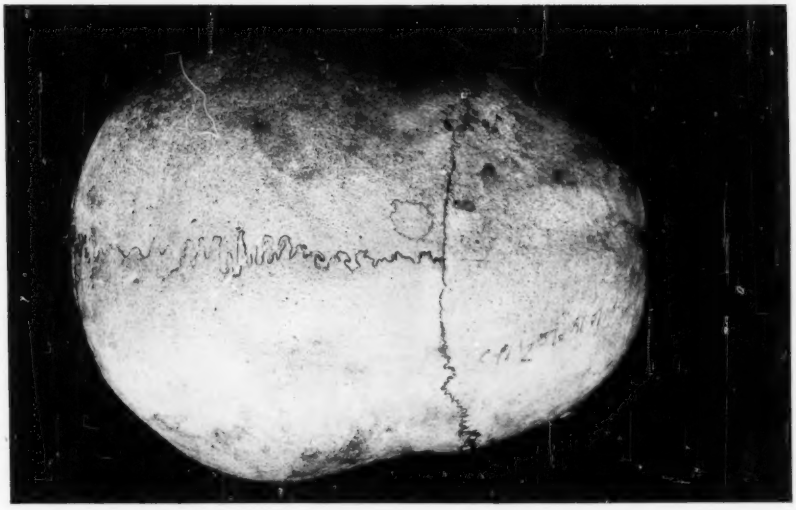


Normal Nepalese Female (1, *i*). *Norma basalis.*



Normal Nepalese Female (1, *i*). *Norma verticalis.*

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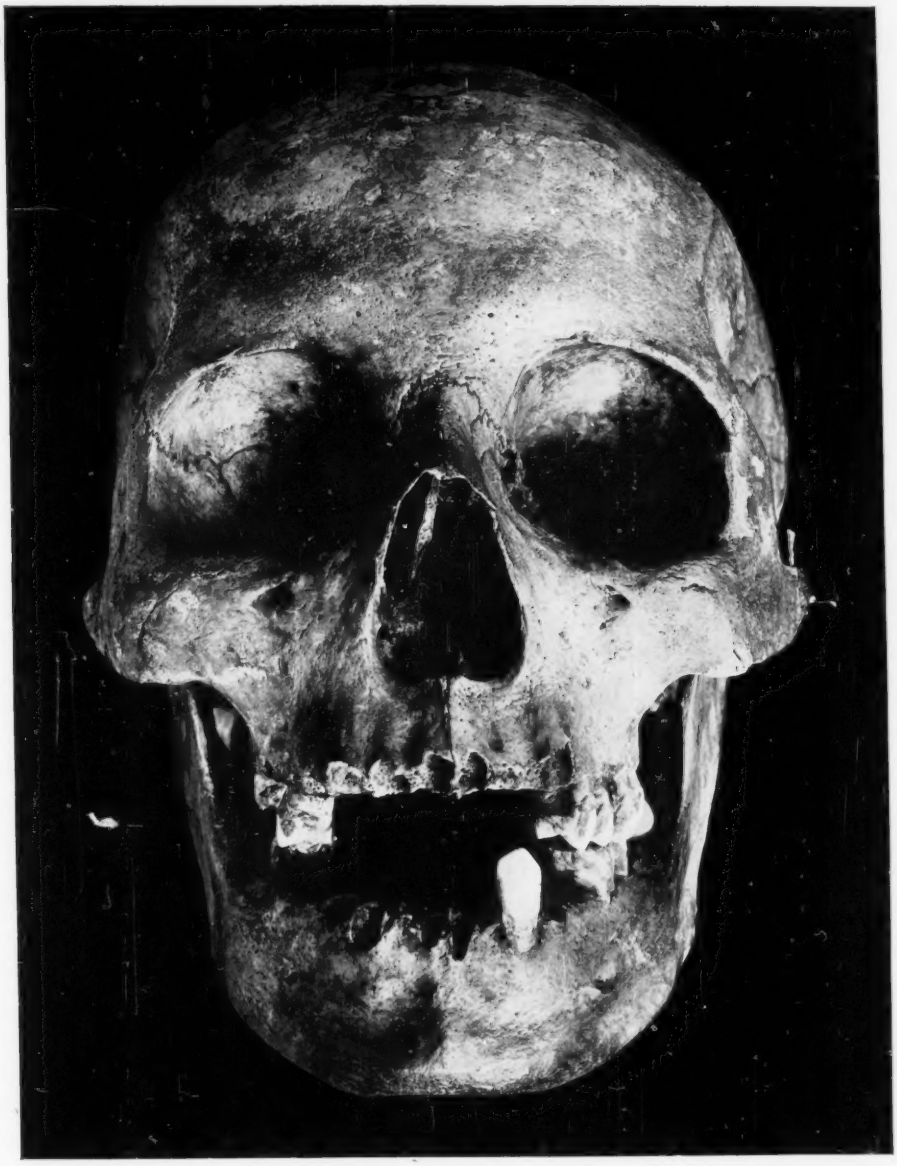

Normal Tibetan Male, Type A (1, *r.r.r.*). *Norma facialis.*

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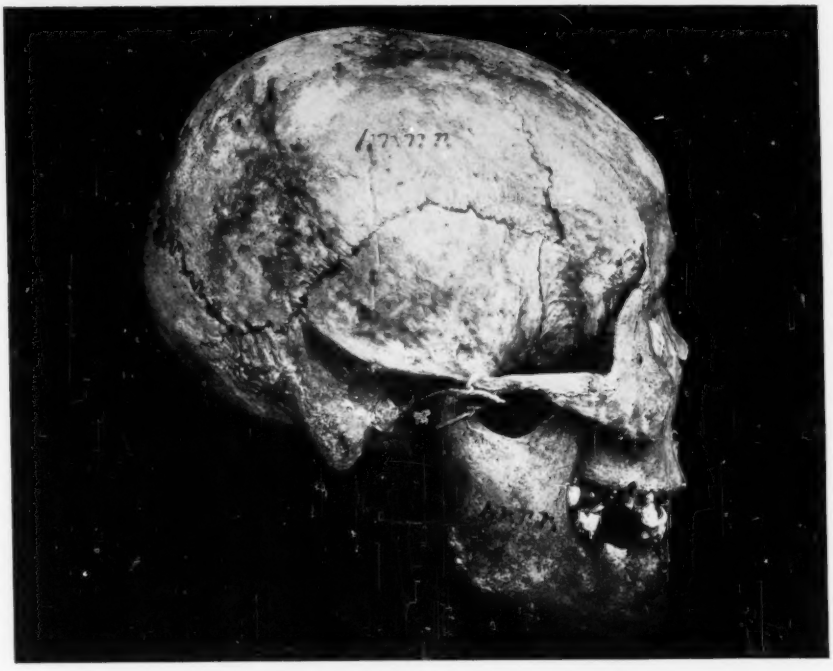
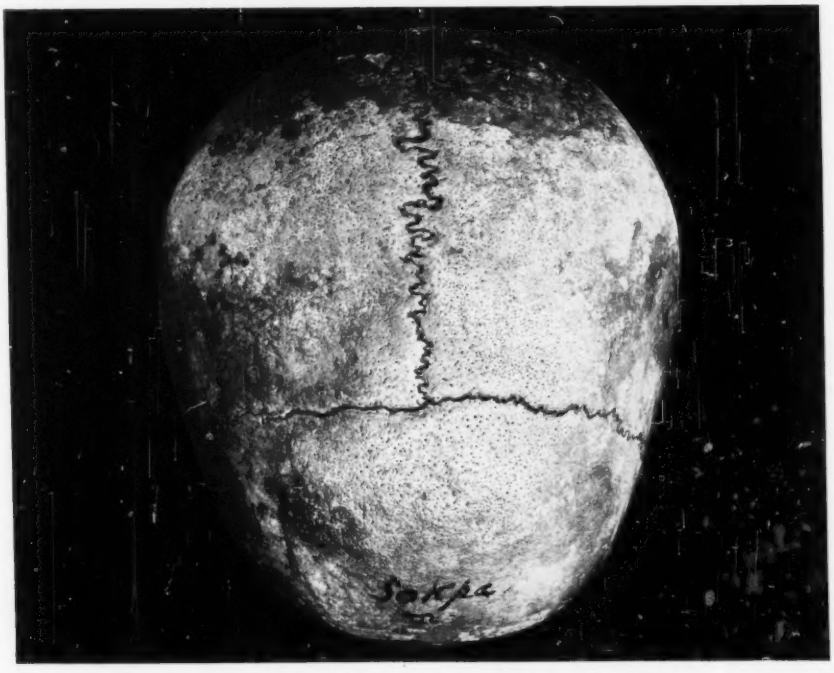


Normal Tibetan Male, Type A (1, *r.r.r.*). *Norma lateralis.*



Normal Tibetan Male, Type A (1, *r.r.r.*). *Norma verticalis.*

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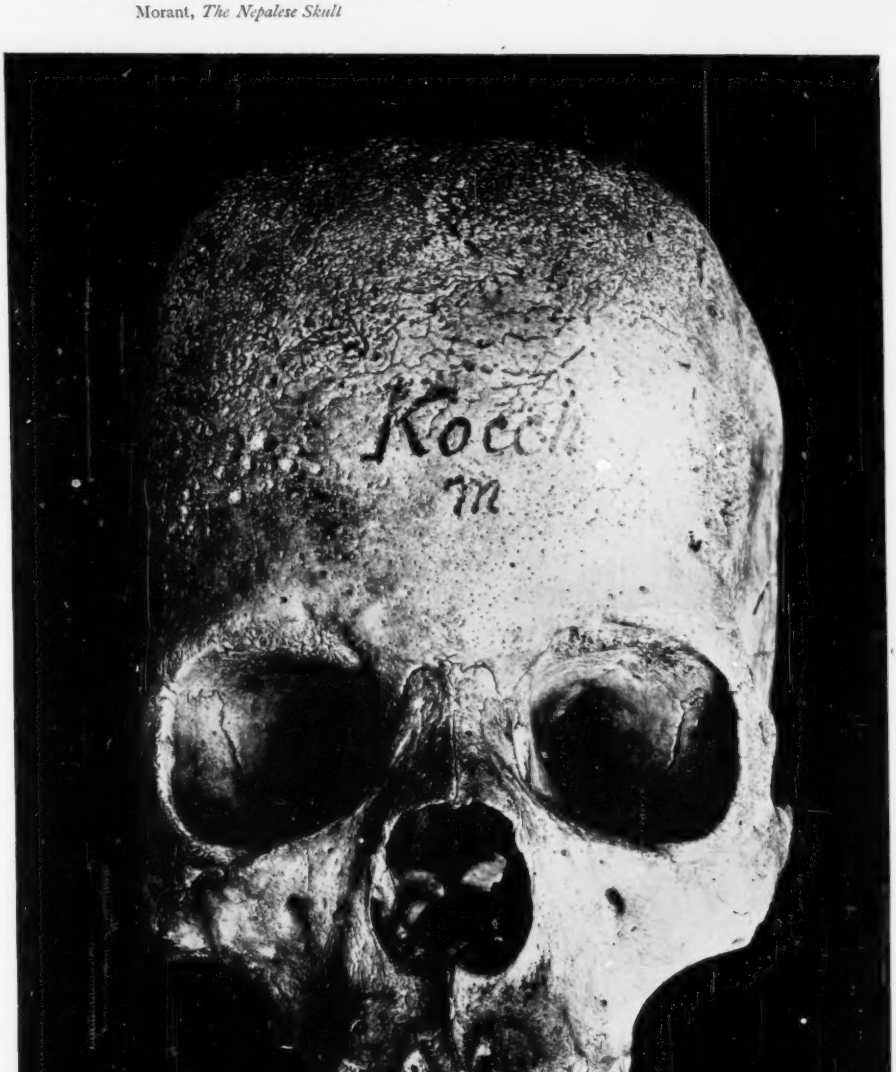

Pathological Nepalese Male Skull (1, *k.k.k.*). Osteitis (see pp. 10, 11). *Norma facialis.*

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Pathological Nepalese Male Skull (1, *k.k.k.*). Osteitis (see pp. 10, 11). *Norma lateralis.*

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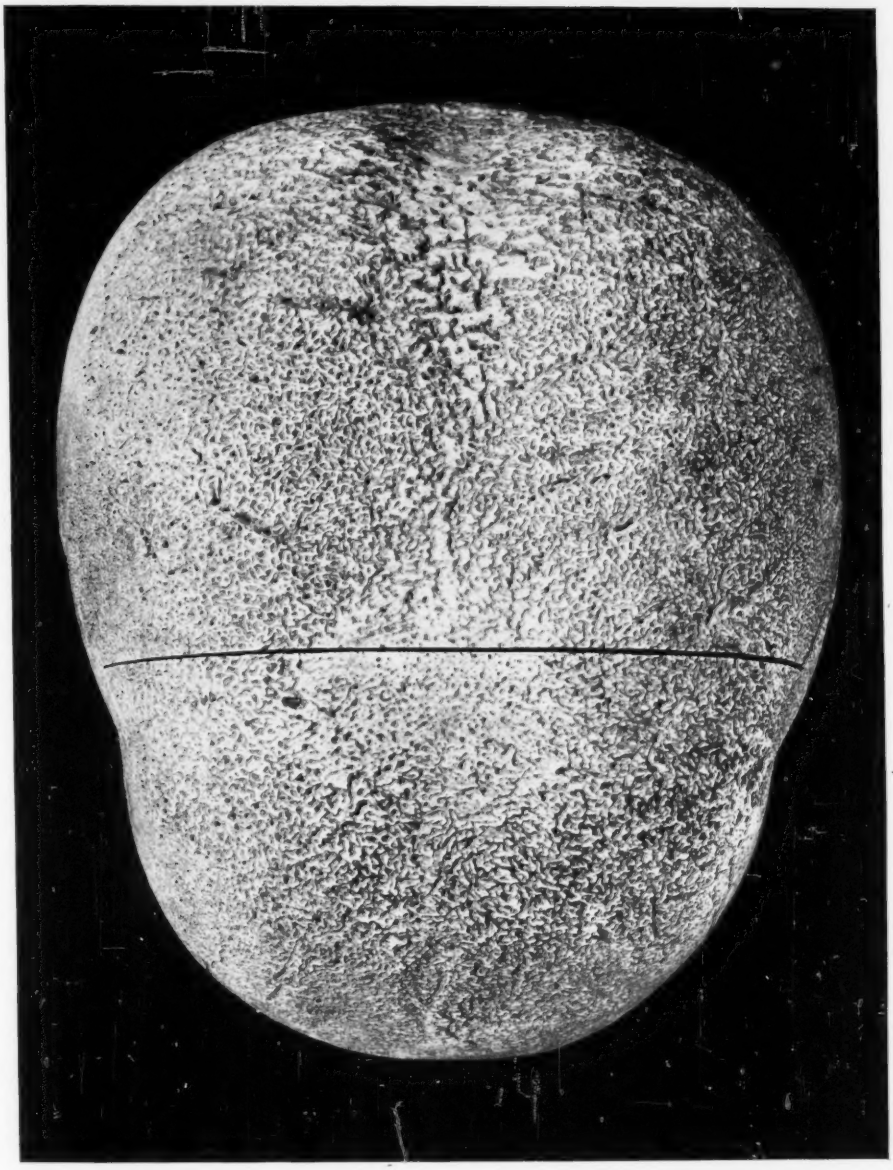

Pathological Nepalese Male Skull (1, *k.k.k.*). Osteitis (see pp. 10, 11). *Norma verticalis.*

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Pathological Nepalese Male Skull (1, *k.k.k.*). Osteitis (see pp. 10, 11).
The scale and section of frontal bone are in the same plane, and parallel to the focal plane.

Norma facialis.

Norma lateralis.

Abnormal Nepalese Skull (1, *v.v.v.v.v.*), showing marked depression of nasal bones and emphasised prognathism.

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Normal Veddah Female (C. 20). (University College, London.) *Norma facialis* (phot. K.P.).

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Normal Veddah Female (C. 20). (University College, London.) *Norma lateralis* (phot. K.P.).

Normal Veddah Female (C. 20). (University College, London.) *Norma verticalis* (phot. K.P.).

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DESCRIPTION OF PLATES.

Plates I to VI give the five aspects of a normal ♂ and a normal ♀ Nepalese skull. Plates VII and VIII are of a normal Tibetan A ♂ skull and they may be compared with the photographs of a similar skull of the same race given in *Biometrika*, Vol. XIV. Plates I, III and VII (following p. 260). Plates IX to XIII are of two anomalous crania in the Hodgson Collection and Plates XIV to XVI give three aspects of a normal Veddah ♀ skull in the Anatomical Museum, University College, London. For the latter I am indebted to Professor Karl Pearson who took the photographs.

Plate I, Normal Nepalese Male (1, *e*), *Norma facialis.*
„ II, „ „ „ *Norma lateralis.*
„ II, „ „ „ *Norma occipitalis.*
„ III, „ „ „ *Norma verticalis.*
„ III, „ „ „ *Norma basalis.*
„ IV, Normal Nepalese Female (1, *i*), *Norma lateralis.*
„ V, „ „ „ *Norma facialis.*
„ V, „ „ „ *Norma occipitalis.*
„ VI, „ „ „ *Norma verticalis.*
„ VI, „ „ „ *Norma basalis.*
„ VII, Normal Tibetan Male, Type A (1, *r.r.r.*), *Norma facialis.*
„ VIII, „ „ „ „ *Norma lateralis.*
„ VIII, „ „ „ „ *Norma verticalis.*
„ IX, Pathological Nepalese Male (1, *k.k.k.*), *Norma facialis.*
„ X, „ „ „ „ *Norma lateralis.*
„ XI, „ „ „ „ *Norma verticalis.*
„ XII, „ „ „ „ With skull-cap removed.
„ XIII, Abnormal Nepalese Male (1, *v.v.v.v v.*), *Norma facialis.*
„ XIII, „ „ „ „ *Norma lateralis.*
„ XIV, Normal Veddah Female (C. 20), *Norma facialis.*
„ XV, „ „ „ *Norma lateralis.*
„ XVI, „ „ „ *Norma verticalis.*

GENERALISATION OF SOME TYPES OF THE FREQUENCY CURVES OF PROFESSOR PEARSON.

By Professor V. ROMANOVSKY (of the University of Turkestan).

I. It is well known that the frequency curves of Prof. Pearson are derived from the differential equation

$$\frac{1}{y}\frac{dy}{dx} = \frac{x+c}{a_0x^2+a_1x+a_2} \quad \text{.....(1).}$$

Denoting by x_0 the mean of the values of x, given by some observations, and by $\mu_1, \mu_2, \mu_3, \ldots$, as usually, their successive moment-coefficients about x_0, y denoting the frequencies of the values of x, we shall obtain from (1) the following equations, which give us the constants c, a_0, a_1, a_2 in terms of $\mu_1 = 0$, μ_2, μ_3, μ_4:

$$\left.\begin{aligned} &a_1 + c = 0 \\ &3a_0\mu_2 + a_2 = -\mu_2 \\ &4a_0\mu_3 + 3a_1\mu_2 + c\mu_2 = -\mu_3 \\ &5a_0\mu_4 + 4a_1\mu_3 + 3a_2\mu_2 + c\mu_3 = -\mu_4 \end{aligned}\right\} \quad \text{.....(2).}$$

Let now $\beta_1 = \mu_3^2/\mu_2^3, \quad \beta_2 = \mu_4/\mu_2^2,$

$$k = \frac{\beta_1(\beta_2+3)^2}{4(2\beta_2 - 3\beta_1 - 6)(4\beta_2 - 3\beta_1)}.$$

Then we know that we shall have the following seven types of the frequency curves due to Prof. Pearson, obtained according to the various corresponding values of the constants β_1, β_2 and k:

I Type: $y = y_0\left(1+\frac{x}{a}\right)^{\nu a}\left(1-\frac{x}{b}\right)^{\nu b}$ $(-\infty < k < 0)$,

II „ $y = y_0\left(1-\frac{x^2}{a^2}\right)^m$ $(k=0,\ \beta_1=0,\ \beta_2 \neq 3)$,

III „ $y = y_0\left(1+\frac{x}{a}\right)^{\nu a} e^{-\nu x}$ $(k = \pm\infty)$,

IV „ $y = y_0\left(1+\frac{x^2}{a^2}\right)^{-m} e^{-\nu\, arctg\frac{x}{a}}$ $(0 < k < 1)$,

V „ $y = y_0 x^{-p}\, e^{-\nu/x}$ $(k=1)$,

VI „ $y = y_0(x-a)^{\alpha}\, x^{-\beta}$ $(1 < k < \infty)$,

VII „ $y = y_0 e^{-x^2/2a^2}$ $(k=0,\ \beta_1=0,\ \beta_2=3)$.

The aim of this article is to show some infinite series of the form

$$y = A_0u_0 + A_1u_1 + A_2u_2 + \ldots,$$

having the properties:

(*a*) $u_0, u_1, u_2, \ldots$ are certain definite one-valued functions of x and $A_0, A_1, A_2, \ldots$ are certain constants.

(*b*) Abridging the series on the right hand side of the above equation and retaining only the first member of it, we shall obtain the equation

$$y = A_0 u_0 \quad \ldots\ldots\ldots\

We shall now consider the function

$$u_0 = (a+x)^\alpha (b-x)^\beta \quad \text{.....(9)},$$

where a and b are any given constants, α and β are constants fulfilling the conditions

$$\alpha > -1, \quad \beta > -1 \quad \text{.....(10)},$$

and x is an independent real variable. This function is well defined, one-valued and continuous in the interval $(-a, b)$.

Denoting the kth derivative of u_0 by $D^k u_0$, we introduce the functions

$$u_k = D^k (a+x)^{\alpha+k} (b-x)^{\beta+k} \quad (k = 1, 2, 3, \ldots) \quad \text{.....(11)}.$$

It is easy to show that

$$u_k = u_0 \phi_k \quad \text{.....(12)},$$

where

$$\phi_k = (\alpha+1)_k (b-x)^k - \frac{k}{1}(\alpha+2)_k (\beta+k)_k (b-x)^{k-1}(a+x)$$
$$+ \frac{k(k-1)}{1.2}(\alpha+3)_k (\beta+k-1)_k (b-x)^{k-2}(a+x)^2 - \ldots$$
$$+ (-1)^k (\beta+1)_k (a+x)^k \quad \text{.....(13)},$$

$$\left.\begin{aligned}(\alpha+i)_k &= (\alpha+i)(\alpha+i+1)\ldots(\alpha+k)\\ (\beta+h)_k &= (\beta+h)(\beta+h+1)\ldots(\beta+k)\end{aligned}\right\} \quad \text{.....(14)}.$$

The polynomials ϕ_k are well known—they are the polynomials of Jacobi in slightly altered form.

Let us show that

$$\left.\begin{aligned}\int_{-a}^{b} u_0 \phi_h \phi_k \, dx &= 0, \quad \text{if } h \neq k\\ &= J_k, \quad \text{if } h = k\end{aligned}\right\} \quad \text{.....(15)},$$

J_k being a quite definite constant, depending on a, b, α, β.

Consider first the case $h \neq k$. Then we can put $h > k$ and write

$$\int_{-a}^{b} u_0 \phi_h \phi_k \, dx = \int_{-a}^{b} \phi_h \, . \, D^k (a+x)^{\alpha+k} (b-x)^{\beta+k} \, . \, dx.$$

Integrating by parts and noting that

$$\left[\phi_h \, . \, D^{k-1} (a+x)^{\alpha+k} (b-x)^{\beta+k}\right]_{-a}^{b} = 0,$$

we shall find

$$\int_{-a}^{b} u_0 \phi_h \phi_k \, dx = -\int_{-a}^{b} D\phi_h \, . \, D^{k-1} (a+x)^{\alpha+k} (b-x)^{\beta+k} \, dx.$$

New integration by parts gives us

$$\int_{-a}^{b} u_0 \phi_h \phi_k \, dx = \int_{-a}^{b} D^2 \phi_h \, . \, D^{k-2} (a+x)^{\alpha+k} (b-x)^{\beta+k} \, dx,$$

and so on. Finally we come to

$$\int_{-a}^{b} u_0 \phi_h \phi_k \, dx = (-1)^{h+1} \int_{-a}^{b} D^{h+1} \phi_h \, . \, D^{k-h-1} (a+x)^{\alpha+k} (b-x)^{\beta+k} \, dx = 0,$$

ϕ_h being a polynomial of degree h and, therefore, $D^{h+1} \phi_h$ being identically zero.

Secondly, let $h = k$. Then, integrating k times by parts, we can obtain

$$\left.\begin{aligned} J_k &= \int_{-a}^{b} u_0 \phi_k \phi_k \, dx = (-1)^k D^k \phi_k \int_{-a}^{b} (a+x)^{\alpha+k} (b-x)^{\beta+k} \, dx \\ &= (-1)^k D^k \phi_k \, . \, J_k' \end{aligned}\right\} \text{.....(16),}$$

where $$J_k' = \int_{-a}^{b} (a+x)^{\alpha+k} (b-x)^{\beta+k} \, dx$$

can be reduced to

$$J_k' = (a+b)^{\alpha+\beta+2k+1} \int_0^1 z^{\alpha+k} (1-z)^{\beta+k} \, dz,$$

putting $z = \dfrac{a+x}{a+b}$. Consequently

$$\begin{aligned} J_k' &= (a+b)^{\alpha+\beta+2k+1} B(\alpha+k+1, \beta+k+1) \\ &= (a+b)^{\alpha+\beta+2k+1} \frac{\Gamma(\alpha+k+1)\,\Gamma(\beta+k+1)}{\Gamma(\alpha+\beta+2k+2)}. \end{aligned}$$

Applying here the fundamental property of Γ-functions

$$\Gamma(x+1) = x\,\Gamma(x),$$

we shall find

$$J_k' = (a+b)^{\alpha+\beta+2k+1} \frac{(\alpha)_k (\beta)_k}{(\alpha+\beta)_{2k+1}} \cdot \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)} \text{ .....(17)}$$

$(\alpha)_k$, $(\beta)_k$ and $(\alpha+\beta)_{2k+1}$ being defined as in (14).

Consider now $(-1)^k D^k \phi_k$. Remembering (13) and differentiating as indicated, we easily obtain:

$$\begin{aligned} (-1)^k D^k \phi_k = k! \Big[&(\alpha+1)_k + \frac{k}{1} (\alpha+2)_k (\beta+k)_k \\ &+ \frac{k(k-1)}{1.2} (\alpha+2)_k (\beta+k-1)_k + \ldots + (\beta+1)_k \Big], \end{aligned}$$

or, after some transformations,

$$(-1)^k D^k \phi_k = k! \, (\alpha+\beta+k+1)_{2k} \text{ .....(18).}$$

With (17) and (18) we get by aid of (16):

$$J_k = \lambda_k \frac{\Gamma(\alpha)\,\Gamma(\beta)}{\Gamma(\alpha+\beta)} \text{ .....(19),}$$

where $$\lambda_k = k! \, (\alpha+\beta+k+1)_{2k} \, (a+b)^{\alpha+\beta+2k+1} \frac{(\alpha)_k (\beta)_k}{(\alpha+\beta)_{2k+1}} \text{ .....(20).}$$

III. The results set forth in the preceding paragraph enable us to calculate the coefficients of the expansion of any given function y of x according to the functions $u_0, u_1, u_2, \ldots$, defined by (9), (12) and (13), assuming that this expansion exists and can be integrated term by term. Indeed, let this expansion be

$$y = A_0 u_0 + A_1 u_1 + A_2 u_2 + \ldots \text{ .....(21),}$$

and suppose that the series on the right hand side is uniformly convergent in the interval $(-a, b)$. Then we can integrate this series by terms after having multiplied it by any of the polynomials $\phi_0 = 1, \phi_1, \phi_2, \ldots$. This gives us an easy method of determining the coefficients $A_0, A_1, A_2, \ldots$ of the expansion (21).

Take A_k; we shall determine it by multiplying both sides of (21) by ϕ_k and then integrating from $-a$ to b. Thus we find:

$$\begin{aligned}\int_{-a}^{b} y\phi_k\,dx &= \int_{-a}^{b} \phi_k \sum_{h=0}^{\infty} A_h u_h\,.\,dx \\ &= \sum_{h=0}^{\infty} A_h \int_{-a}^{b} u_0 \phi_h \phi_k\,dx \\ &= A_k J_k,\end{aligned}$$

according to (15). Suppose that we can evaluate the integral $\int_{-a}^{b} y\phi_k\,dx$ and that we denote it by S_k. Then we get

$$A_k = \frac{S_k}{J_k} \quad (k = 0, 1, 2, \ldots) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(22),$$

and our expansion (21) takes the form

$$y = \frac{S_0}{J_0} u_0 + \frac{S_1}{J_1} u_1 + \frac{S_2}{J_2} u_2 + \ldots \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(23),$$

where

$$S_k = \int_{-a}^{b} y\phi_k\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24).$$

Remembering (19), we can write (23) in the form

$$y = \frac{S_0}{\lambda_0} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} u_0 \left[1 + \frac{\lambda_0}{\lambda_1} \frac{S_1}{S_0} \phi_1 + \frac{\lambda_0}{\lambda_2} \frac{S_2}{S_0} \phi_2 + \ldots\right] \ldots\ldots\ldots\ldots(25).$$

We shall not linger over the conditions under which the expansion (25) can exist. The reader, wishing to know them, can refer to modern books on the theory of integral equations (such as Kneser's, Hilbert's, Volterra's and others) or to a very valuable memoir of the Russian mathematician, W. Stekloff: *Sur les expressions asymptotiques de certaines fonctions, définies par les équations différentielles linéaires du second ordre, et leurs applications au problème du développement d'une fonction arbitraire en série procédant suivant les-dites fonctions*, published in the *Journal of the Mathematical Society of Kharkoff* (Сообщенія Харьковскаго Математическаго Общества), Vol. x. 1909. We shall indicate only that the expansion (25) is admissible for functions that are continuous in the interval $(-a, b)$, i.e. for such functions as are chiefly considered in the applications of mathematical statistics.

IV. If we stop the series with its first member, we shall obtain from (25) the equation

$$y = \frac{S_0}{\lambda_0} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} u_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26),$$

which can be regarded as an approximation to the given function y. If we do not stop with the first member, but retain more members, we shall obtain a finite series

which will approximate indefinitely to the function y if we increase the number of members retained. So, at any rate theoretically, we can fit as closely as we please any given distribution, taking the expansion (25) and retaining therein a sufficient number of members. This procedure may be expected to be the more successful, if the distribution in question is of Type I. But to this remark we shall return later, and we shall consider now (26)—the first approximation to the given function y.

We can write (26) as follows:

$$y = S_0 (a+b)^{\alpha+\beta+1} \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\,\Gamma(\beta+1)} (a+x)^{\alpha} (b-x)^{\beta},$$

if we remember that

$$\lambda_0 = (a+b)^{\alpha+\beta+1} \frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)},$$

and attend also to the fundamental property of the Γ-function. Further we shall suppose that

$$\frac{\alpha}{a} = \frac{\beta}{b} = \nu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (27),$$

and make

$$y_0 = \frac{S_0}{a+b} \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}} \frac{\Gamma(\alpha+\beta+2)}{\Gamma(\alpha+1)\,\Gamma(\beta+1)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (28).$$

Then our first approximation takes the form

$$y = y_0 \left(1 + \frac{x}{a}\right)^{\alpha} \left(1 - \frac{x}{b}\right)^{\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots (29),$$

i.e. represents the equation of the frequency curve of Type I.

We shall now show, that supposing

$$\alpha = \nu a, \quad \beta = \nu b,$$

and determining ν, a, b by the aid of equations (5) and (6), we obtain

$$S_1 = S_2 = S_3 = S_4 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30),$$

so that the expansion (25) for such values of ν, a, b takes the form

$$y = \frac{S_0}{\lambda_0} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)} u_0 \left[1 + \frac{\lambda_0}{\lambda_5} \frac{S_5}{S_0} \phi_5 + \frac{\lambda_0}{\lambda_6} \frac{S_6}{S_0} \phi_6 + \ldots\right] \ldots\ldots\ldots\ldots (31).$$

Let

$$S_0 \mu_k' = \int_{-a}^{b} (a+x)^k \, y \, dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (32),$$

then

$$S_0 \mu_k' = \sum_{h=0}^{\infty} A_h \int_{-a}^{b} (a+x)^k \, u_h dx,$$

where the A's are coefficients of the expansion (25). But, for $h > k$, we shall have

$$\int_{-a}^{b} (a+x)^k \, u_h dx = 0,$$

as is easily shown, if we use the same method as that by which we obtained the first of relations (15). So

$$S_0 \mu_k' = \sum_{h=0}^{k} A_h \int_{-a}^{b} (a+x)^k \, u_h dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (33).$$

Now consider

$$\int_{-a}^{b}(a+x)^k u_h dx = \int_{-a}^{b}(a+x)^k D^h (a+x)^{\alpha+h}(b-x)^{\beta+h} dx$$

for $h \leqq k$. Integrating by parts, we easily obtain

$$\int_{-a}^{b}(a+x)^k u_h\, dx = (-1)^h k(k-1)\dots(k-h+1)\int_{-a}^{b}(a+x)^{k-h}(a+x)^{\alpha+h}(b-x)^{\beta+h}\, dx$$

$$= (-1)^h k(k-1)\dots(k-h+1)\int_{-a}^{b}(a+x)^{\alpha+k}(b-x)^{\beta+h} dx$$

$$= (-1)^h k(k-1)\dots(k-h+1)(a+b)^{\alpha+\beta+k+h+1}\int_0^1 z^{\alpha+k}(1-z)^{\beta+h} dz,$$

or

$$\int_{-a}^{b}(a+x)^k u_h\, dx = (-1)^h k(k-1)\dots(k-h+1)(a+b)^{\alpha+\beta+k+h+1}\frac{\Gamma(\alpha+k+1)\Gamma(\beta+h+1)}{\Gamma(\alpha+\beta+k+h+2)}.$$

Remembering further the value of λ_h in

$$A_h = \frac{S_h}{\lambda_h}\cdot\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)},$$

it is easy to see that for $0 \leqq h \leqq k$

$$A_h\int_{-a}^{b}(a+x)^k u_h\, dx = (-1)^h \frac{S_h}{(\alpha+\beta+h+1)_{2h}}\cdot\frac{k(k-1)\dots(k-h+1)}{1.2.3\dots h}\cdot(a+b)^{k-h}$$
$$\times\frac{(\alpha+h+1)_k}{(\alpha+\beta+2h+2)_{k+h+1}} \qquad \dots\dots\dots\dots\dots\dots\dots\dots(34).$$

Summing these values from $h=0$ to $h=k$, we get

$$S_0\mu_k' = (a+b)^k\frac{(\alpha+1)_k}{(\alpha+\beta+2)_{k+1}}S_0 - \frac{k}{1}(a+b)^{k-1}\frac{(\alpha+2)_k}{(\alpha+\beta+4)_{k+2}}\frac{S_1}{\alpha+\beta+2}$$
$$+\frac{k(k-1)}{1.2}(a+b)^{k-2}\frac{(\alpha+3)_k}{(\alpha+\beta+6)_{k+2}}\frac{S_2}{(\alpha+\beta+3)_4} - \dots + (-1)^k\frac{S_k}{(\alpha+\beta+k+1)_{2k}}$$
$$\dots\dots\dots\dots\dots\dots(35).$$

Whence

$$\left.\begin{aligned}
S_0\mu_1' &= (a+b)\frac{\alpha+1}{\alpha+\beta+2}S_0 - \frac{S_1}{\alpha+\beta+2}\\
S_0\mu_2' &= (a+b)^2\frac{(\alpha+1)_2}{(\alpha+\beta+2)_3}S_0 - 2(a+b)\frac{\alpha+2}{\alpha+\beta+4}\frac{S_1}{\alpha+\beta+2} + \frac{S_2}{(\alpha+\beta+3)_4}\\
S_0\mu_3' &= (a+b)^3\frac{(\alpha+1)_3}{(\alpha+\beta+2)_4}S_0 - 3(a+b)^2\frac{(\alpha+2)_3}{(\alpha+\beta+4)_5}\frac{S_1}{\alpha+\beta+2}\\
&\quad + 3(a+b)\frac{\alpha+3}{\alpha+\beta+6}\frac{S_2}{(\alpha+\beta+3)_4} - \frac{S_3}{(\alpha+\beta+4)_6}\\
S_0\mu_4' &= (a+b)^4\frac{(\alpha+1)_4}{(\alpha+\beta+2)_5}S_0 - 4(a+b)^3\frac{(\alpha+2)_4}{(\alpha+\beta+4)_6}\frac{S_1}{\alpha+\beta+2}\\
&\quad + 6(a+b)^2\frac{(\alpha+3)_4}{(\alpha+\beta+6)_7}\frac{S_2}{(\alpha+\beta+3)_4} - 4(a+b)\frac{\alpha+4}{\alpha+\beta+8}\frac{S_3}{(\alpha+\beta+4)_6}\\
&\quad + \frac{S_4}{(\alpha+\beta+5)_8}
\end{aligned}\right\}\ (36).$$

Note now, that the μ_k''s are moments about $x = -a$ and the μ_k's are moments about $x = x_0$, and therefore are connected by the relation

$$\mu_k' = (a+x_0)^k + \frac{k}{1}(a+x_0)^{k-1}\mu_1 + \ldots + \mu_k,$$

whence, as $\mu_1 = 0$,

$$\mu_1' = a + x_0, \quad \mu_2' = (a+x_0)^2 + \mu_2, \quad \mu_3' = (a+x_0)^3 + 3(a+x_0)\mu_2 + \mu_3,$$
$$\mu_4' = (a+x_0)^4 + 6(a+x_0)^2\mu_2 + 4(a+x_0)\mu_3 + \mu_4.$$

Substituting here in the place of μ_2, μ_3, μ_4 their values from (7), we shall obtain

$$\mu_1' = \frac{(a+b)(\alpha+1)}{\alpha+\beta+2}, \qquad \mu_2' = \frac{(a+b)^2(\alpha+1)_2}{(\alpha+\beta+2)_3},$$
$$\mu_3' = \frac{(a+b)^3(\alpha+1)_3}{(\alpha+\beta+2)_4}, \quad \mu_4' = \frac{(a+b)^4(\alpha+1)_4}{(\alpha+\beta+2)_5}.$$

Multiplying these equations with S_0 and then subtracting from (36) correspondingly, we find:

$$\frac{S_1}{\alpha+\beta+2} = 0, \quad -2(a+b)\frac{\alpha+2}{\alpha+\beta+4}\frac{S_1}{\alpha+\beta+2} + \frac{S_2}{(\alpha+\beta+3)_4} = 0,$$
$$-3(a+b)^2\frac{(\alpha+2)_3}{(\alpha+\beta+4)_5}\frac{S_1}{\alpha+\beta+2} + 3(a+b)\frac{\alpha+3}{\alpha+\beta+6}\frac{S_2}{(\alpha+\beta+3)_4} - \frac{S_3}{(\alpha+\beta+4)_6} = 0,$$
$$-4(a+b)^3\frac{(\alpha+2)_4}{(\alpha+\beta+4)_6}\frac{S_1}{\alpha+\beta+2} + 6(a+b)^2\frac{(\alpha+3)_4}{(\alpha+\beta+6)_7}\frac{S_2}{(\alpha+\beta+3)_4}$$
$$-4(a+b)\frac{\alpha+4}{\alpha+\beta+8}\frac{S_3}{(\alpha+\beta+4)_6} + \frac{S_4}{(\alpha+\beta+5)_8} = 0.$$

Then, we see that really

$$S_1 = S_2 = S_3 = S_4 = 0,$$

if the constants a, b, α, β in u_0 are determined as they are for the frequency curve of Type I.

V. We see that the expansion (25) fulfils all the statements (a), (b), (c) made in paragraph I. In particular, we see that, choosing the constants appropriately, we can bring it into the form

$$y = y_0\left(1+\frac{x}{a}\right)^{\alpha}\left(1-\frac{x}{b}\right)^{\beta}\left[1 + \frac{\lambda_0}{\lambda_5}\frac{S_5}{S_0}\phi_5 + \frac{\lambda_0}{\lambda_6}\frac{S_6}{S_0}\phi_6 + \ldots\right] \ldots\ldots (36)\ bis.$$

The applications of this result depend on the evaluation of moments of orders 5, 6 and so on, which is not always desirable seeing the high probable errors of these moments. But in some cases, if we abridge the series on the right hand side of our equation to two or three terms, the equations obtained may be of use. To this point I shall return in a later article, when I have the time for the necessary calculations.

I would make another remark about the expansion (25):

$$y = \frac{S_0}{\lambda_0}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}u_0\left[1 + \frac{\lambda_0}{\lambda_1}\frac{S_1}{S_0}\phi_1 + \frac{\lambda_0}{\lambda_2}\frac{S_2}{S_0}\phi_2 + \ldots\right] \ldots\ldots\ldots\ldots (25).$$

One knows that by evaluating the constants a, b, α, β entering in the coefficients of this series according to the relations (6), one finds as a rule non-integer numbers, which must be taken with many decimals in order to secure the needed accuracy of the results to be drawn from the equation of the curve. I now suggest that one can use the expansion (25) in this way: we may take instead of the usual values of a, b, α, β, those calculated to their nearest integer. Then S_1, S_2, S_3 and S_4 will not vanish, but the calculation while more complicated in its form may be simpler. This simplification may be considerable, if we tabulate the polynomials ϕ_1, ϕ_2, ϕ_3, ϕ_4 (and possibly ϕ_5 and ϕ_6) for integer values of a, b, α, β appropriate to practice. But all these remarks are only suggestions and I shall consider them in another article.

VI. I shall not stay long over the curve of Type II:

$$y = y_0\left(1 - \frac{x^2}{a^2}\right)^m \quad \text{.....(37),}$$

for this curve is only a special case of the foregoing one: make $a = b$, $\alpha = \beta = m$ and we obtain (37). On this assumption the expansion (25) gives a generalised form of the curve (37). If we abridge it to its first member, we shall obtain (37). If we choose the constants as follows:

$$a = \sqrt{\frac{2\mu_2\beta_2}{3-\beta_2}}, \quad \alpha = \frac{5\beta_2 - 9}{2(3-\beta_2)}, \quad y_0 = \frac{S_0\Gamma(2\alpha+2)}{2^{2\alpha+1}\,a\,[\Gamma(\alpha+1)]^2},$$

then in the expansion (25), duly rewritten, S_1, S_2, S_3 and S_4 will be zero.

VII. Consider now the curves of Type III:

$$y = y_0\left(1 + \frac{x}{a}\right)^{\nu a} e^{-\nu x} \quad \text{.....(38).}$$

They are fitted to the distribution given by aid of the relations

$$a = \frac{2\mu_2^2}{\mu_3} - \frac{\mu_3}{2\mu_2}, \quad \nu = \frac{2\mu_2}{\mu_3}, \quad y_0 = \frac{S_0}{a}\,\frac{\alpha^{\alpha+1}}{e^{\alpha}\,\Gamma(\alpha+1)} \quad \text{.....(39),}$$

where

$$\alpha = \nu a \quad \text{.....(40).}$$

We shall take in this case

$$u_0 = (a+x)^{\alpha} e^{-\nu x}, \quad u_k = D^k (a+x)^{\alpha+k} e^{-\nu x} \quad \text{.....(41),}$$

where

$$\alpha > -1, \quad \nu > 0 \quad \text{.....(42).}$$

We find

$$u_k = u_0 \phi_k \quad \text{.....(43)}$$

$$\phi_k = (\alpha+1)_k - \frac{k}{1}(\alpha+2)_k\,\nu\,(a+x) + \frac{k(k-1)}{1.2}(\alpha+3)_k\,\nu^2(a+x)^2 - \ldots + (-1)^k \nu^k (a+x)^k \quad \text{.....(44).}$$

It is known that the polynomials, defined by the relation

$$\psi_k = e^x D^k (x^k e^{-x}),$$

are the polynomials of Laguerre. Thus the ϕ_k are the same polynomials in generalised form.

By the same method as was described above we easily find that

$$\left.\begin{aligned}\int_{-a}^{\infty} u_0\phi_h\phi_k dx &= 0\ (h,\ k=0,\ 1,\ 2,\ldots,\ h\neq k)\\ \int_{-a}^{\infty} u_0\phi_k\phi_k dx &= k!\,\nu^k\int_{-a}^{\infty}(a+x)^{\alpha+k}e^{-\nu x}dx\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(45),$$

$$= \frac{k!\,(\alpha)_k}{\nu^{\alpha+1}}e^{\nu a}\,\Gamma(\alpha) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(46).$$

We can show that for a certain class of functions y of the independent variable x, one-valued and definite in the interval $(-a,\ \infty)$, an expansion of the form

$$y = A_0u_0 + A_1u_1 + A_2u_2 + \ldots$$

exists uniformly convergent in the above interval.

Among these functions are the continuous functions, i.e., of course, continuous in the interval $(-a,\ \infty)$. The coefficients $A_0,\ A_1,\ A_2,\ldots$ of this series are readily obtainable: multiply both sides with ϕ_k and integrate from $-a$ to ∞, then

$$A_k = \frac{\nu^{\alpha+1}S_k}{k!\,e^{\nu a}\,(\alpha)_k\,\Gamma(\alpha)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(47),$$

where

$$S_k = \int_{-a}^{\infty} y\phi_k\,dx \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(48).$$

Thus we have

$$y = \frac{\nu^{\alpha+1}S_0}{e^{\nu a}\,\alpha\,\Gamma(\alpha)}\,u_0\left[1 + \sum_{k=1}^{\infty}\frac{S_k}{k!\,(\alpha+1)_k\,S_0}\phi_k\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(49).$$

It is easy to show that for the constants ν, α, a defined by the aid of equations (39) and (40), we shall have

$$S_1 = S_2 = S_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(50),$$

and, therefore, the expansion (49) takes in this case the form

$$y = y_0\left(1 + \frac{x}{a}\right)^{\nu a}e^{-\nu x}\left[1 + \sum_{k=4}^{\infty}\frac{S_k}{k!\,(\alpha+1)_k\,S_0}\phi_k\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(51).$$

If we abridge this infinite series and take only its first member, we shall obtain the frequency curve of Prof. Pearson's Type III.

We can repeat now all the remarks that were made in paragraph V. We may add to them, that in the case of distributions of Type III we work under more favourable conditions than in the case of distributions of Types I or II. Indeed, for the evaluation of constants in the present case we need first only three moments of distribution. The fourth one can be evaluated in most cases with sufficient accuracy and thus S_4 can be obtained with accuracy of the same order; therefore we can expect a better fit to the data by aid of the curve

$$y = y_0\left(1 + \frac{x}{a}\right)^{\nu a}e^{-\nu x}\left[1 + \frac{S_4}{4!\,(\alpha+1)_4\,S_0}\phi_4\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots(52)$$

than by the usual curve of Type III. Here

$$\phi_4 = (\alpha+1)_4 - 4(\alpha+2)_4 \nu(a+x) + 6(\alpha+3)_4 \nu^2 (a+x)^2 - 4(\alpha+4)_4 \nu^3 (a+x)^3 + \nu^4 (a+x)^4,$$

and

$$S_4 = \int_{-a}^{\infty} y \phi_4 \, dx,$$

which is easily expressed in terms of μ_2, μ_3, μ_4.

VIII. There remains now the curve of Type VII:

$$y = y_0 e^{-x^2/2a^2}.$$

But we omit its discussion, for the expansion, that generalises the above equation, is well known and has been the subject of researches and applications to statistics by a great number of authors.

IX. We note in conclusion that for the polynomials ϕ_k, considered above for the curves of the first three types, there exist simple reduction formulae, which can be of great use in calculations. But they will be considered in another article, which is to deal with numerical applications of the present theory.

BERLIN, 5. XI. 1923.

Note on Professor Romanovsky's Generalisation of my Frequency Curves.

BY KARL PEARSON.

I have published Professor Romanovsky's paper because it seems to me that it may interest readers of *Biometrika* to see that functions may be derived from my curves in precisely the same way as Thiele and Charlier have derived functions from the normal curve $y = y_0 e^{-\frac{1}{2}(x/\sigma)^2}$; and accordingly those who wish to do so need not be confined to the endeavour to expand frequency curves in tetrachoric functions. Whether, however, such functions are appropriate to statistical frequencies remains entirely unproven. It is certain that tetrachoric functions do not give good results (see J. Henderson, *Biometrika*, Vol. XIV. pp. 157—185) in the case of distributions which closely approach actual statistical experience. There is no *a priori* reason to suppose except for an increase in the number of available constants that these series will give the best form of curve any more than any other series. To illustrate my meaning consider (52); we choose Type III, because the relation $2\beta_2 - 3\beta_1 - 6 = 0$ is satisfied. Presumably if this relation is satisfied $S_4 = 0$, because we ought to fall back on Type III. If it is not satisfied we have to make an allowance for the correct value of β_2. There is no evidence that using (52) *retaining one term only* will give a distribution with the true β_2, but if I use the appropriate curve, I shall get a theoretical distribution with the true fourth moment, in a much more manageable form, and I am prepared to say a better fit to the observations than (52) will provide. Again, if the first four moments do not

give a reasonable fit, *allowing for random sampling*, I should anticipate better results from integrating

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x+a}{c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots} \quad \dots\dots\dots\dots\dots\dots(\epsilon),$$

where c_3 and higher constants introduce the higher moments, than by adopting (36) *bis*. Whether we introduce c_3, or c_3 and c_4, we have at once a definite criterion for using a curve of this type, while in the case of series like (36) *bis* we have no criterion at all as to their suitability. The theory of curves like (ϵ) was worked out a number of years ago by Dr David Heron; it has not been published because (*a*) good fits in the case of homogeneous material are as a rule found from using the first four moments, and because (*b*) of my profound distrust—based largely on experience of the different results for the frequency curves obtained from different samples of the same material, when high moment coefficients are involved—of the use of high moments at all.

For example, suppose we had an observational population, and therefore one involving considerable irregularities of distribution. We will now take samples, relatively small, out of this population and we require to know the distribution of the variability in these samples—the problem has recently arisen in a very important practical case. Well, we can actually experiment by taking a large number of samples and finding the distribution of their σ^2, or we can use the formulae provided by Professor Tchouproff* involving μ_5, μ_6 and μ_8. If we give to these constants the values observed in the original population, which is itself a sample of an indefinitely large population, the probable errors of these moment coefficients are so large, that the resulting constants of the distribution of standard deviations may be widely different from what the original population would provide. I have a very firm conviction that the mathematician who uses high moments may make interesting contributions to mathematics, but he removes his work from any contact with practical statistics.

* *Biometrika*, Vol. XII. pp. 193—4.

ON STABILITY OF THE CEPHALIC INDICES WITHIN THE RACE.

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(1) *Introductory.* A considerable number of anthropologists seem to believe that all racial problems can be solved by merely ascertaining the mean cephalic index within a given group or district. As a matter of fact such a belief can lead only to a deceptive simplicity of treatment; no group can be defined without the study of a multiplicity of characters. In recent times a strong attack has been delivered on the almost sacrosanct character of the cephalic index by Professor Franz Boas*, who has asserted that the children of Jewish parents when born in America have cephalic indices significantly different from those of their brothers and sisters born in Europe; the cephalic indices of these American born Jews are stated to differ from those of their European brethren and to approach more nearly those of the American bred white population. In other words environment is asserted to produce—not a selective but—an immediate physical change in head shape.

Still more recently Miss R. M. Fleming† has stated that according to her observations the cephalic index changes with growth. She writes of children:

"Other points noted are that up to the age of 8 years the girls showed rapid increase in cephalic index...while from 9 years onward the changes were much slower and less marked. The boys up to the age of 10 showed only slight alterations...in increase of cephalic index, but from 10 years onward changes were rapid and marked" (p. 71).

While Miss Fleming states that she has measured two to three thousand children, her conclusions appear to be based on 419 cases, of which 187 were males and 232 females, "remeasured at intervals of from one to two years." It does not appear from this whether she remeasured each child only once, or whether she followed it through its growth, say from 4 to 14, measuring it every year.

Anyhow it is clear that if the views of Professor Boas and Miss Fleming be correct there would not be much value left in cephalic index as a racial character. Miss Fleming would have demonstrated that it varies with growth and Professor Boas would have indicated that growth being a function of environment the cephalic index of the Jew could become rapidly that of the native-born American.

Unfortunately, if these views were correct, we should have to find another explanation of the close equality of cephalic index in English, German and Russian

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† *Man*, Vol. xxii. pp. 69—72.

Jews; we should have to explain why centuries have failed in Europe to strip the Jew of those racial characters, which a voyage across the Atlantic accomplishes in the American born Jew.

The Biometric and Galton Laboratories possess several long series of head measurements, and from these it is possible to determine the manner in which both cephalic indices change with growth. If we endeavour in the present paper to reinstate the cephalic index as a stable racial character, it is not intended to proclaim for it any unique position,—such as the followers of Retzius have assumed it to hold,—among many cranial indices and characters. All that we would suggest is that like a good many other cranial characters it is determined in the first place by heredity, and if modified by environment, this change must be due to the long selection of ages.

The series of head measurements referred to are the following:

(i) Measurements on school children about 4500 in number and nearly equally divided among the sexes. The ages range from 4 to 19, and the children are almost entirely those of the English professional classes from Kindergarten to the great public girls' and boys' schools. The measurements taken were Breadth (B), Length (L) and Auricular Height (H). The two cephalic indices $100B/L$ and $100H/L$ were formed. The material was collected between 1895 and 1900, and growth curves were formed at the latter date to reduce the material for other purposes. All the growth curves have now been recalculated.

(ii) A series of measurements taken by and for Galton in his Anthropometric Laboratory. These range from children of 5 to adults of 80. But the males predominate largely over the females especially in the head measurements, which at first were not taken on the women. Only head length and head breadth were recorded. The measurements were made about 1884, but the visitors to the Laboratory were rather heterogeneous in class ranging from skilled workmen to ministers of state, e.g. Mr Gladstone. Still the material gives a fairly good idea of the extent to which the cephalic index changes with age in a crowd of mixed English classes and of mixed districts of origin.

(iii) A third series is at present being collected in the Anthropometric Department of the Galton Laboratory, partly on children and partly on students. The results of these measurements will not at present be referred to.

(2) *Growth Curve for Cephalic Index* $100B/L$ *in Boys.* Table I gives the cephalic index for boys ranging from 4 to 20. Table II gives the mean cephalic index for each year of age.

The only one of these array values which differs significantly from the mean of the whole is that corresponding to 11·5 years. None of the other values seems of any significance. The odds against such a value as 78·38 occurring are of the order 5 to 1, since we have 32 chances of a deviation occurring which differs

TABLE I.

Age and First Cephalic Index in Boys.

Central Ages.

Central Cephalic Indices.	4·5	5·5	6·5	7·5	8·5	9·5	10·5	11·5	12·5	13·5	14·5	15·5	16·5	17·5	18·5	19·5	20·5	Totals
67·45	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
68·45	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
69·45	—	—	—	—	3	—	—	2	—	—	—	—	—	—	—	—	—	5
70·45	—	—	—	—	—	—	4	2	2	1	2	—	—	—	1	—	—	12
71·45	—	—	—	—	2	2	—	4	2	4	4	2	—	1	—	—	—	21
72·45	—	—	1	2	3	—	1	5	3	4	4	4	2	—	—	—	—	29
73·45	1	1	1	3	4	10	8	12	8	9	6	6	3	3	2	—	—	77
74·45	—	2	3	2	5	7	9	13	8	12	9	10	4	1	1	—	—	86
75·45	1	2	1	7	6	13	22	24	20	20	14	9	16	8	4	—	—	167
76·45	—	3	4	9	26	15	19	27	41	21	21	15	11	7	5	2	—	226
77·45	—	3	3	5	17	24	28	34	35	36	26	31	19	8	3	—	—	272
78·45	1	—	5	17	17	34	43	35	42	50	31	29	17	9	8	3	1	342
79·45	3	2	6	9	10	27	24	34	38	40	28	22	13	8	1	—	1	266
80·45	—	3	6	7	17	23	26	31	32	23	19	16	10	9	4	4	—	230
81·45	—	1	2	9	14	15	17	23	19	28	22	18	9	7	4	—	—	188
82·45	—	—	1	7	11	8	16	18	17	19	23	8	5	6	4	2	—	145
83·45	1	3	4	5	7	8	5	12	13	14	10	6	5	4	2	—	—	99
84·45	—	—	1	4	6	4	2	7	13	12	4	2	4	—	1	1	—	61
85·45	—	—	1	1	3	1	3	1	8	8	4	1	4	3	1	—	—	39
86·45	—	—	1	1	2	1	2	—	4	4	2	5	3	—	—	—	—	25
87·45	—	—	—	—	1	—	2	1	1	2	2	1	—	—	—	—	—	10
88·45	—	—	—	—	—	—	2	—	—	4	—	1	1	—	—	—	—	8
89·45	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
90·45	—	—	—	—	1	—	—	—	—	—	—	1	—	—	—	—	—	2
Totals	7	20	40	89	156	192	233	286	306	311	231	187	126	74	41	12	2	2313

TABLE II.

First Cephalic Index at each Central Age, Boys.

Age	100 B/L	Age	100 B/L
4·5	78·45 ± ·825	14·5	78·95 ± ·143
5·5	78·35 ± ·488	15·5	78·79 ± ·159
6·5	79·25 ± ·344	16·5	78·92 ± ·194
7·5	79·05 ± ·231	17·5	78·99 ± ·254
8·5	78·99 ± ·175	18·5	78·77 ± ·341
9·5	78·68 ± ·157	19·5	79·81 ± ·565
10·5	78·69 ± ·143	20·5	
11·5	78·38 ± ·129		
12·5	79·01 ± ·125	Mean of whole	78·87 ± ·045
13·5	79·23 ± ·124		

$2{\cdot}56\sigma$ from the mean 78·87. The graph, Diagram I, gives the growth changes and on either side of the mean value is placed 2·5 times the probable error of a mean*. It will be seen that only the mean at age 11·5 years falls outside this zone. It

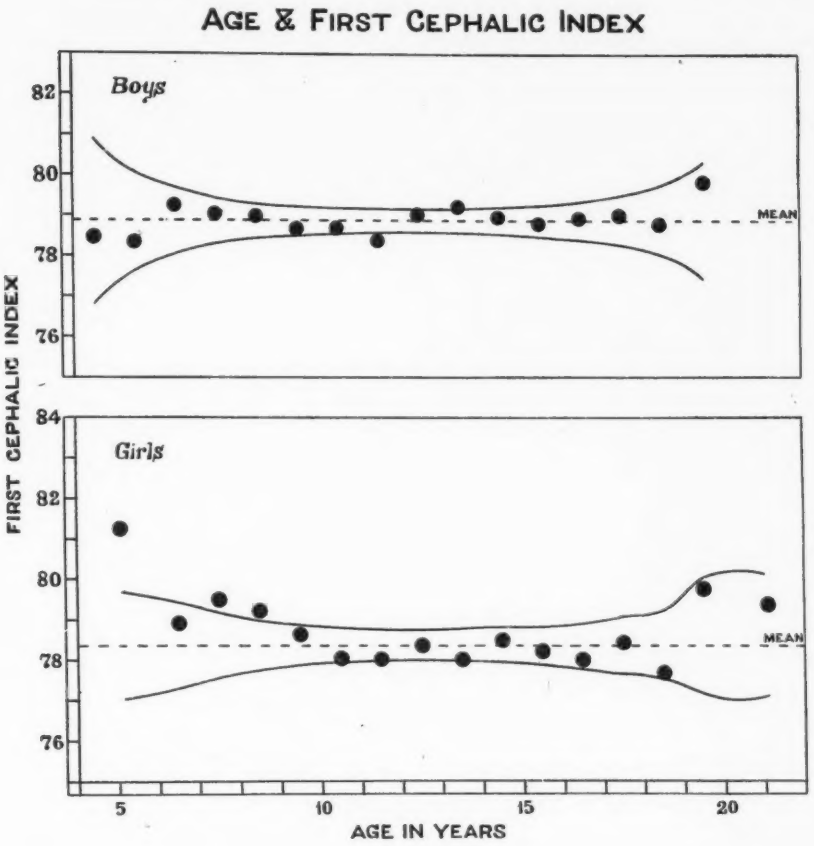

Diagram I.

might be suggested that the cephalic index rises slightly from 4·5 to 6·5 then falls to 11·5, after which the years of puberty mark a rapid rise and are followed by a stable value from 12 or 13 to 20. But far larger numbers than our 2313 would be required to justify the assumption of any such refined changes. We can demonstrate this in another way. The correlation coefficient of age and cephalic index is

$$r_{aC_1} = \cdot 0198 \pm \cdot 0177,$$

* The S.D. of the total series, σ, is $3{\cdot}238 \pm {\cdot}032$, and we may take at age a, $2{\cdot}5 \times 3{\cdot}238 \times {\cdot}67449/\sqrt{n_a}$, where n_a is the frequency of the array at age a, for the semi-breadth of the belt or zone.

which is non-significant: and we have for the correlation ratio $\eta_{C_1.a}$ of index on age

$$\eta^2_{C_1.a} = \cdot 006{,}730,$$

while the mean correlation ratio for no association is

$$\bar{\eta}^2_{C_1.a} = \cdot 006{,}485.$$

Or again, $$\eta_{C_1.a} = \cdot 0820 \pm \cdot 026,$$

and does not differ significantly from

$$\bar{\eta}_{C_1.a} = \cdot 0805.$$

(3) *Second Cephalic Index in Boys* ($100H/L$). Table III gives the actual data, while Table IV provides the means of the arrays for each central age. Here again there is only a single outstanding mean index, that at the central age 6·5. This, as the outstanding values in other series, has been carefully checked, but the value seems correct. We cannot account for its divergence and it does not belong to any systematic deviation of boys round this age from the general mean. We are therefore compelled to consider it as a divergence peculiar to our individual

TABLE III.

Age and Second Cephalic Index in Boys.

Central Ages.

Central Cephalic Indices.	4·5	5·5	6·5	7·5	8·5	9·5	10·5	11·5	12·5	13·5	14·5	15·5	16·5	17·5	18·5	19·5	20·5	Totals
57·45	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	2
58·45	—	—	—	—	—	1	—	—	—	—	1	—	—	—	1	—	—	3
59·45	—	—	—	1	—	—	—	—	3	1	1	—	—	1	—	—	—	7
60·45	—	1	—	—	2	3	2	2	2	1	2	—	1	1	—	1	—	18
61·45	1	—	—	—	1	2	3	2	3	4	2	1	—	1	—	—	—	20
62·45	1	1	—	—	3	3	4	3	2	5	1	4	3	—	3	—	—	33
63·45	—	1	—	5	3	6	8	6	7	13	3	4	3	1	1	—	—	61
64·45	—	1	1	2	5	4	14	11	10	9	10	10	3	5	2	—	—	87
65·45	1	1	1	6	7	14	15	22	30	16	12	14	9	5	2	2	—	157
66·45	—	1	2	4	11	20	14	24	23	32	22	16	11	7	2	—	—	189
67·45	—	1	5	11	17	15	27	34	26	33	20	9	13	3	4	1	—	219
68·45	—	3	2	12	19	20	18	35	29	34	28	27	16	6	3	2	—	254
69·45	1	3	2	10	19	23	25	28	27	29	24	23	11	4	1	1	—	231
70·45	—	1	6	13	21	27	31	33	31	31	29	15	14	7	7	1	—	267
71·45	1	2	6	3	16	13	17	27	27	21	25	17	9	11	2	2	2	201
72·45	—	—	3	4	7	19	21	22	28	20	14	11	7	5	4	1	—	166
73·45	1	—	5	5	6	8	12	11	19	23	17	8	11	2	3	—	—	131
74·45	—	1	1	3	5	9	6	8	13	14	4	7	3	7	1	—	—	82
75·45	—	1	1	5	6	—	8	11	7	9	6	6	7	5	2	—	—	74
76·45	—	—	3	1	5	—	4	5	13	6	4	6	1	1	1	—	—	50
77·45	1	1	1	1	—	2	3	—	4	4	—	6	1	1	—	1	—	26
78·45	—	—	—	3	2	2	1	—	2	4	—	—	1	1	—	—	—	16
79·45	—	—	1	—	1	1	—	1	—	1	1	—	1	—	2	—	—	9
80·45	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	2
81·45	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	1
82·45	—	—	—	—	—	—	—	—	—	—	2	1	—	—	—	—	—	3
83·45	—	1	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	3
84·45	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
85·45	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	1
Totals	7	20	40	89	156	192	233	286	306	311	231	187	126	74	41	12	2	2313

TABLE IV.

Second Cephalic Index at each Central Age, Boys.

Age	100 H/L	Age	100 H/L
4·5	69·23 ± ·487	*14·5*	69·63 ± ·166
5·5		*15·5*	69·61 ± ·185
6·5	71·24 ± ·400	*16·5*	69·61 ± ·225
7·5	69·65 ± ·268	*17·5*	69·86 ± ·294
8·5	69·56 ± ·202	*18·5*	69·60 ± ·395
9·5	69·11 ± ·183	*19·5*	69·38 ± ·675
10·5	69·25 ± ·166	*20·5*	
11·5	69·28 ± ·149		
12·5	69·60 ± ·145	Mean of Whole	69·49 ± ·053
13·5	69·49 ± ·144		

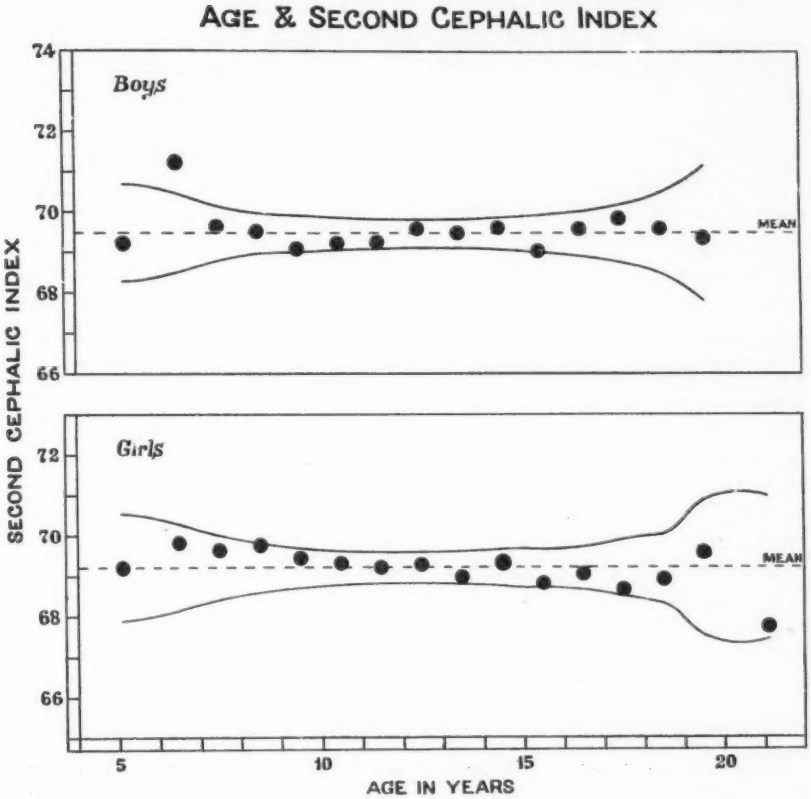

Diagram II.

sample. Diagram II shows the mean values for each array of age lying well within the zone of random sampling, except for this anomaly*.

The correlation coefficient for Table II is given by

$$r_{aC_2} = \cdot 0076 \pm \cdot 0140,$$

and is therefore wholly insignificant. The correlation ratio of second cephalic index on age $\eta_{C_2 . a}$ is given by

$$\eta^{\circ}_{C_2 . a} = \cdot 0061,$$

while the mean value of η for zero association is determined by

$$\overline{\eta}^{\circ}_{C_2 . a} = \cdot 0061.$$

Thus the correlation ratio is non-significant, or the insignificance of r_{aC_2} is not associated with the existence of a significant skew correlation. Accordingly as far as our data go we have found for boys no measurable change in either of the two chief cephalic indices from ages 4 to 20.

(4) *First Cephalic Index in Girls* ($100B/L$). Table V gives the actual data for 2189 girls of ages 3·5 to 22·5. The means for each central age are given in Table VI. The graph is provided in Diagram I. They are slightly more irregular than the results for the boys, but the variability for the girls is roughly 22 % greater than in the case of the boys†. It may be questioned whether this difference is real; it is not met with when measurements are taken on the skull, and is possibly due to the greater difficulty of taking the head measurements in the case of girls from the far greater amount of hair‡.

The correlation coefficient $r_{C_1 a}$ is given by

$$r_{C_1 a} = - \cdot 0603 \pm \cdot 0144.$$

This indicates a small but significant amount of correlation, the cephalic index very slightly *decreasing* with age. Miss Fleming states that up to the age of 8 years the girls showed rapid increase in cephalic index while from 9 years onward changes were much slower and less marked. This agrees no more with our observations than what she writes of the boys, namely: that up to the age of 10 they showed only slight *increases* of cephalic index, but from 10 years onward changes were rapid and marked. If any stress can be laid at all on the irregularities of our means of the arrays, boys would appear to reach a maximum at 6·5 and then to *decrease*, not increase. On the other hand girls start with a high value from 3 to 5 years and then rapidly decrease, not increase, in cephalic index. The correlation ratio equally indicates a slight but significant association. We have

$$\eta_{C_1 . a} = \cdot 1331 \pm \cdot 0210,$$

* The standard deviation of the whole population is $\sigma_{C_2} = 3 \cdot 757 \pm \cdot 038$.

† The standard deviation σ_{C_1} for the girls is $3 \cdot 895 \pm \cdot 040$ as against $3 \cdot 238 \pm \cdot 032$ in the case of the boys.

‡ In the second cephalic index girls are also, if to a slighter extent, more variable than boys. It is more common to find girls nervous about the insertion of the ear-plugs than is the case with boys; so that there is more difficulty in placing them adequately in the auricular passages.

TABLE V.

Age and First Cephalic Index in Girls.

Central Ages.

Central Cephalic Indices.	3·5	4·5	5·5	6·5	7·5	8·5	9·5	10·5	11·5	12·5	13·5	14·5	15·5	16·5	17·5	18·5	19·5	20·5	21·5	22·5	Totals
61·45	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	2
62·45	—	—	—	—	—	—	—	1	1	1	—	—	—	—	—	—	—	—	—	—	3
63·45	—	—	—	—	—	—	—	2	1	—	—	1	—	—	—	1	—	—	—	—	5
64·45	—	—	—	—	—	—	—	1	—	1	—	1	2	1	—	—	—	—	—	—	6
65·45	—	—	—	—	—	—	2	1	—	1	—	—	—	—	—	—	—	—	—	—	4
66·45	—	—	—	—	—	—	1	—	1	—	—	1	—	2	—	—	—	—	—	—	5
67·45	—	—	—	—	—	—	—	1	1	—	3	—	—	1	1	—	—	—	—	—	7
68·45	—	—	—	—	—	2	—	—	1	2	3	2	2	—	1	1	—	—	—	—	14
69·45	—	—	—	—	—	—	—	2	3	3	5	1	2	1	—	1	—	—	—	—	18
70·45	—	—	—	—	1	2	2	3	6	4	2	2	1	3	—	1	—	—	—	—	27
71·45	—	—	—	—	—	1	—	5	4	6	1	1	9	3	4	1	—	—	—	—	35
72·45	—	—	—	1	—	—	4	4	6	6	10	5	3	8	1	3	—	—	—	—	51
73·45	—	—	—	2	3	3	8	4	11	5	13	7	7	1	3	2	—	—	1	—	70
74·45	1	—	—	1	1	3	8	9	16	15	10	13	10	7	3	2	—	—	—	—	99
75·45	—	—	—	3	6	9	10	8	14	18	16	14	13	12	4	7	1	—	—	—	135
76·45	—	—	—	5	4	7	21	24	31	22	16	12	16	13	7	6	1	1	1	—	187
77·45	—	—	3	5	6	10	12	20	33	31	35	22	29	13	9	5	1	1	2	—	237
78·45	—	—	1	3	9	12	25	38	33	46	34	28	20	27	14	7	4	1	—	—	302
79·45	—	—	1	3	12	10	17	33	15	26	34	20	26	16	11	7	2	2	2	—	237
80·45	—	2	3	3	5	21	23	29	25	29	25	14	25	15	7	1	2	—	—	—	229
81·45	—	—	6	4	10	12	19	12	26	27	15	14	14	9	9	6	—	—	—	—	183
82·45	—	2	—	3	5	2	10	9	11	15	11	11	10	10	7	10	3	—	—	—	119
83·45	—	1	1	2	3	8	6	6	6	8	12	11	5	5	2	—	—	—	1	—	77
84·45	—	1	1	3	5	5	3	3	9	11	7	3	8	2	3	—	—	—	—	—	64
85·45	—	—	—	—	—	—	4	3	4	5	1	7	4	2	1	—	—	—	—	—	31
86·45	—	—	1	—	1	1	—	2	3	3	3	3	1	2	—	—	1	1	—	1	23
87·45	—	—	—	—	1	1	1	—	1	—	1	2	—	1	—	1	—	—	—	—	9
88·45	—	1	—	—	—	—	—	—	2	—	—	1	—	—	—	—	—	—	—	—	4
89·45	—	—	—	—	—	1	2	—	1	—	—	—	—	—	—	—	—	—	—	—	4
90·45	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
91·45	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	1
Totals	1	7	17	38	72	111	178	220	266	285	258	196	208	154	87	62	15	6	7	1	2189

TABLE VI.

First Cephalic Index at each Central Age, Girls.

Age	100 *B/L*	Age	100 *B/L*
3·5	81·25 ± ·525	*14·5*	78·50 ± ·188
4·5		*15·5*	78·21 ± ·182
5·5		*16·5*	78·00 ± ·212
6·5	78·90 ± ·426	*17·5*	78·43 ± ·281
7·5	79·49 ± ·310	*18·5*	77·69 ± ·333
8·5	79·22 ± ·249	*19·5*	79·78 ± ·678
9·5	78·64 ± ·197	*20·5*	79·38 ± ·700
10·5	78·06 ± ·177	*21·5*	
11·5	78·03 ± ·161	*22·5*	
12·5	78·37 ± ·155		
13·5	78·01 ± ·163	Mean of Whole	78·37 ± ·056

while $\eta^2_{C_1 \cdot a} = \cdot 017{,}736$ and $\bar{\eta}^2_{C_1 \cdot a} = \cdot 006{,}396$ or, $\bar{\eta}_{C_1 \cdot a} = \cdot 0800$, so that $\eta_{C_1 \cdot a}$ is probably significantly different from $\bar{\eta}_{C_1 \cdot a}$. The graph shows, however, no systematic divergences from linearity, which could be adequately described by a smooth curve.

(5) *Second Cephalic Index in Girls* ($100H/L$). Table VII provides the material and Table VIII gives the mean second cephalic index for each central age. The graph is provided in Diagram II. Here again the same method was adopted of plotting off on either side of the general mean a zone of 2·5 times the probable error, calculated on the assumption that each array was merely a random sample of the general population of the given size of the array. Again there appear no deviations of real significance.

The correlation $r_{C_2 a} = -\cdot 0532 \pm \cdot 0144$ and thus though very small is significant. The graph shows a slight decrease with age. On the other hand

$$\eta_{C_2 \cdot a} = \cdot 0813 \pm \cdot 0212,$$

while

$$\eta_{C_2 \cdot a} = \cdot 0800,$$

and accordingly no stress can be laid on $\eta_{C_2 \cdot a}$ for it does not differ sensibly from the mean value it would take if there were no association between the second cephalic index and age.

We conclude therefore that the shape of the head in both boys and girls remains practically fixed from 4 or 5 years onward, and the cephalic indices are essentially stable. We do not think that anthropologists need be troubled by any fear that in a given class of the community it is of any great importance to correct for the age of the subjects measured. Such *slight* evidences of association with age as we have come across in the case of girls are far more probably due to difficulties associated with measurement of the head of school girls with luxuriant supplies of hair, than to a sexual difference in the growth of the male and female skulls.

It would of course be of great value to obtain a considerable series of growth curves of the head for the same individuals between the ages of 3 and 20, accompanied by a proper record of the variation of the same individual on making repeated measurements—the personal error of measurement on the living subject being too often disregarded. But the anthropologist is chiefly concerned with the type, and there is sufficient evidence in the present paper to show that the shape of the head of the professional classes in England remains substantially of the same type whether measured at 7 or 14 or 21 years of age.

(6) The Series (ii)—that from Galton's Anthropometric Laboratory—has been reduced by Dr Karl J. Holzinger of the School of Education, University of Chicago, who has placed his material at our disposal. He was able to deal with 3377 male individuals ranging from 5 to 80 years of age. He found for the mean cephalic index 78·547 and for its standard deviation 2·9800. The correlation table of first cephalic index and age in three year groups is given in Table IX, and the mean cephalic index for each central age in Table X.

TABLE VII.

Age and Second Cephalic Index in Girls.

Central Ages.

Central Cephalic Indices.	3·5	4·5	5·5	6·5	7·5	8·5	9·5	10·5	11·5	12·5	13·5	14·5	15·5	16·5	17·5	18·5	19·5	20·5	21·5	22·5	Totals
56·45	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
57·45	—	—	—	—	—	—	—	—	1	—	1	1	—	—	—	—	—	—	—	—	3
58·45	—	—	—	—	—	—	1	1	—	—	1	2	—	1	—	—	—	—	—	—	6
59·45	—	—	—	—	—	—	—	—	—	1	1	—	1	—	—	—	1	—	—	—	4
60·45	—	—	—	—	—	—	1	1	1	1	1	4	2	3	2	—	—	—	—	—	16
61·45	—	—	—	2	—	1	—	2	2	2	4	1	10	—	1	2	—	—	—	—	27
62·45	—	—	—	1	1	—	1	1	7	5	5	6	5	5	3	3	—	2	—	—	45
63·45	—	—	1	1	2	5	4	7	10	10	9	2	8	5	1	3	—	—	1	—	69
64·45	—	1	1	2	3	5	12	10	17	14	10	9	6	11	10	5	2	—	—	—	118
65·45	—	—	2	1	3	4	16	15	17	25	26	16	13	14	4	3	—	—	—	—	159
66·45	—	3	1	3	9	9	12	19	21	26	27	13	17	10	9	4	—	1	1	—	185
67·45	1	—	—	2	6	15	14	24	25	22	17	28	12	13	8	3	2	—	3	—	195
68·45	—	1	1	2	6	11	21	26	34	37	22	12	31	19	9	6	—	2	—	—	240
69·45	—	—	3	5	10	10	18	25	27	32	29	23	25	13	13	7	2	1	1	—	244
70·45	—	—	4	3	12	10	19	25	26	26	20	14	16	9	2	6	2	—	—	—	194
71·45	—	—	2	5	5	8	17	18	18	12	25	17	19	13	4	6	—	—	—	—	169
72·45	—	—	—	4	3	8	15	11	15	17	24	8	16	9	7	4	1	—	—	—	142
73·45	—	—	—	1	2	10	7	16	13	16	16	7	9	10	5	6	1	—	—	—	119
74·45	—	—	1	1	6	5	9	5	7	18	6	12	9	6	4	1	—	—	1	1	92
75·45	—	—	1	3	—	5	4	5	9	10	3	7	3	4	3	1	1	—	—	—	59
76·45	—	1	—	1	3	3	1	3	8	3	1	5	2	5	2	1	1	—	—	—	40
77·45	—	1	—	—	—	2	3	2	2	2	4	4	2	1	—	—	—	—	—	—	23
78·45	—	—	—	—	—	—	1	1	2	3	3	1	2	1	—	—	—	—	—	—	14
79·45	—	—	—	—	—	—	1	—	2	2	—	1	—	—	—	—	1	—	—	—	7
80·45	—	—	—	—	—	—	—	1	1	—	2	1	—	1	—	1	1	—	—	—	8
81·45	—	—	—	—	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	2
82·45	—	—	—	1	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	2
83·45	—	—	—	—	—	—	1	1	—	1	—	2	—	—	—	—	—	—	—	—	5
84·45	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	1	7	17	38	72	111	178	220	266	285	258	196	208	154	87	62	15	6	7	1	2189

TABLE VIII.

Second Cephalic Index at each Central Age, Girls.

Age	100 H/L	Age	100 H/L
3·5	69·21 ± ·530	14·5	69·34 ± ·189
4·5		15·5	68·83 ± ·266
5·5		16·5	69·07 ± ·213
6·5	69·83 ± ·430	17·5	68·67 ± ·234
7·5	69·64 ± ·312	18·5	68·92 ± ·336
8·5	69·77 ± ·252	19·5	69·58 ± ·685
9·5	69·44 ± ·199	20·5	67·74 ± ·708
10·5	69·32 ± ·179	21·5	
11·5	69·21 ± ·162	22·5	
12·5	69·29 ± ·157		
13·5	68·97 ± ·165	Mean of Whole	69·21 ± ·057

TABLE IX.

Francis Galton's Data, Age and First Cephalic Index, Males.

Central Ages.

First Cephalic Indices.	74·5	76·5	78·5	80·5	82·5	84·5	86·5	88·5	90·5	92·5	Totals
68·5	—	—	—	—	—	—	—	—	—	—	—
69·5	—	—	—	—	—	—	—	—	—	—	—
70·5	—	—	—	—	—	—	—	—	—	—	—
71·5	—	—	—	—	—	—	—	—	—	—	—
72·5	1	—	—	—	—	—	—	—	—	—	1
73·5	—	1	1	—	—	—	—	—	—	—	1
74·5	—	1	2	1	—	2	—	—	—	—	6
75·5	1	3	2	1	3	2	—	—	—	—	12
76·5	3	1	3	2	—	2	—	—	—	—	11
77·5	1	1	2	3	—	2	—	—	—	—	9
78·5	3	1	2	2	1	2	—	—	—	—	11
79·5	2	2	3	2	1	2	—	—	—	—	12
80·5	1	2	2	2	1	2	—	—	—	—	10
81·5	2	1	2	2	1	2	—	—	—	—	10
82·5	3	2	2	2	1	2	—	—	—	—	12
83·5	—	1	2	2	1	2	—	—	—	—	8
84·5	2	1	2	2	1	2	—	—	—	—	10
85·5	2	1	2	2	1	2	—	—	—	—	10
86·5	—	1	2	2	1	2	—	—	—	—	8
87·5	—	—	1	2	1	2	—	—	—	—	6
88·5	—	—	1	2	1	2	—	—	—	—	6
89·5	—	—	1	2	1	2	—	—	—	—	6
90·5	—	—	1	2	1	2	—	—	—	—	6
91·5	—	—	1	2	1	2	—	—	—	—	6
92·5	—	—	1	2	1	2	—	—	—	—	6
Totals	21	112	290	495	799	467	293	213	132	112	85	73	58	61	44	25	21	26	13	19	5	5	3	3	2	3377

TABLE X.

First Cephalic Index at each Central Age, Males.

Age	100 B/L	Age	100 B/L
7·5	80·02	*46·5*	78·47
10·5	79·82	*49·5*	79·06
13·5	78·88	*52·5*	78·53
16·5	78·46	*55·5*	77·59
19·5	78·37	*58·5*	80·03
22·5	78·22	*61·5*	80·99
25·5	78·32	*64·5*	79·13
28·5	78·54	*67·5*	75·85
31·5	78·71	*70·5*	78·05
34·5	78·91	*76·25*	76·82
37·5	78·56		
40·5	78·83	Mean of Whole	78·55
43·5	78·57		

The correlation coefficient is

$$r_{C_1 a} = \cdot 0012 \pm \cdot 0116,$$

and is therefore quite insensible.

On the other hand $\bar{\eta}^2_{C_1 \cdot a} = \cdot 007{,}107$, but $\eta^2_{C_1 \cdot a} = \cdot 020{,}650$; thus the latter is substantially greater than the former and

$$\eta_{C_1 \cdot a} = \cdot 1437 \pm \cdot 0114$$

is significant. The distribution is thus slightly but definitely skew.

Diagram III serves to demonstrate the same point. The zone given by the mean for all ages $(78\cdot547) \pm 2\cdot5 \times$ probable error of a mean determined for the array, i.e. $\cdot 67449 \times 2\cdot9800/\sqrt{n_a}$, where n_a is the number in the array, is drawn. It will be seen that except in the case of very young children the means of the arrays keep well within the limits of this zone. There is some evidence, however, that the younger children may have been somewhat more brachycephalic, and that in extreme age there is more dolichocephaly, which is possibly due to a difference between longitudinal and transverse rates of shrinkage. These points are indicated in the cubic fitted by Dr Holzinger. Throughout the bulk of life, however, there appears to be no change in the cephalic index, and it is remarkable how closely measurements on the living head of English subjects *of the same social class* are in accordance. When the social class is changed the cephalic index is changed. We do not look upon this as due to environment and nutrition, but to a slight amount of racial difference in the various social strata. Table XI gives some collected British results.

It will be seen that no result falls badly out of line. The general weighted mean for 12,255 heads is 78·55 agreeing exactly with Galton's data and very close indeed to the habitual and non-habitual criminals of Goring and Macdonell. On the whole

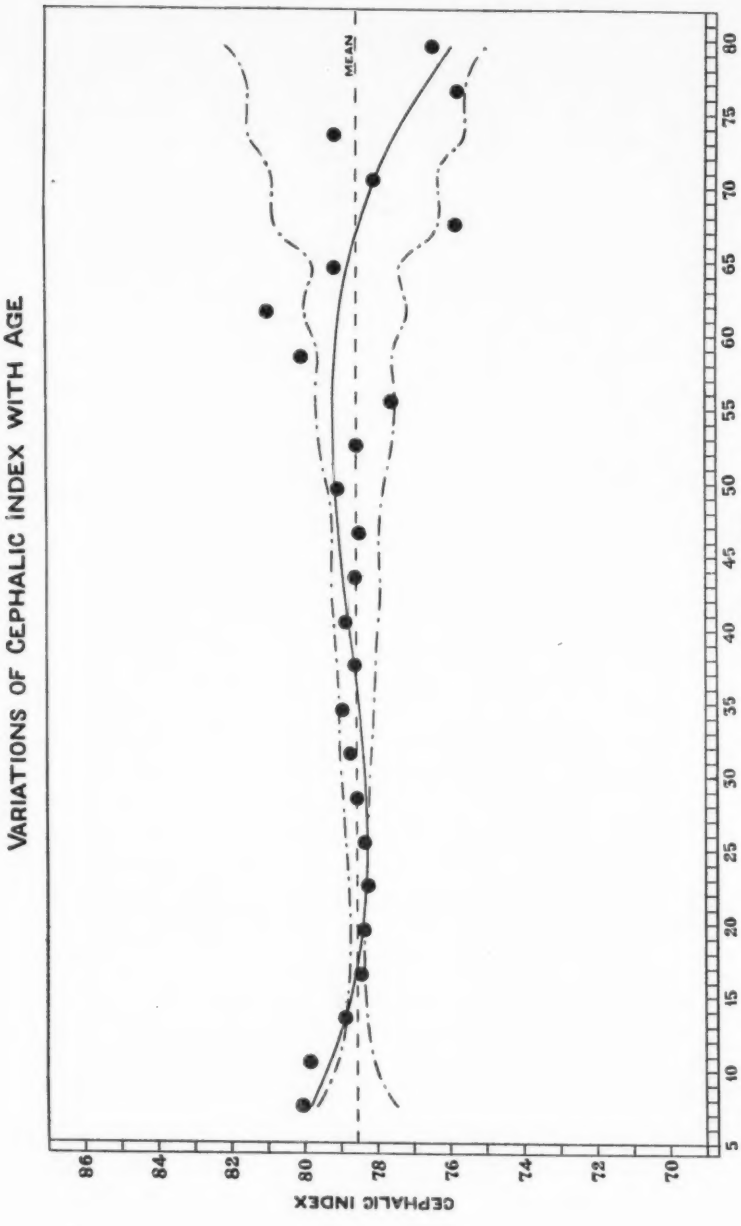

Diagram III.

The equation to the cubic, if y be the cephalic index, is

$$y = 78{\cdot}8965 + {\cdot}135{,}331x - {\cdot}007{,}11699x^2 - {\cdot}002{,}06666x^3,$$

where x is in three-year units measured from central age 43·5 as origin.

TABLE XI.

Mean Cephalic Index 100 *B/L.*

Class	Sex	Age	Collector	Value	Number
Children of Professional Classes ...	Male	5—20	K. Pearson	78·87	2313
" " " ...	Female	5—20	"	78·37	2189
Artizan to Professional Classes ...	Male	5—80	Sir Francis Galton	78·55	3377
Criminal Class, Non-habitual... ...	Male	16—70	W. R. Macdonell	78·54	3000
" " Convicts	Male	16—70	Charles Goring	78·50	800
Cambridge Students	Male	18—25	Venn	78·28	1095
Oxford Students	Male	18—25	Schüster	78·02	959
Aberdeen Students	Male	18—25	Reid	78·80	493
Royal Engineers (Non-coms. and men)	Male	20—30	Benington	78·10	118
General Hospital Population (London)	Male	20—70	Gladstone	78·48	100
British Association Members ...	Male	—	B.A. Anthrop. Committee	78·20	—
Staff, University College	Male	—	K. Pearson	78·15	—

the results appear to indicate great stability in the average cephalic index of this island*.

(7) It appeared worth while as the data were under reduction to determine whether in a highly hybridised population like that of England there exists any appreciable association between pigmentation and the shape of the head. Interracially there is of course a high correlation between shape of head and eye or hair colour. But this interracial correlation does not involve a linkage between these characters which will be maintained after hybridisation. Even if we find association between pigmentation and shape of head in England, it might only signify that we had a mixture of individuals of pure races, and not necessarily a hybrid race with a linkage. Should we find no association between head shape and pigmentation it will be hard to accept the view (*a*) that the English are a conglomerate of pure races or

* This stability is not, however, without some curious and hitherto unexplained exceptions. The value for the Cambridge students is deduced from Dr Venn's figures published in 1886 in the *Journal of the Anthropological Institute*, Vol. XVIII. p. 150. He worked on the early measurements of the Cambridge Anthropometric Laboratory, and his results are in excellent accordance with each other. Nearly twenty years later Pearson obtained copies of more than 1000 schedules from the Cambridge Laboratory and determined the head measurements from these cards. His results were (*R. Soc. Proc.* Vol. LXVI. p. 28, *Biometrika*, Vol. v. p. 124) Head Length: 194·00, Head Breadth: 154·21, Cephalic Index: 79·58, and his different groups were all in accordance with these values. Macdonell (*Biometrika*, Vol. I. pp. 188, 190) working on a portion of this material obtained: Head Length: 193·51, Head Breadth: 153·96, Cephalic Index: 79·56. Careful re-examination of these data has again been made and the above results confirmed. Three independent tablings thus give 79·57 about for the index, differing very markedly from Venn's 78·28. A further test was made on 275 Cambridge Female Students measured by Miss Cave-Brown-Cave at Girton. These gave a Cephalic Index of 80·13 being in fairly close accordance with the male returns. No solution of this discrepancy between Cambridge Students and the rest of the population or between the two Cambridge series appears to be forthcoming. Somewhat the same result arises in the Second Cephalic Index, where Pearson's data for schools give for males 69·49, for females 69·21, the Oxford data giving the reasonable accordance of 69·58, but the Cambridge men 70·00 and the Girton women 71·02. The Cambridge men appear at first to have been measured with a bad choice of horizontal plane, namely from the axis of the auricular passages to the *upper* edges of the orbits, but the Girton women were measured precisely as in the case of Pearson's material. A change from glabella to ophryon in measuring the length might account for the difference.

(*b*) that intraracially there is a linkage of head shape with pigmentation. On the contrary we should be driven to the conclusion that the English are a thoroughly hybridised race, and that in such a race there is no intraracial correlation of pigmentation and head shape. Biologically the important point elucidated would be that the definite interracial association between head shape and pigmentation is not maintained intraracially on hybridisation. Of course if pigmentation characters and cranial lengths blended on crossing we might account for the destruction of an association which existed interracially equally well with the hypothesis of unlinked unit factors. However, it is difficult on either hypothesis to provide adequately for the multitude of hair tints and eye colours which appear in a race presumably the result of hybridisation between races with very definite associations of hair colour with eye colour. The present investigation is therefore not intended as an inquiry into the truth of blended or alternate inheritance, but rather as a criterion of whether the English professional classes must be looked upon as a mixture of racial types, or as a mixed race, that is as an "emulsion or as a solution."

(*a*) We deal first with the relation of pigmentation in boys to the First Cephalic Index. It has been shown that there is a small correlation between age and pigmentation in school children*; we have just seen that there is no correlation of any significance between the cephalic indices and age. Correction accordingly for age is not needful, the partial correlation coefficients of cephalic indices and pigmentation for constant age are for all practical purposes identical with the crude correlations; and accordingly we shall only record these. Table XII gives the data.

None of the means of the arrays is significantly different from the mean of the general population.

Looked at otherwise the correlation ratio for First Cephalic Index on hair colour is

$$\eta_{C_1.h} = \cdot 0514 \pm \cdot 027,$$

while

$$\bar{\eta}_{C_1.h} = \cdot 0416.$$

Thus $\eta_{C_1.h}$ does not differ significantly from the mean correlation ratio when there is no association.

For eye colour the crude correlation ratio (without correction for class-index) for First Cephalic Index on eye colour is

$$\eta_{C_1.e} = \cdot 0213 \pm \cdot 028,$$

while

$$\bar{\eta}_{C_1.e} = \cdot 0294,$$

or, $\eta_{C_1.e}$ is less than the mean value of the correlation ratio on the hypothesis of no association.

Thus we see that in either way of approaching the subject the relationship between pigmentation and cephalic index for boys must be extremely slight, so slight that our 2000 cases are inadequate to show indication of it.

* *Biometrika*, Vol. III. pp. 462—465.

TABLE XII.

First Cephalic Index (Boys) and Pigmentation.

First Cephalic Index	Hair Colour					Totals	Eye Colour		
	Red	Fair	Light Brown	Dark Brown	Jet Black		Light	Medium	Dark
67·45	—	—	1	—	—	1	—	1	—
68·45	—	1	—	—	—	1	—	1	—
69·45	—	3	1	1	—	5	2	2	1
70·45	—	4	4	4	—	12	6	3	3
71·45	1	8	5	7	—	21	6	10	5
72·45	1	9	9	9·5	0·5	29	11	9	9
73·45	3	30	23	19	2	77	35	28	14
74·45	4·5	29·5	28·5	21·5	2	86	36·5	30·5	19
75·45	11·5	54·5	48·5	51·5	1	167	60·5	63	43·5
76·45	12	75	76·5	57·5	5	226	85·5	91·5	49
77·45	10·5	93·5	98·5	64	5·5	272	114·5	105·5	52
78·45	10·5	125·5	110	93	3	342	131	151·5	59·5
79·45	10	82·5	95	73·5	5	266	93·5	118	54·5
80·45	12·5	70·5	83·5	60·5	3	230	88	92	50
81·45	4	66	61·5	50·5	6	188	74	65·5	48·5
82·45	3	46·5	58·5	36	1	145	54	55	36
83·45	4	39	33·5	21·5	1	99	42·5	38·5	18
84·45	2	15	28	16	—	61	20·5	22·5	18
85·45	1·5	10·5	11	14	2	39	11·5	15·5	12
86·45	1	6·5	9·5	7·5	0·5	25	9·5	13·5	2
87·45	—	5	2	3	—	10	3	4	3
88·45	—	2	5	1	—	8	2	5·5	0·5
89·45	—	—	1	—	—	1	—	1	—
90·45	—	1	1	—	—	2	1	1	—
Totals	92	777·5	794·5	611·5	37·5	2313	887·5	928	497·5

Taking the mean index for each array we have:

First Cephalic Index (Boys).

Hair Colour		Eye Colour	
Red	78·39 ± ·228	Light	78·78 ± ·073
Fair	78·76 ± ·078	Medium	78·93 ± ·072
Light Brown ...	79·07 ± ·077	Dark	78·90 ± ·098
Dark Brown ...	78·82 ± ·088		
Jet Black	78·89 ± ·361		

Whole Population ... 78·87 ± ·045

(*b*) *First Cephalic Index in Girls and Pigmentation.*

Table XIII gives our data, and below it are the means of the arrays. Again we find no association whatever between either hair or eye colour and the First Cephalic Index.

The differences have still less significance than we found in the case of the boys.

We have $\eta_{C_1.h} = \cdot0242 \pm \cdot0213$,

$\bar{\eta}_{C_1.h} = \cdot0438$,

and $\eta_{C_1.e} = \cdot0043 \pm \cdot0213$,

$\bar{\eta}_{C_1.e} = \cdot0302$.

TABLE XIII.

First Cephalic Index (Girls) and Pigmentation.

First Cephalic Index	Hair Colour					Totals	Eye Colour		
	Red	Fair	Light Brown	Dark Brown	Jet Black		Light	Medium	Dark
61·45	1	—	—	1	—	2	—	2	—
62·45	—	2	1	—	—	3	—	3	—
63·45	2	1	2	—	—	5	3	1	1
64·45	—	3	—	3	—	6	2	2	2
65·45	—	—	2	2	—	4	1	3	—
66·45	1	3	—	1	—	5	1	3	1
67·45	—	3	4	—	—	7	1	3·5	2·5
68·45	—	5	5	4	—	14	5	6	3
69·45	1	3	10	4	—	18	1	9	8
70·45	1	8·5	13·5	4	—	27	10·5	10	6·5
71·45	—	14	13	8	—	35	13	15	7
72·45	1	17	21	10	2	51	14	20·5	16·5
73·45	3	21	27	18·5	0·5	70	19·5	31·5	19
74·45	4·5	38·5	37·5	17·5	1	99	37	44	18
75·45	4	50	58·5	21	1·5	135	57	58·5	19·5
76·45	12	59·5	73·5	40	2	187	62·5	72	52·5
77·45	10	90·5	84·5	48	4	237	69	115	53
78·45	13	123·5	113	51·5	1	302	116·5	127·5	58
79·45	10	79·5	92	53·5	2	237	78	97	62
80·45	7	83	89	49	1	229	87·5	95·5	46
81·45	7	83	62	28	3	183	77	65	41
82·45	5·5	39	37	35·5	2	119	34	51	34
83·45	3	22	38	14	—	77	24	35	18
84·45	4	25	20	15	—	64	14·5	35·5	14
85·45	—	9	14	8	—	31	11	15	5
86·45	2	7	7	6	1	23	7	13	3
87·45	—	4·5	3	1·5	—	9	1	5	3
88·45	—	1	2	1	—	4	1	2	1
89·45	—	—	3	1	—	4	—	2	2
90·45	—	—	—	1	—	1	—	—	1
91·45	—	—	1	—	—	1	1	—	—
Totals	92	795·5	833·5	447	21	2189	749	942·5	497·5

Means of the arrays:

First Cephalic Index (Girls).

Hair Colour		Eye Colour	
Red	78·03 ± ·274	Light	78·39 ± ·096
Fair	78·40 ± ·093	Medium	78·36 ± ·086
Light Brown ...	78·32 ± ·091	Dark	78·35 ± ·118
Dark Brown ...	78·48 ± ·124		
Jet Black ...	78·35 ± ·573		

Whole Population ... 78·37 ± ·056.

Thus the crude correlation ratios are in both cases less than the mean value of those ratios, if there were no association at all. Again we conclude that the association, if any, is so small that over 2000 cases are inadequate to demonstrate its existence.

(*c*) We now turn to the Second Cephalic Index starting with boys. Table XIV provides the data, and below it are given the means for the arrays. It is clear that, having regard to the probable errors, no stress can be laid on the slight differences

TABLE XIV.

Second Cephalic Index (Boys) and Pigmentation.

Second Cephalic Index	Hair Colour					Totals	Eye Colour		
	Red	Fair	Light Brown	Dark Brown	Jet Black		Light	Medium	Dark
57·45	—	1	1	—	—	2	—	2	—
58·45	—	2	1	—	—	3	2	1	—
59·45	1	3	2	1	—	7	1	4	2
60·45	1	10	2	5	—	18	8	6	4
61·45	—	5	7·5	6·5	1	20	6	9	5
62·45	3	10	10	10	—	33	7	21	5
63·45	1	25	15	20	—	61	23	19	19
64·45	2	36·5	24·5	22	2	87	39·5	35·5	12
65·45	5	52·5	54·5	45	—	157	67	63	27
66·45	9	66	60	51	3	189	64	80·5	44·5
67·45	4·5	82·5	76	54	2	219	90	88	41
68·45	11	91	83	64	5	254	102·5	97	54·5
69·45	6	71·5	88	58·5	7	231	75	98	58
70·45	6	82	104·5	71	3·5	267	102·5	100·5	64
71·45	11	63·5	67	54·5	5	201	81·5	70	49·5
72·45	10	55·5	52	46	2·5	166	62·5	76·5	27
73·45	9	44·5	46·5	27	4	131	49·5	58·5	23
74·45	6	22	33·5	20·5	—	82	34·5	28·5	19
75·45	2	22·5	31·5	15·5	2·5	74	24	34·5	15·5
76·45	3·5	10·5	15	21	—	50	20	17·5	12·5
77·45	—	8	11	7	—	26	10	9	7
78·45	—	6	5	5	—	16	8	4	4
79·45	1	4	1	3	—	9	5	2	2
80·45	—	—	1	1	—	2	—	1	1
81·45	—	—	—	1	—	1	1	—	—
82·45	—	1	2	—	—	3	3	—	—
83·45	—	1	—	2	—	3	—	2	1
84·45	—	—	—	—	—	—	—	—	—
85·45	—	1	—	—	—	1	1	—	—
Totals	92	777·5	794·5	611·5	37·5	2313	887·5	928	497·5

Means of the arrays:

Second Cephalic Index (Boys).

Hair Colour		Eye Colour	
Red	69·97 ± ·264	Light	69·56 ± ·085
Fair	69·21 ± ·091	Medium	69·38 ± ·083
Light Brown ...	69·68 ± ·090	Dark	69·58 ± ·114
Dark Brown ...	69·51 ± ·102		
Jet Black ...	69·88 ± ·419		

Whole Population ... 69·49 ± ·053.

which occur. This is confirmed by a study of the crude correlation ratios. We have

$$\eta_{C_2.h} = \cdot0601 \pm \cdot0213,$$

$$\bar{\eta}_{C_2.h} = \cdot0416,$$

and

$$\eta_{C_2.e} = \cdot0243 \pm \cdot0213,$$

$$\bar{\eta}_{C_2.e} = \cdot0249.$$

Clearly neither η is significantly greater than its $\bar{\eta}$, or in the Second Cephalic Index for boys there is no more association with pigmentation than in the case of the first.

(*d*) *Second Cephalic Index* (*Girls*) *and Pigmentation.*

The data are provided in Table XV.

TABLE XV.

Second Cephalic Index (*Girls*) *and Pigmentation.*

Second Cephalic Index	Hair Colour					Totals	Eye Colour		
	Red	Fair	Light Brown	Dark Brown	Jet Black		Light	Medium	Dark
56·45	—	—	1	—	—	1	—	1	—
57·45	—	1	2	—	—	3	1	2	—
58·45	1	4	1	—	—	6	1	3·5	1·5
59·45	1	2	1	—	—	4	1	2	1
60·45	1	4	7·5	3·5	—	16	2	10	4
61·45	5	7	8	7	—	27	10	13	4
62·45	1·5	17·5	14	11	1	45	18·5	16·5	10
63·45	3	31	23·5	11	0·5	69	22	34·5	12·5
64·45	8·5	31·5	50	27	1	118	37	52	29
65·45	8	50	70	31	—	159	47·5	65·5	46
66·45	9·5	73·5	68·5	32·5	1	185	67·5	77	40·5
67·45	6	79·5	74·5	32	3	195	66·5	81·5	47
68·45	3	96·5	91	48·5	1	240	79	103·5	57·5
69·45	6·5	89·5	93·5	53·5	1	244	90	102	52
70·45	5	71	84	31	3	194	53	95	46
71·45	9	50	65·5	41	3·5	169	60·5	71·5	37
72·45	10	43·5	58·5	27	3	142	59·5	49·5	33
73·45	4	39	41·5	32·5	2	119	32	58	29
74·45	3	42	28·5	18·5	—	92	36·5	38	17·5
75·45	3	21	21	14	—	59	23·5	27·5	8
76·45	2	21	7	10	—	40	18	14	8
77·45	1	9	8	4	1	23	9	13	1
78·45	—	5	5	4	—	14	6	5	3
79·45	1	2	2	2	—	7	4	2	1
80·45	—	2	3	3	—	8	2	—	6
81·45	—	1	1	—	—	2	—	2	—
82·45	—	1	—	1	—	2	1	1	—
83·45	—	1	2	2	—	5	1	1	3
84·45	—	—	1	—	—	1	—	1	—
Totals	92	795·5	833·5	447	21	2189	749	942·5	497·5

Means of arrays:

Second Cephalic Index (Girls).

Hair Colour		Eye Colour	
Red	68·55 ± ·276	Light	69·40 ± ·097
Fair	69·22 ± ·094	Medium	69·11 ± ·086
Light Brown ...	69·01 ± ·086	Dark	69·12 ± ·119
Dark Brown ...	69·52 ± ·125		
Jet Black ...	69·93 ± ·578		

Whole Population ... 69·21 ± ·06

Here again there is no deviation in the mean of any array from the mean of the whole population which makes their difference more than 2·5 times the probable error of their differences. We cannot assert that the cephalic index for auricular height to length of head for any particular pigmentation is characteristic of that pigmentation. The same point is reached by consideration of the crude correlation ratios. We have

$$\eta_{C_2.h} = \cdot0558 \pm \cdot0213,$$

$$\bar{\eta}_{C_2.h} = \cdot0438,$$

and

$$\eta_{C_2.e} = \cdot0349 \pm \cdot0213,$$

$$\bar{\eta}_{C_2.e} = \cdot0302.$$

The correlation ratios are thus both low and do not differ significantly from the mean zero association correlation ratios.

We conclude therefore that as far as our present data—a large sample of the children of the English professional classes—go there is no evidence of any relation of the slightest importance between pigmentation and shape of head.

Conclusions.

(i) Dealing with a large amount of data, we are unable to find any change of real significance in the cephalic indices for school children from 5 to 20. It may be taken as reasonably certain that at least from 8 to 20 there is no significant change in the shape of the head. The indices might be equally well determined from children of 10 or 15 or from adolescents of 20.

(ii) Dealing with Galton's material ranging from age 6 to age 80, we find again no evidence of a continuous change. The erratic values in extreme childhood and extreme age are largely due to means based upon inadequate numbers.

We seem therefore justified when dealing anthropometrically with the shape of the head as determined by the two cephalic indices in neglecting any correction whatever for age. We do not think that the confidence of anthropologists in the cephalic indices need be shaken by Miss Fleming's results.

(iii) From Table XI (p. 131) we see how remarkably stable is the cephalic index when we pass from one class of the community to a second, notwithstanding the great diversity of environment involved when we pass from the children of the professional classes to habitual criminals.

(iv) The stability of the cephalic indices in this country is then investigated from the standpoint of hybridisation. Our population had originally undoubtedly

a mixed racial origin. In the various components there was just as undoubtedly a close association of head shape and pigmentation. If the race were still a mixture of these components, we should undoubtedly expect to find cephalic index correlated with hair and eye colour. Our investigation of the children of the professional classes shows no such association; we do not find dolichocephaly associated with a light pigmentation, or brachycephaly with a dark pigmentation. The conclusion we would draw from this is that the English people, whatever its original components may have been, is now thoroughly hybridised, and indeed may venture to call itself a 'race,' as distinguished from the American population which is at present only a mixture of races. Those who will study the pedigree of individuals of the professional and upper classes—say, those of Sir Francis Galton, Charles Darwin, or Sir Thomas Browne—will be surprised to find how very rapidly as we ascend in ancestry the ancestors are scattered over the whole face of this country, even from parts which 800 to 1000 years ago must have had different racial populations.

(v) Having regard to the fact that extraordinary environmental differences in this country appear to make no significant change in the shape of the head, it is very difficult to accept Professor Boas' view that the child born to Jewish parents in Europe differs in head shape from the child born to the same parents after their arrival in America. The cephalic index of the Jews is much the same in the most diverse environments in Europe, and we do not believe that anything but hybridisation, or long selective action, can change the type, or make the Jew born in America akin to the European population there.

NOTE ON THE RELATION OF VITAL CAPACITY TO CERTAIN PSYCHICAL CHARACTERS.

By KARL J. HOLZINGER, Ph.D., University of Chicago, U.S.A.

1. *Introductory.* Physical measurements as dealt with in Biometry may be considered as of two types. On the one hand there are the ordinary anthropometric characters such as height, weight, head measurements, etc., which Professor Pearson has termed "*Static Physical Characters*," inasmuch as their determination does not involve any immediate functioning of the body. Contrasted with these, there are the "*Dynamic Physical Characters*," which include such traits as pulse-beat, blood-pressure, strength of grip, pull, and vital capacity. In this second group the character is clearly dependent upon bodily functioning, and is indeed a measure of some special type of bodily fitness.

The above distinction is rather an important one, because in the general problem of the relationship between physical and psychical characters, it is only the former group of physical traits which has received adequate attention. Static physical characters have been shown to have only a slight positive correlation with such traits as intelligence, and the association is so small as to furnish no sufficient basis for prediction. Some of the most interesting results of this type are included in a memoir, "On the Relationship of Intelligence to Size of Head," and may be briefly set forth in Table I* as typical of such correlations.

TABLE I.

Correlation Ratio between Shape and Size of Head and Ability.

Intelligence and	Cambridge Graduates	School Boys at Twelve years	School Girls at Twelve years
Cephalic Index ...	(1011)† − ·061 ± ·021	(2345) − ·041 ± ·014	(2226) ·067 ± ·014
Length of Head ...	(1011) ·111 ± ·020	(2298) ·139 ± ·014	(2188) ·084 ± ·014
Breadth of Head ...	(1011) ·097 ± ·021	(2299) ·109 ± ·014	(2165) ·113 ± ·014
Auricular Height ...	—	(2290) ·073 ± ·014	(2166) ·055 ± ·014

When we turn to the question of the relationship between dynamic physical characters and psychical traits we find that very little has been done. In the present paper a beginning has been made in the consideration of vital capacity as related to four primary psychical characters. These include reaction time to sight, reaction time to sound, keenness of vision, and highest audible pitch discernible. The general results obtained are in close harmony with those cited above for

* Karl Pearson, "On the Relationship of Intelligence to Size of Head," *Biometrika*, Vol. v. p. 105.

† The numbers in brackets give the sizes of the populations dealt with.

static characters, i.e. a low but significant positive correlation is found between the dynamic and psychic traits, but it is too small for reliable prediction. [At the beginning of this study I had intended to include strength of grip as an additional dynamic factor, but pressure of time prevented my completion of the tabulations at the Galton Laboratory, and the results for grip will have to be given in a future note.]

In the present paper we are primarily interested in the possibility of predicting change in psychical character for a given change in vital capacity inasmuch as it is the latter which must be the conditioning factor in association. Prediction curves in the reverse direction are given, however, in the case of two of the traits.

2. *The Data Employed.* Very fortunately for the purposes of this study excellent data were at hand in the Galton series employed by Koga and Morant in their recent paper*: "On Association between Reaction Times for Different Senses." These data had already been corrected for age so far as the four psychical characters were concerned, so it only remained to make similar corrections for vital capacity.

For a complete account of Galton's method of determining reaction time and sense acuity, the reader is referred to Professor Pearson's *Life, Letters, and Labours of Francis Galton* (Vol. II. now in the press). For the present purpose, however, it will be sufficient to enumerate the variables once more with a brief description of their method of measurement.

(*a*) *Reaction Time to Sight* as determined by Galton's myograph pendulum giving a sight signal by brushing against a mirror reflecting a light on to a screen.

(*b*) *Reaction Time to Sound* determined by a similar pendulum giving a sound signal by a light weight on the pendulum striking against a piece of metal.

(*c*) *Acuteness of Hearing* in thousands of vibrations per second by Galton's whistle.

(*d*) *Acuteness of Sight* as furnished by the number of inches at which diamond type could be read by the unaided eye.

(*e*) Age to the nearest month.

(*f*) Vital Capacity in cubic inches.

Cards for 3377 males were used in obtaining the correction of vital capacity on age, but this number was later reduced to 3373 owing to missing data.

3. *Correction of Vital Capacity for Age and Heteroscedasticity.* The fact that none of the variables studied has a linear regression on age makes the partial correlation method useless for such cases and necessitates the laborious correction of each card entry. In the present paper it seemed desirable to correct not only for age but also for lack of homoscedasticity in the manner described below. For those who think such labour unnecessary it may be well to note the extreme non-linearity as shown in Diagram 1. The correlation coefficient between age and vital

* Y. Koga and G. M. Morant, "On the Degree of Association between Reaction Times in the Case of Different Senses," *Biometrika*, Vol. xv. p. 346.

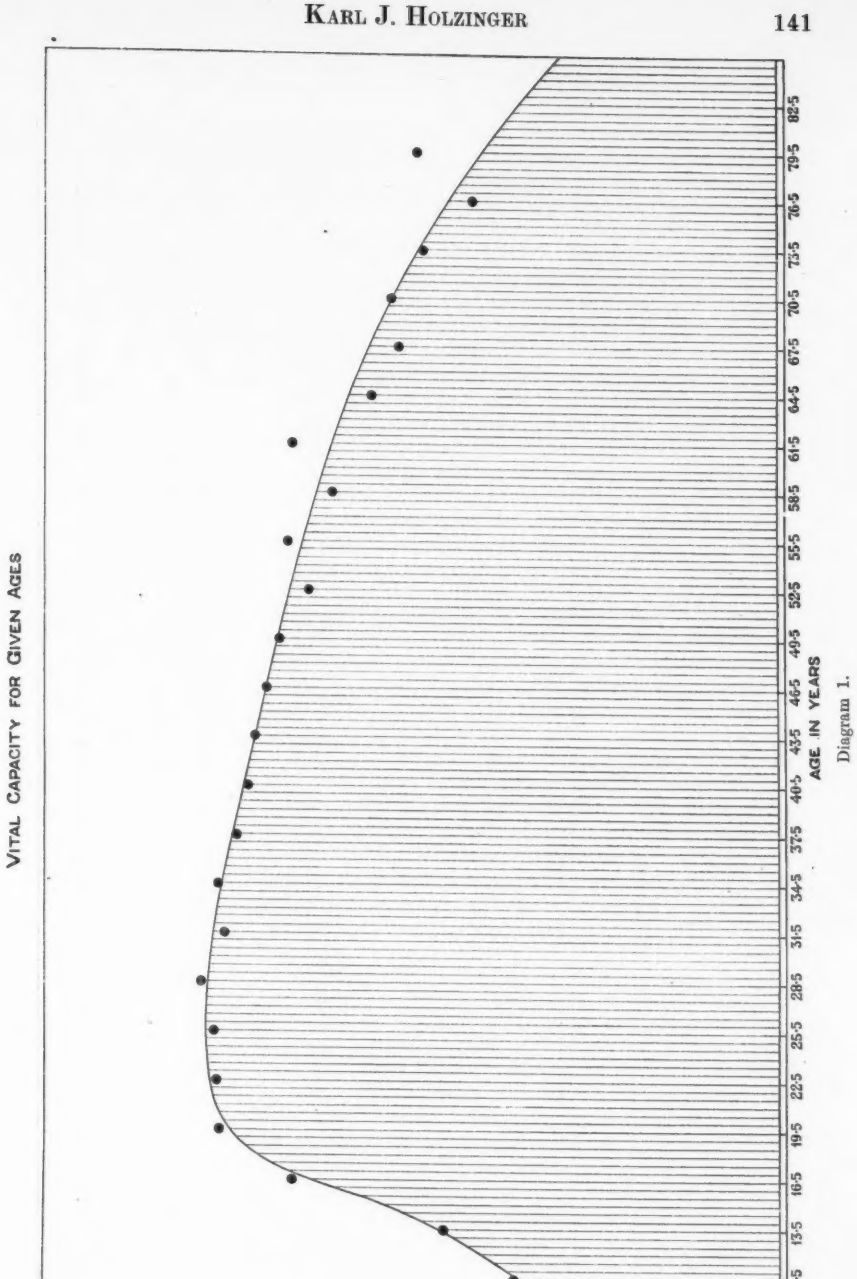
VITAL CAPACITY FOR GIVEN AGES
VITAL CAPACITY IN CUBIC INCHES
300
250
200
150
100
50
0
7·5
10·5
13·5
16·5
19·5
22·5
25·5
28·5
31·5
34·5
37·5
40·5
43·5
46·5
49·5
52·5
55·5
58·5
61·5
64·5
67·5
70·5
73·5
76·5
79·5
82·5
AGE. IN YEARS

Diagram 1.

TABLE II.

Vital Capacity and Age.

Vital Capacity in cubic inches.

Age in Years.	42·5—57·5	57·5—72·5	72·5—87·5	87·5—102·5	102·5—117·5	117·5—132·5	132·5—147·5	147·5—162·5	162·5—177·5	177·5—192·5	192·5—207·5	207·5—222·5	222·5—237·5	237·5—252·5	252·5—267·5	267·5—282·5	282·5—297·5	297·5—312·5	312·5—327·5	327·5—342·5	342·5—357·5	357·5—372·5	372·5—387·5	387·5—402·5	Totals
6— 8	3	5	5	5	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	21
9—11	—	6	14	24	26	29	8	2	1	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	112
12—14	1	1	2	29	51	59	51	47	18	13	9	4	3	1	—	1	—	—	—	—	—	—	—	—	290
15—17	—	—	—	3	6	13	27	50	51	84	48	70	60	39	20	14	4	3	2	1	—	—	—	—	495
18—20	—	—	—	3	1	2	10	10	29	70	84	151	138	112	76	54	25	22	5	6	—	—	—	1	799
21—23	—	—	—	—	1	2	2	22	12	34	39	89	73	82	37	35	16	16	3	3	1	—	—	—	467
24—26	—	—	—	—	—	2	1	8	9	33	19	53	41	51	27	28	10	8	2	1	—	—	—	—	293
27—29	—	—	—	—	—	1	2	2	4	18	19	36	30	42	14	22	9	6	3	4	1	—	—	—	213
30—32	—	—	—	2	—	1	5	2	2	12	10	29	13	26	10	8	5	5	2	—	—	—	—	—	132
33—35	—	1	1	—	—	—	1	2	2	11	11	16	16	22	12	12	2	2	—	1	—	—	—	—	112
36—38	—	—	—	—	—	3	—	5	2	12	8	17	13	7	6	5	1	3	—	2	1	—	—	—	85
39—41	—	1	—	—	—	2	1	3	1	12	4	12	15	12	4	5	—	—	—	1	—	—	—	—	73
42—44	—	—	—	—	—	1	1	2	2	14	4	12	7	8	2	3	—	2	—	—	—	—	—	—	58
45—47	—	—	—	1	—	2	2	4	7	10	5	2	9	11	2	3	1	—	1	—	1	—	—	—	61
48—50	—	—	—	—	—	—	3	4	4	6	7	6	7	2	3	—	2	—	—	—	—	—	—	—	44
51—53	—	—	—	—	2	1	2	2	2	1	4	4	4	2	—	1	—	—	—	—	—	—	—	—	25
54—56	—	—	—	—	—	2	1	2	—	3	1	7	2	—	2	1	—	—	—	—	—	—	—	—	21
57—59	—	—	—	—	1	2	1	4	2	3	6	5	2	—	—	—	—	—	—	—	—	—	—	—	26
60—62	—	—	—	—	—	—	1	1	1	2	4	1	1	1	1	—	—	—	—	—	—	—	—	—	13
63—65	—	—	—	—	—	4	2	1	5	4	1	2	—	—	—	—	—	—	—	—	—	—	—	—	19
66—68	—	—	—	—	1	1	—	1	—	1	1	—	—	—	—	—	—	—	—	—	—	—	—	—	5
69—71	—	—	—	—	—	—	1	3	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	5
72—74	—	—	—	—	—	1	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
75—77	—	1	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
78—80	—	—	—	—	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	4	15	22	67	93	128	124	177	156	346	284	516	435	418	216	192	75	67	18	19	4	0	0	1	3377

capacity was +·1767 while the correlation ratio based on the regression curve drawn was ·6737. It is evident, therefore, that more elaborate methods are required than the mere substitution of product-moment correlation coefficients in multiple correlation formulae.

From Table II the following constants were determined:

Vital Capacity (y)	Age (x)
Mean = 209·097 ± ·595 cubic inches	Mean = 24·303 ± ·127 years
S.D. = 51·234 ± ·420 " "	S.D. = 10·959 ± ·090 "
$\eta_{y.x}$ = ·6737 ± ·0063	$\eta_{x.y}$ = ·2656 ± ·0107

It is evident that prediction of vital capacity from age is much more reliable than the reverse operation.

The smooth curve in Diagram 1 has been drawn with the aid of a spline. Several attempts were made to fit the data by means of biquadratics with adjusted values for the coefficients in higher powers of x, but all the curves failed to give a reasonable fit in the neighbourhood of age fourteen at which place a considerable dip in the curve as drawn will be noted. This dip occurs during the ages of puberty and is of some significance inasmuch as it is also found in the curves for grip and other dynamic factors. In splining the data, therefore, the biquadratic values were followed over most of the curve, but the dip was allowed for so as to bring the curve within the limits which might be expected from random sampling, a result which could not be secured by the least square method applied to a biquadratic.

It will be noted that the maximum vital capacity given by the smoothed curve is found near age twenty-five, a result quite in agreement with the findings in the Biometric Laboratory for other physical characters. Man reaches the full prime of his dynamic power near the age of twenty-five and from then on the decrease is probably gradual until near death.

In Table III are given the smoothed values of vital capacity for each age from six to eighty-two. If these be denoted at age t by $\bar{y}_t$ and any observation value by y_t it is clear that a correction to standard age s might be given by the simple formula:

$$y_s = \bar{y}_s + (y_t - \bar{y}_t).$$

But this assumes that the growth curve of an individual will be sensibly parallel to the mean curve at every age. In other words, it assumes homoscedasticity. At the suggestion of Professor Pearson the standard deviations of the arrays were worked out and found when plotted against age to reveal a condition far from linear as is clear from Diagram 2. The smooth curve is again splined in so as to fall within the limits which might easily arise from random sampling. From the values in Table IV which have been read off from the curve, the complete correction of y may now be accomplished.

If σ_x and σ_s denote the standard deviations at age x and at standard age s, respectively, the full corrective formula becomes

$$y_s = \bar{y}_s + (y - \bar{y})\,\sigma_s/\sigma_x.$$

TABLE III.

Probable Vital Capacity at Central Ages.

Age	Vital Capacity	Age	Vital Capacity	Age	Vital Capacity	Age	Vital Capacity
6·5	74	*25·5*	234	*44·5*	212	*63·5*	178
7·5	81	*26·5*	234	*45·5*	211	*64·5*	176
8·5	88	*27·5*	234	*46·5*	210	*65·5*	173
9·5	97	*28·5*	233	*47·5*	208	*66·5*	170
10·5	106	*29·5*	233	*48·5*	206	*67·5*	167
11·5	116	*30·5*	232	*49·5*	205	*68·5*	164
12·5	127	*31·5*	231	*50·5*	203	*69·5*	161
13·5	139	*32·5*	230	*51·5*	202	*70·5*	158
14·5	153	*33·5*	229	*52·5*	200	*71·5*	154
15·5	170	*34·5*	228	*53·5*	198	*72·5*	151
16·5	190	*35·5*	226	*54·5*	196	*73·5*	147
17·5	206	*36·5*	225	*55·5*	194	*74·5*	143
18·5	217	*37·5*	223	*56·5*	193	*75·5*	139
19·5	224	*38·5*	222	*57·5*	191	*76·5*	134
20·5	228	*39·5*	220	*58·5*	189	*77·5*	130
21·5	231	*40·5*	219	*59·5*	187	*78·5*	125
22·5	232	*41·5*	217	*60·5*	185	*79·5*	121
23·5	233	*42·5*	215	*61·5*	183	*80·5*	116
24·5	234	*43·5*	214	*62·5*	180	*81·5*	111

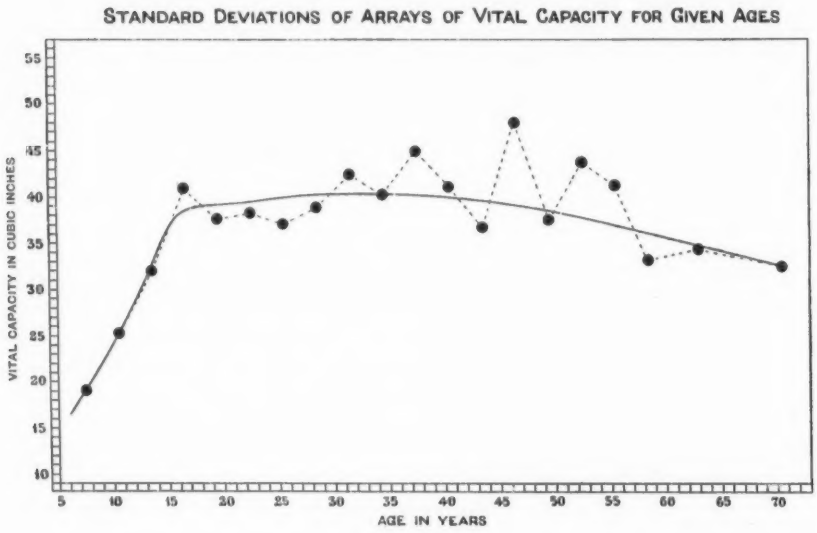

Diagram 2.

TABLE IV. *Standard Deviations of Arrays in Vital Capacity at Central Ages.*

Age	Standard Deviation	Age	Standard Deviation	Age	Standard Deviation	Age	Standard Deviation
6·5	17·7	25·5	39·9	44·5	39·3	63·5	34·8
7·5	19·2	26·5	39·9	45·5	39·0	64·5	34·5
8·5	21·0	27·5	39·9	46·5	39·0	65·5	34·2
9·5	22·8	28·5	40·2	47·5	38·7	66·5	33·9
10·5	25·2	29·5	40·2	48·5	38·4	67·5	33·6
11·5	27·3	30·5	40·2	49·5	38·4	68·5	33·3
12·5	29·7	31·5	40·2	50·5	38·1	69·5	33·0
13·5	32·4	32·5	40·2	51·5	37·8	70·5	33·0
14·5	35·4	33·5	40·2	52·5	37·5	71·5	32·7
15·5	37·5	34·5	40·2	53·5	37·5	72·5	32·4
16·5	38·4	35·5	40·2	54·5	37·2	73·5	32·1
17·5	38·7	36·5	40·2	55·5	36·9	74·5	31·8
18·5	39·0	37·5	40·2	56·5	36·6	75·5	31·5
19·5	39·3	38·5	39·9	57·5	36·3	76·5	31·5
20·5	39·3	39·5	39·9	58·5	36·0	77·5	31·2
21·5	39·6	40·5	39·9	59·5	35·7	78·5	30·9
22·5	39·6	41·5	39·6	60·5	35·7	79·5	30·6
23·5	39·6	42·5	39·6	61·5	35·4	80·5	30·6
24·5	39·6	43·5	39·3	62·5	35·1	81·5	30·3

In order to bring the adjusted values into harmony with those obtained by Professor Koga, the same standard age was chosen, viz. 40·5. From Table IV it is evident that the second correction mentioned above becomes quite important up to the age of twenty-five, a period including the great majority of the data.

It also seems advisable to use the same symbols for abbreviation as were employed in the paper cited, since four of the variables studied are the same. Thus for the corrected variables we have:

b = breathing capacity at age 40·5
p = highest audible pitch " "
k = keenness of vision " "
h = reaction time to sound " "
v = reaction time to sight " "

4. *The Relation of Vital Capacity and Reaction Time to Sight.* Table V furnishes the correlations with both variables corrected for age, and the associational values may be set down as follows:

	r_{bv}	$\eta_{v.b}$	$\eta_{b.v}$
At age 40·5	$-\cdot1231 \pm \cdot0114$	$\cdot1506 \pm \cdot0113$	$\cdot1404 \pm \cdot0114$

The resulting values are all small and apparently significant according to their probable errors. Professor Pearson has shown that when we sample from material for which the actual value of η is zero, and k is the number of arrays, then the mean value of $\overline{\eta}^2$ is $\frac{k-1}{N}$*, where N is the size of the sample. By calculation we

* Karl Pearson, "On the Correction Necessary for the Correlation Ratio," *Biometrika*, Vol. XIV. p. 412.

TABLE V.

Corrected Vital Capacity with Corrected Reaction Time to Sight.

Corrected Vital Capacity in cubic inches.

Corrected Reaction Time to Sight in hundredths of a second.	67·6— 72·6	72·6— 87·6	87·6—102·6	102·6—117·6	117·6—132·6	132·6—147·6	147·6—162·6	162·6—177·6	177·6—192·6	192·6—207·6	207·6—222·6	222·6—237·6	237·6—252·6	252·6—267·6	267·6—282·6	282·6—297·6	297·6—312·6	312·6—327·6	327·6—342·6	342·6—357·6	357·6—372·6	372·6—387·6	387·6—402·6	402·6—417·6	Totals
5— 6*	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
7— 8	—	—	—	—	—	—	1	—	1	—	2	3	—	1	—	—	—	—	—	—	—	—	—	—	9
9—10	—	—	—	—	—	5	3	8	1	9	22	18	11	5	4	6	3	2	1	—	—	—	—	—	107
11—12	—	—	—	—	—	1	7	7	8	17	17	15	22	14	9	5	3	4	2	—	—	—	—	—	125
13—14	—	—	2	—	—	3	6	17	19	33	39	36	33	23	14	10	6	4	2	1	—	—	—	—	246
15—16	—	1	—	—	—	11	10	48	58	91	96	98	79	57	35	21	10	8	3	1	—	—	—	1	633
17—18	—	—	—	4	1	9	21	41	60	96	102	95	84	45	34	25	10	4	2	—	1	—	—	—	633
19—20	2	—	5	3	7	12	20	54	88	117	138	166	98	73	46	24	9	8	1	1	—	—	—	—	873
21—22	—	—	2	—	6	8	11	23	32	39	58	50	36	30	12	9	2	1	—	—	1	—	—	—	319
23—24	—	—	1	1	1	4	11	22	24	34	37	31	27	10	9	5	1	—	—	—	—	—	—	—	219
25—26	—	—	—	2	3	3	6	11	13	20	16	21	10	7	2	2	—	—	—	—	—	—	—	—	116
27—28	—	—	—	—	1	—	4	2	7	4	2	5	2	5	3	—	—	—	—	—	—	—	—	—	33
29—30	—	—	—	—	—	1	1	2	6	8	5	6	2	4	3	—	—	—	—	—	—	—	—	—	40
31—32	—	—	—	1	—	—	—	2	3	2	2	—	1	1	—	—	—	—	—	—	—	—	—	—	10
33—34	—	—	—	—	—	—	1	—	1	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	4
35—36	—	—	—	1	—	—	—	—	—	3	—	—	—	—	—	—	—	—	—	—	—	—	—	—	3
37—38	—	—	—	—	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	2
Totals	2	1	12	11	25	57	102	241	397	474	536	546	405	275	161	107	43	30	11	2	3	1	0	1	3373

* Here as elsewhere 5—6 means 4·995 to 6·995 or the mid-point is 5·995.

find $\bar{\eta}^2_{v.b} = \cdot0068$ and $\bar{\eta}^2_{b.v} = \cdot0047$, while $\eta^2_{v.b} = \cdot0227$ and $\eta^2_{b.v} = \cdot0197$. Since η^2 in each case is significantly larger than the corresponding $\bar{\eta}^2$, we have to consider how the observed η differs from $\sqrt{\frac{k-1}{N}} + \cdot6745\frac{1}{\sqrt{N}}$. Now $\eta_{v.b} - \sqrt{\frac{k-1}{N}} = \cdot0680$ and $\eta_{b.v} - \sqrt{\frac{k-1}{N}} = \cdot0715$, both of which are approximately six times the value of $\frac{\cdot6745}{\sqrt{N}} = \cdot0116$. We therefore conclude that it is extremely unlikely that the correlation ratios above should have arisen from fluctuations in uncorrelated material.

It will be noted that the negative sign for the correlation coefficient indicates that shorter reaction time is associated with greater vital capacity. There is thus a slight but undeniable tendency for the robust person with large lung capacity to possess also a relatively short reaction time to sight. The conditioning factor is the vital capacity, improvement in which function probably implies improvement in general bodily fitness, and, as a resultant, a sharpening of the mental characters such as reaction time.

The regression of reaction time to sight on vital capacity has been fitted by a cubic with the resulting equation

$$\text{Probable } v = 18\cdot78 - \cdot128014\tilde{b} - \cdot00695688\tilde{b}^2 - \cdot00226233\tilde{b}^3,$$

where $\tilde{b} = (b - 215)/15$, and probable v is measured from the origin of absolute reaction time to sight. See Diagram 3.

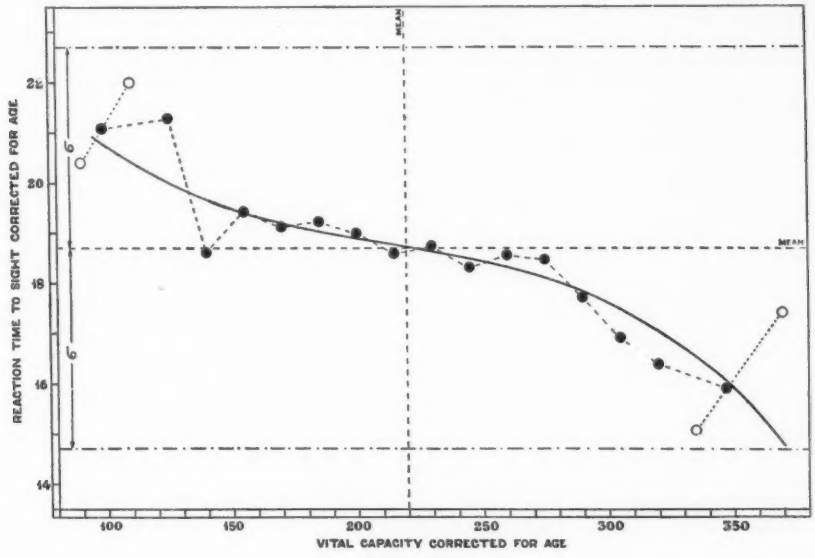

Diagram 3.

It is also of interest to note that the standard deviation for vital capacity has been reduced from 51·234 to 39·362 after correction for age and heteroscedasticity. Mean vital capacity, on the other hand, has been raised from 209·097 to 219·856 by correction. The net effect of the double correction, then, was to narrow the distribution and to shift it upward on the scale.

Mean reaction time to sight was found to be 18·711 ± ·046 and the standard deviation 3·9911 ± ·0328, the unit being one one-hundredth of a second. These results agree very closely with those in the Koga-Morant paper—as indeed they should, since the calculations are based on the same data with six fewer cases.

5. *Vital Capacity and Reaction Time to Sound.* The correlational values for vital capacity and reaction time to sound both corrected for age were then investigated. The coefficients may be set forth as follows:

	r_{bh}	$\eta_{h.b}$	$\eta_{b.h}$
At age 40·5	−·1159 ± ·0115	·1438 ± ·0114	·1502 ± ·0114.

It will be noted that these results are very similar to those obtained for reaction time to sight. The correlation is of the same order and sign and again indicates that large vital capacity is to a slight extent a conditioning factor in improvement in reaction time to sound.

By the formula

$$\sigma_d = \sqrt{\sigma_1^2 + \sigma_2^2 - 2r_{12}\sigma_1\sigma_2}, \quad \text{where } r_{12} = ·452,$$

the probable error of the difference between r_{bh} and r_{bv} may be found, and we have the difference = ·0072 ± ·0081, which shows that the two values are not significantly different, i.e. the relationship between vital capacity and both types of reaction time is the same. This of course assumes that the association is here adequately measured by the correlation coefficient.

Again fitting cubics to the regressions by the method of least squares we find

$$\text{Probable } h = 15·78 - ·1079\tilde{b} + ·011335\tilde{b}^2 - ·00155122\tilde{b}^3.$$

$$\text{Probable } b = 220·68 - 3·5212\tilde{h} - ·56441\tilde{h}^2 + ·0964663\tilde{h}^3,$$

where $\tilde{b}$ is again $(b-215)/15$ and $\tilde{h} = (h-16)/2$, b and h being the absolute measurements in the units given in Table VI.

From Diagrams 3, 4 and 5 it will be noted that the cubics for probable reaction times to sight and sound on vital capacity are very similar, while prediction for breathing capacity is furnished by a cubic of quite different appearance. It is evident from the latter cubic that a maximum vital capacity of about 225 cubic inches is found at a reaction time to sound of 12 hundredths of a second, while a minimum of 200 inches is given at a reaction time of 28 hundredths of a second.

The mean reaction time to sound was 15·802 ± ·037, and the standard deviation 3·1484 ± ·0259, the unit, as before, being one one-hundredth of a second.

TABLE VI.

Corrected Vital Capacity and Corrected Reaction Time to Sound.

Corrected Vital Capacity in cubic inches.

Corrected Reaction Time to Sound in $\frac{1}{100}$ second.	57·5— 72·5	72·5— 87·5	87·5—102·5	102·5—117·5	117·5—132·5	132·5—147·5	147·5—162·5	162·5—177·5	177·5—192·5	192·5—207·5	207·5—222·5	222·5—237·5	237·5—252·5	252·5—267·5	267·5—282·5	282·5—297·5	297·5—312·5	312·5—327·5	327·5—342·5	342·5—357·5	357·5—372·5	372·5—387·5	387·5—402·5	402·5—417·5	Totals
3— 4	—	—	—	—	—	—	—	—	—	—	1	—	—	1	1	—	—	—	—	—	—	—	—	—	3
5— 6	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
7— 8	—	—	1	—	—	—	1	1	1	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	5
9—10	—	—	—	—	2	3	3	5	14	9	38	27	20	7	2	4	2	—	1	—	—	—	—	—	137
11—12	—	—	—	—	1	7	7	23	30	49	56	68	56	41	21	13	8	8	2	—	1	—	—	—	391
13—14	—	—	1	—	4	8	20	68	79	122	137	127	114	67	40	32	16	6	5	—	1	—	—	1	848
15—16	2	—	3	4	7	21	37	64	90	157	157	187	125	86	52	36	11	11	2	2	—	1	—	—	1055
17—18	—	—	4	3	3	6	16	33	58	65	68	67	47	29	30	13	4	3	1	—	—	—	—	—	450
19—20	—	—	2	3	3	7	10	26	33	43	56	49	36	26	12	7	—	1	—	—	—	—	—	—	314
21—22	—	1	1	—	2	2	3	5	8	14	8	12	3	5	2	1	2	—	—	—	1	—	—	—	70
23—24	—	—	—	—	2	2	—	11	10	12	13	4	2	6	1	—	—	—	—	—	—	—	—	—	63
25—26	—	—	—	1	—	1	3	2	2	3	2	4	2	2	—	—	—	—	—	—	—	—	—	—	22
27—28	—	—	—	—	1	—	1	1	1	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	5
29—30	—	—	—	—	—	—	1	1	1	—	—	—	—	2	—	—	—	—	—	—	—	—	—	—	5
31—32	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
33—34	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	1
35—36	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
37—38	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	0
39—40	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	1
41—42	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	1
Totals	2	1	12	11	25	57	102	241	327	474	536	546	405	275	161	107	43	30	11	2	3	1	0	1	3373

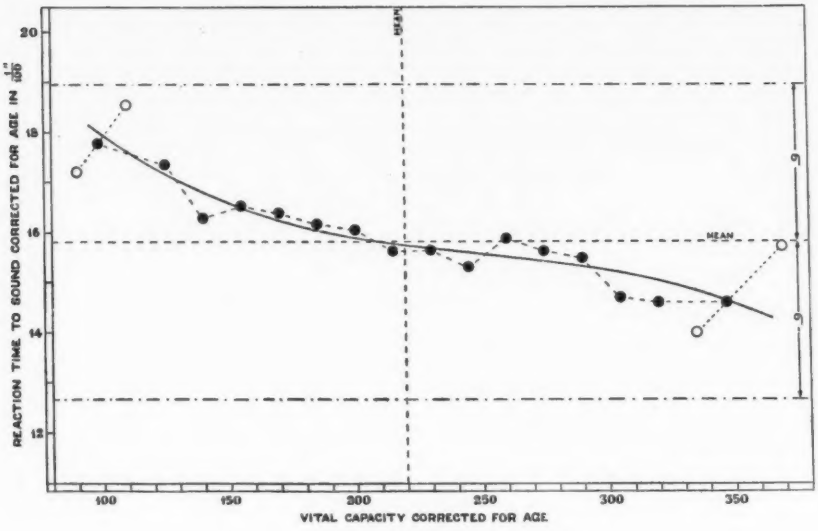

Diagram 4.

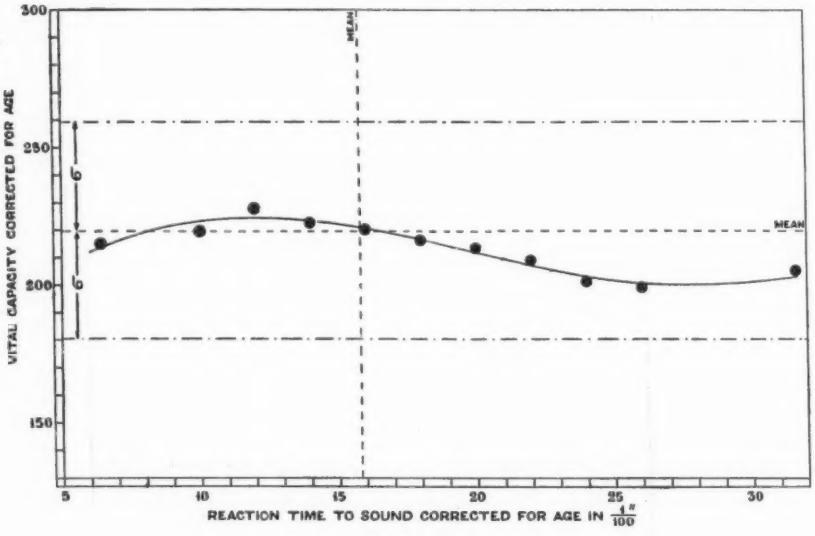

Diagram 5.

TABLE VII.

Corrected Vital Capacity and Acuity of Sight.

Corrected Vital Capacity in cubic inches.

Corrected Visual Acuity (in inches at which diamond type can be read).	57·5— 72·5	72·5— 87·5	87·5—102·5	102·5—117·5	117·5—132·5	132·5—147·5	147·5—162·5	162·5—177·5	177·5—192·5	192·5—207·5	207·5—222·5	222·5—237·5	237·5—252·5	252·5—267·5	267·5—282·5	282·5—297·5	297·5—312·5	312·5—327·5	327·5—342·5	342·5—357·5	357·5—372·5	372·5—387·5	387·5—402·5	402·5—417·5	Totals
0— 2	1	—	—	—	2	3	3	13	9	18	21	17	8	6	4	4	1	—	1	—	—	—	—	—	111
3— 5	—	—	3	1	2	4	3	11	18	20	19	22	31	11	5	4	—	—	1	—	—	—	—	—	155
6— 8	—	—	—	—	2	9	10	22	37	36	44	42	33	19	11	6	3	3	—	1	1	—	—	—	279
9—11	—	1	2	—	4	6	13	15	21	31	25	45	29	17	12	5	1	1	—	—	—	—	—	—	228
12—14	—	—	3	2	1	5	22	43	37	68	74	79	42	35	13	12	6	2	—	—	—	—	—	—	444
15—17	—	—	1	3	8	12	28	67	69	115	137	102	72	56	30	20	5	4	1	—	1	—	—	1	732
18—20	1	—	1	3	2	9	14	29	62	84	83	104	69	52	35	19	7	8	2	1	—	—	—	—	585
21—23	—	—	1	1	3	7	7	27	51	75	77	84	71	53	25	19	12	3	3	—	1	—	—	—	520
24—26	—	—	1	1	1	2	2	12	17	20	39	42	35	18	22	10	5	8	2	—	—	1	—	—	238
27—29	—	—	—	—	—	—	—	2	4	6	13	8	11	4	4	7	1	1	1	—	—	—	—	—	62
30—32	—	—	—	—	—	—	—	—	2	—	3	1	3	3	—	1	2	—	—	—	—	—	—	—	15
33—35	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1
36—38	—	—	—	—	—	—	—	—	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	2
39—41	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
42—44	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
45—47	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	—	—	—	—	1
Totals	**2**	**1**	**12**	**11**	**25**	**57**	**102**	**241**	**327**	**474**	**536**	**546**	**405**	**275**	**161**	**107**	**43**	**30**	**11**	**2**	**3**	**1**	**0**	**1**	**3373**

6. *Corrected Vital Capacity and Acuity of Sight.* Having noted the close similarity of the reaction times to sensations of sight and sound in their relationship to vital capacity, it will be well next to investigate acuity in these two senses. From Table VII (p. 151) the associational values for acuity of sight have been derived and may be set forth for comparison as before:

	r_{bk}	$\eta_{k.b}$	$\eta_{b.k}$
At age 40·5	+·1358 ± ·0114	·1489 ± ·0114	·1631 ± ·0113.

These constants are of much the same order as those found in the case of reaction times, the positive sign of the correlation indicating that increase in vital capacity is associated with increased keenness of sight. Remembering that the negative sign to the correlation coefficient for reaction times really indicates positive association of mental and physical fitness, it is evident that the results for all three of the above psychical factors are in very close agreement.

The fact that r and η are more nearly of the same size than in the case of reaction times indicates greater approach to linearity of regression for visual acuity on vital capacity. This is graphically evident in Diagrams 6 and 7, the equations for the respective cubics being given as follows:

$$\text{Probable } k = 16{\cdot}20 + {\cdot}33561\tilde{b} - {\cdot}00068255\tilde{b}^2 - {\cdot}00047106\tilde{b}^3,$$

$$\text{Probable } b = 217{\cdot}75 + 2{\cdot}90286\tilde{k} + {\cdot}502026\tilde{k}^2 + {\cdot}0030456\tilde{k}^3,$$

where $\tilde{k} = (k - 16{\cdot}5)/3$. From the second cubic a minimum vital capacity of about 213·5 cubic inches is found at $k = 7{\cdot}5$.

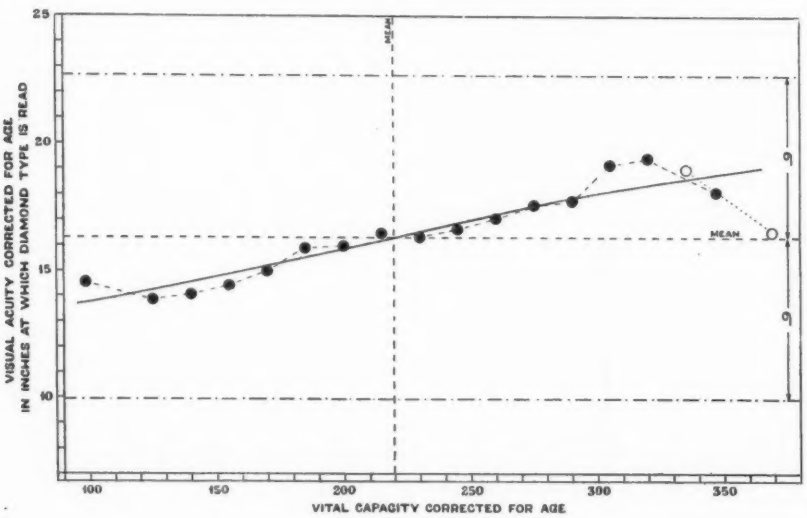

Diagram 6.

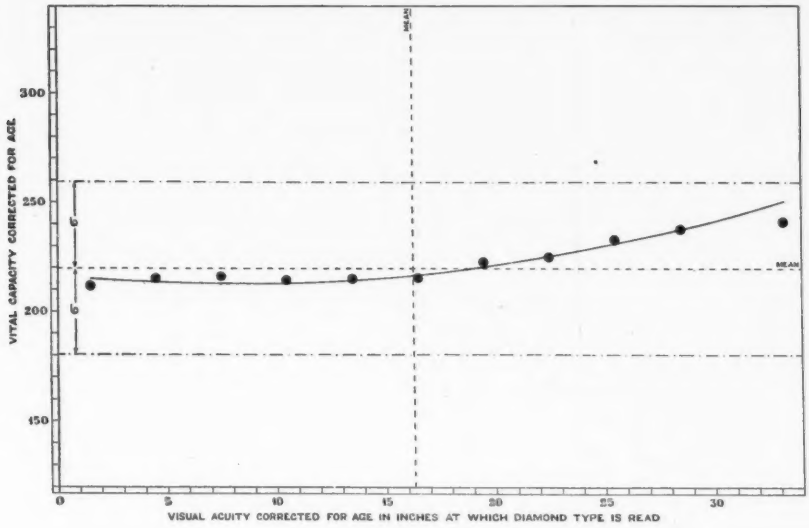

Diagram 7.

The mean and standard deviation for keenness of vision are $16{\cdot}299 \pm {\cdot}074$ and $6{\cdot}3758 \pm {\cdot}0524$ respectively, the unit being inches at which diamond type could be read.

7. *Corrected Vital Capacity and Pitch.* The remaining associations are in distinct contrast with those previously found. For vital capacity and highest audible pitch the constants as deduced from Table VIII are:

	r_{bp}	$\eta_{p.b}$	$\eta_{b.p}$
At age 40·5	$+{\cdot}0216 \pm {\cdot}0116$	$\cdot0875 \pm {\cdot}0115$	$\cdot0840 \pm {\cdot}0116$.

The correlation coefficient is at once seen to be insignificant. Furthermore if $\bar{\eta}$, the value expected from uncorrelated material, be computed, it is found by the formula cited above that

$$\eta_{p.b} - \bar{\eta}_{p.b} = {\cdot}0049 \pm {\cdot}0116,$$

$$\eta_{b.p} - \bar{\eta}_{b.p} = {\cdot}0196 \pm {\cdot}0116,$$

from which it may be concluded that the correlation ratios found might easily have arisen from material in reality uncorrelated. Clearly no prediction is possible in estimating pitch from vital capacity, and we may assert that increased or decreased vital capacity brings about no change in acuity of hearing.

It thus appears that while physical robustness as measured by vital capacity produces an appreciable effect upon the quality of sight, it does not do so in the case of hearing. This result suggests that the former correlation is due to factors other than mere sensory fitness. Quality of vision is more intimately related to condition of the nervous system than is hearing, and general fitness of the nerves

TABLE VIII.

Corrected Vital Capacity and Highest Audible Pitch.

Corrected Vital Capacity in cubic inches.

Corrected Highest Audible Pitch in thousands of Vibrations per second.	57·5— 72·5	72·5— 87·5	87·5—102·5	102·5—117·5	117·5—132·5	132·5—147·5	147·5—162·5	162·5—177·5	177·5—192·5	192·5—207·5	207·5—222·5	222·5—237·5	237·5—252·5	252·5—267·5	267·5—282·5	282·5—297·5	297·5—312·5	312·5—327·5	327·5—342·5	342·5—357·5	357·5—372·5	372·5—387·5	387·5—402·5	402·5—417·5	Totals
1— 2	—	—	—	—	—	—	—	—	—	—	—	—	—	—	1	—	—	—	—	—	—	—	—	—	1
3— 4	—	—	—	—	—	—	—	—	—	2	1	1	1	3	1	1	—	—	—	—	—	—	—	—	11
5— 6	—	—	—	—	—	—	—	—	—	2	1	—	—	—	—	—	—	—	—	—	—	—	—	—	3
7— 8	—	—	—	—	—	—	—	—	4	3	1	3	3	3	1	—	—	—	—	—	—	—	—	—	19
9—10	—	—	—	1	1	1	2	—	6	5	3	4	10	6	1	—	2	4	1	—	—	—	—	—	50
11—12	—	1	—	3	3	2	9	6	14	26	31	34	19	20	11	9	4	2	3	1	—	—	—	—	209
13—14	—	—	4	2	5	12	21	17	57	75	84	88	73	39	28	11	4	7	—	—	—	—	—	—	562
15—16	—	—	6	4	8	15	35	50	112	155	195	178	120	85	56	33	11	6	2	—	1	—	—	1	1098
17—18	2	—	—	1	5	17	22	69	90	158	161	160	138	91	41	38	16	12	5	1	2	—	—	—	1030
19—20	—	—	—	1	1	6	5	15	39	37	45	67	27	17	19	12	5	2	—	—	—	—	—	—	297
21—22	—	—	2	—	1	—	3	7	4	6	11	6	7	9	2	2	1	—	—	—	—	—	—	—	64
23—24	—	—	—	1	—	2	1	—	1	3	3	5	4	2	—	1	—	—	—	—	—	—	—	—	20
25—26	—	—	—	—	—	1	—	—	—	1	—	—	2	—	—	—	—	—	—	—	—	—	—	—	6
27—28	—	—	—	—	—	—	1	—	—	—	—	1	1	—	—	—	—	—	—	—	—	—	—	—	0
29—30	—	—	—	—	—	—	—	—	—	1	—	1	—	—	—	—	—	—	—	—	—	—	—	—	3
Totals	2	1	12	11	25	57	102	241	327	474	536	546	405	275	161	107	43	30	11	2	3	1	0	1	3373

is probably correlated with vital capacity. The observed correlation may be then due to the influence of some such intermediate factors as nervous or mental fitness.

From Table VIII the mean and standard deviation for acuity of hearing are found to be 16·382 ± ·030 and 2·5943 ± ·0213 respectively, the unit being one thousand vibrations per second. Diagram 8 shows the smallness of the correlation in graphic form.

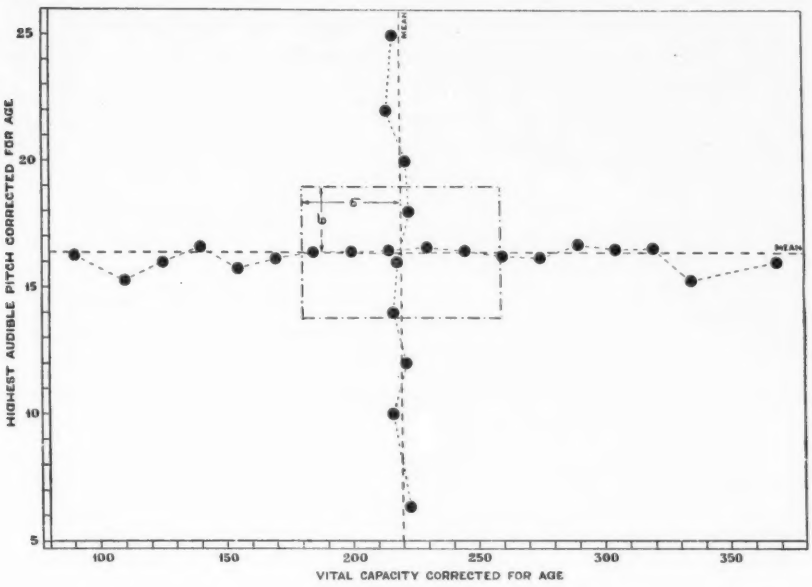

Diagram 8.

For convenience of reference I have added Table IX giving the means, standard deviations and coefficients of variation for the characters dealt with corrected for age.

TABLE IX.

Means, Standard Deviations and Coefficients of Variation for Age-Corrected Characters.

Character	Mean	Standard Deviation	Coefficient of Variation
Vital Capacity	219·854 ± ·457	39·362 ± ·323	17·904 ± ·151
Reaction Time to Sight...	18·711 ± ·046	3·9911 ± ·0328	21·331 ± ·183
Reaction Time to Sound	15·802 ± ·037	3·1484 ± ·0259	19·924 ± ·169
Visual Acuity	16·299 ± ·074	6·3758 ± ·0524	39·118 ± ·369
Auditory Acuity ...	16·382 ± ·030	2·5943 ± ·0213	15·836 ± ·133

SUMMARY.

From the above results the following general conclusions may be drawn.

(1) There is a significant and essentially positive correlation of ·12 to ·16 between vital capacity and the three psychical factors, reaction time to sight, reaction time to sound, and acuity of sight, when each of the variables has been suitably corrected for age.

(2) There is no correlation between vital capacity and acuity of hearing.

(3) The order of the significant correlations is somewhat greater than that found between static physical characters and such mental traits as intelligence.

(4) All of the associational constants are too small for reliable prediction, but yet indicate that vital capacity is to a significant extent a conditioning factor for three of the psychical characters dealt with.

(5) It is probable that the correlation found is partly due to the influence of intermediate factors such as nervous and mental fitness which are in turn related to vital capacity.

(6) The range of age and the size of sample here employed seem to form an adequate basis for reliable statistical conclusions.

ON THE MOMENTS OF THE HYPERGEOMETRICAL SERIES.

By KARL PEARSON, F.R.S.

In 1899 I published in the *Philosophical Magazine** expressions for the first five moments of the special hypergeometrical series $F(\alpha, \beta, \gamma, 1)$. Those values were obtained by a rather long process involving transference from the start of the series to the mean. In 1895 I had published probably for the first time the first *four* moments about the mean of the binomial †. The growing importance not only of the first four moments of these series, but of higher moments for statistical purposes, has been recognised and relatively easy methods of finding the moment-coefficients are very desirable. Professor V. Romanovsky has recently published a remarkable formula‡ by which the successive moments of a binomial may be fairly easily obtained. The object of the present paper is to provide a formula by aid of which the successive moment-coefficients about its mean of the hypergeometrical series $F(\alpha, \beta, \gamma, 1)$ may be written down without any differentiation, but by a certain finite difference relation: this will include that of the binomial as a special case.

Given the hypergeometrical series $F(\alpha, \beta, \gamma, x)$ we lose nothing in generality by writing $\alpha = -r$, $\beta = -qn$, $\gamma = pn - r + 1$, $x = e^y$; we merely replace the arbitrary constants α, β, γ, x by the arbitrary constants r, n, p and y, taking $q = 1 - p$. This transformation, however, enables us to write the general hypergeometric series in the form

$$1 + r \cdot \frac{qn}{pn - r + 1} e^y + \frac{r(r-1)}{1.2} \cdot \frac{qn(qn-1)}{(pn-r+1)(pn-r+2)} e^{2y}$$
$$+ \frac{r(r-1)(r-2)}{1.2.3} \cdot \frac{qn(qn-1)(qn-2)}{(pn-r+1)(pn-r+2)(pn-r+3)} e^{3y} + \ldots \quad \text{(i)},$$

where we see that if multiplied by the factor $\dfrac{M \cdot pn(pn-1)\ldots(pn-r+1)}{n(n-1)\ldots(n-r+1)}$ it represents, when $y = 0$, the distribution of frequency of M random samples of size r drawn from a population of n containing pn marked and qn non-marked individuals.

The mean of these samples will clearly have rq marked individuals. Multiply the above expression by e^{-rqy}; then the $(t+1)$th term will contain the exponential $e^{(t-rq)y}$, and if we differentiate with regard to y s times, the $(t+1)$th term will be multiplied by $(t - rq)^s$, which is clearly the sth power of the distance of the $(t+1)$th term from the mean. Accordingly if we put $y = 0$, we have an expression

* February, 1899, p. 239. † *Phil. Trans.* Vol. 186, p. 347.
‡ *Biometrika*, Vol. xv. pp. 410—412.

for the sth moment of the hypergeometrical series about its mean or we may, writing

$$F_1(\alpha, \beta, \gamma, e^y) = \frac{pn(pn-1)\dots(pn-r+1)}{n(n-1)\dots(n-r+1)} F(\alpha, \beta, \gamma, x),$$

say that

$$\mu_s = \frac{d^s}{dy^s}\{e^{-rqy} F_1(\alpha, \beta, \gamma, e^y)\}_{y=0} \quad \dots\dots\dots\dots\dots\dots\dots\dots\text{(ii)}$$

is the sth moment-coefficient of the hypergeometrical series multiplied by a constant so chosen that the sum of the terms for $y=0$ is unity.

Now consider the series

$$\mu_0 + \mu_1 \frac{y}{1!} + \mu_2 \frac{y^2}{2!} + \dots + \mu_s \frac{y^s}{s!} + \dots ;$$

substituting for the values of μ_s we see that this series is the expansion by Maclaurin's theorem of

$$e^{-rqy} F_1(\alpha, \beta, \gamma, e^y).$$

Using the operator E to denote the operation $E\mu_s = \mu_{s+1}$, we have

$$e^{yE}\mu_0 = e^{-rqy} F_1(\alpha, \beta, \gamma, e^y).$$

Accordingly

$$e^{y(rq+E)}\mu_0 = F_1(\alpha, \beta, \gamma, e^y) \quad \dots\dots\dots\dots\dots\dots\dots\dots\text{(iii)},$$

where of course μ_0 must not be put unity until we have freed it from the operator E.

Now $F(\alpha, \beta, \gamma, x)$ satisfies a certain differential equation, namely*

$$x(1-x)\frac{d^2F}{dx^2} + \{\gamma - (\alpha+\beta+1)x\}\frac{dF}{dx} - \alpha\beta F = 0 \quad \dots\dots\dots\dots\text{(iv)},$$

and if we transform this equation by writing $y = \log x$, we have

$$(1-e^y)\frac{d^2F_1}{dy^2} + \{\gamma - 1 - (\alpha+\beta)e^y\}\frac{dF_1}{dy} - \alpha\beta e^y F_1 = 0 \quad \dots\dots\text{(iv } bis\text{)},$$

which again may be rewritten

$$(1-e^y)\left\{\frac{d^2F_1}{dy^2} - (r+qn)\frac{dF_1}{dy} + rqnF_1\right\} = rqnF_1 - n\frac{dF_1}{dy},$$

or using Equation (iii) we have

$$(1-e^y)\{(rq+E)^2 e^{y(rq+E)}\mu_0 - (r+qn)(rq+E)e^{y(rq+E)}\mu_0 + rqne^{y(rq+E)}\mu_0\} = -ne^{y(rq+E)}\mu_1.$$

We can now divide out by e^{yrq}, and we have

$$(1-e^y)\{E^2 + E(2rq - r - qn) + r^2q^2 - r^2q + nrq^2 + nrq\}e^{yE}\mu_0 = -ne^{yE}\mu_1.$$

This may again be written

$$(1-e^y)\{E^2 - (nq + r(p-q))E + rqp(n-r)\}e^{yE}\mu_0 = -ne^{yE}\mu_1.$$

Operating with the first factor before the second

$$(1-e^y)e^{yE}\{\mu_2 - (nq + r(p-q))\mu_1 + rqp(n-r)\mu_0\} = -ne^{yE}\mu_1,$$

* Forsyth, *A Treatise on Differential Equations*, 1885, p. 185.

where of course we must not put $\mu_1 = 0$ and $\mu_0 = 1$ until the operator e^{yE} has been used. We will now expand the whole equation in powers of y, and it will provide a relation between successive μ's. We have

$$\left(y + \frac{y^2}{2!} + \ldots + \frac{y^s}{s!} + \ldots\right)\left\{\mu_2 + y\mu_3 + \ldots + \frac{y^s}{s!}\mu_{s+2} + \ldots\right.$$
$$- \{nq + r(p-q)\}\left(\mu_1 + y\mu_2 + \ldots + \frac{y^s}{s!}\mu_{s+1} + \ldots\right)$$
$$\left. + rqp(n-r)\left(1 + y\mu_1 + \ldots + \frac{y^s}{s!}\mu_s + \ldots\right)\right\}$$
$$= n\left(\mu_1 + y\mu_2 + \ldots + \frac{y^s}{s!}\mu_{s+1} + \ldots\right).$$

Equate powers of y^{s+1} on both sides,

$$\frac{1}{s!}\mu_{s+2} + \frac{1}{(s-1)!}\frac{1}{2!}\mu_{s+1} + \frac{1}{(s-2)!}\frac{1}{3!}\mu_s + \ldots$$
$$- \{nq + r(p-q)\}\left\{\frac{\mu_{s+1}}{s!} + \frac{\mu_s}{(s-1)!\,2!} + \frac{\mu_{s-1}}{(s-2)!\,3!} + \ldots\right\}$$
$$+ rqp(n-r)\left\{\frac{\mu_s}{s!} + \frac{\mu_{s-1}}{(s-1)!\,2!} + \frac{\mu_{s-2}}{(s-2)!\,3!} + \ldots\right\}$$
$$= n\frac{\mu_{s+2}}{(s+1)!}.$$

Multiply up by $(s+1)!$,

$$(s+1)\mu_{s+2} + \frac{(s+1)s}{1.2}\mu_{s+1} + \frac{(s+1)s(s-1)}{1.2.3}\mu_s + \ldots$$
$$- \{nq + r(p-q)\}\left\{(s+1)\mu_{s+1} + \frac{(s+1)s}{1.2}\mu_s + \frac{(s+1)s(s-1)}{1.2.3}\mu_{s+1} + \ldots\right\}$$
$$+ rqp(n-r)\left\{(s+1)\mu_s + \frac{(s+1)s}{1.2}\mu_{s-1} + \frac{(s+1)s(s-1)}{1.2.3}\mu_{s-2} + \ldots\right\}$$
$$= n\mu_{s+2} \quad \ldots\ldots\ldots\

Put $s=2$,

$$(1+2E)\{\mu_2-(nq+r(p-q))\mu_1+rqp(n-r)\mu_0\}=n\mu_2 \quad \text{.....(ix)},$$

$$n\mu_2+2\mu_2-2\{nq+r(p-q)\}\mu_2=n\mu_2,$$

$$\mu_2\{n(1-2q)-2r(p-q)\}=(n-2)\mu_2,$$

or

$$\mu_2=\frac{rqp(p-q)(n-r)(n-2r)}{(n-1)(n-2)} \quad \text{.....(x)}.$$

Put $s=3$,

$$(1+3E+3E^2)\{\mu_2-(nq+r(p-q))\mu_1+rqp(n-r)\mu_0\}=n\mu_4,$$

or by (vii) and (ix),

$$n\mu_2+\tfrac{3}{2}n(\mu_2-\mu_2)+3\mu_4-3(nq+r(p-q))\mu_2+3rqp(n-r)\mu_2=n\mu_4.$$

Thus

$$(n-3)\mu_4=\mu_2\{\tfrac{3}{2}n-3(nq+r(p-q))\}+\mu_2\{-\tfrac{1}{2}n+3rqp(n-r)\} \quad \text{...(xi)},$$

or by (x),

$$(n-3)\mu_4=\mu_2\left\{\frac{(p-q)(n-2r)}{n-2}\{\tfrac{3}{2}n-3(nq+r(p-q))\}-\tfrac{1}{2}n+3rpq(n-r)\right\}$$

$$=\mu_2\left\{\frac{3(p-q)^2(n-2r)^2}{2(n-2)}-\tfrac{1}{2}n+3rpq(n-r)\right\}.$$

Replacing $(p-q)^2$ by $1-4pq$, we easily find

$$\mu_4=\frac{rqp(n-r)}{(n-1)(n-2)(n-3)}\{n(n+1)-6r(n-r)$$

$$+3pq(n^2(r-2)-nr^2+6r(n-r))\} \quad \text{.....(xi } bis\text{)}.$$

This will be found to agree with the form given in *Biometrika*, Vol. v. p. 174, after the needful reductions.

The higher moments can be readily expressed in terms of the lower in equations like (xi), and for practical purposes it is better to use numerical values of the constants and of the lower moments rather than proceed by reducing the formula to the ultimate constants only by inserting algebraical values for the lower moments.

Special Cases. Let n be indefinitely large. The hypergeometrical series reduces to the binomial series

$$(p+q)^n,$$

and (vi) becomes, after dividing by n,

$$\{(1+E)^s-E^s\}(rqp\mu_0-q\mu_1)=\mu_{s+1} \quad \text{.....(xii)}.$$

If $s=0$, the equation is identically satisfied since $\mu_1=0$.

If $s=1$,

$$rqp\mu_0=\mu_2,$$

or,

$$\mu_2=rqp \quad \text{.....(xiii)}.$$

If $s=2$,

$$(1+2E)(rqp\mu_0-q\mu_1)=\mu_2,$$

or,

$$rqp\mu_0-2q\mu_2=\mu_2,$$

i.e.

$$\mu_2=rqp(1-2q)=rqp(p-q) \quad \text{.....(xiv)}.$$

If $s=3$,

$$(1+3E+E^2)(rqp\mu_0-q\mu_1)=rqp\mu_0-3q\mu_2+3rqp\mu_2-q\mu_3=\mu_4,$$

or,

$$\mu_4=\mu_2(1-3q+3rqp-3q+6q^2)$$
$$=\mu_2\{1+3qp(r-2)\} \quad \text{.....(xv).}$$

If $s=4$,

$$\mu_5=(1+4E+6E^2+4E^3)(rqp\mu_0-q\mu_1)$$
$$=rqp(1+6\mu_2+4\mu_3)-2q(2\mu_2+3\mu_3+2\mu_4).$$

Substituting we have

$$\mu_5=rqp\{1-8q-6q(p-q)+24q^2p+rqp(6+4(p-q)-12q)\}$$
$$=rqp(p-q)\{1+2pq(5r-6)\} \quad \text{.....(xvi),}$$

which agrees with the result given in *Biometrika*, Vol. XII. p. 270.

Again make q indefinitely small and rq finite $=m$, then

$$\mu_{s+1}=\{(1+E)^s-E^s\}m\mu_0 \quad \text{.....(xvii).}$$

If $s=1$,

$$\mu_2=m \quad \text{.....(xviii).}$$

If $s=2$,

$$\mu_3=m\mu_0+2m\mu_1=m \quad \text{.....(xix).}$$

If $s=3$,

$$\mu_4=m+3m^2 \quad \text{.....(xx).}$$

If $s=4$,

$$\mu_5=m(1+10m) \quad \text{.....(xxi),}$$

and so on, giving the moment-coefficients of the Poisson series.

As a last illustration turn back to (vi) and multiply both sides by c^{s+1}. We have

$$c^{s-1}\{(1+E)^s-E^s\}(rqpc^2\mu_0-qc\,.\,c\mu_1)=c^{s+1}\mu_{s+1}.$$

Now let us write $rqpc^2=\sigma^2$, and suppose r to become indefinitely large, while c grows indefinitely small, σ^2 remaining finite. We shall then obtain the limit to a binomial of an infinite number of terms plotted at infinitely small distances apart, but with a finite standard deviation. We find

$$c^{s-1}\{(1+E)^s-E^s\}(\sigma^2\mu_0-qc^2\mu_1)=\tilde{\mu}_{s+1},$$

where $c^{s+1}\mu_{s+1}=\tilde{\mu}_{s+1}$ will be the $(s+1)$th moment-coefficient of the resulting curve.

Now it is clear that when we operate with E^t, $t<s-1$,

$$c^{s-1}E^t(\sigma^2\mu_0-qc^2\mu_1)=c^{s-1-t}\sigma^2\tilde{\mu}_t-qc^{s-t}\tilde{\mu}_{t+1},$$

and since $t<s-1$, this vanishes with c. For the highest power of E, i.e. E^{s-1}, we have

$$c^{s-1}sE^{s-1}(\sigma^2\mu_0-qc^2\mu_1)=s\sigma^2\tilde{\mu}_{s-1}-qc\tilde{\mu}_s,$$

and the latter term vanishes with c.

Accordingly our equation reduces to

$$s\sigma^2\tilde{\mu}_{s-1}=\tilde{\mu}_{s+1}.$$

When $s=1$, $\sigma^2=\tilde{\mu}^2$,

$s=2$, $2\sigma^2\tilde{\mu}_1=0=\tilde{\mu}_3$,

$s=3$, $3\sigma^2\tilde{\mu}_2=\tilde{\mu}_4=3\sigma^4$.

Hence generally $\tilde{\mu}_{2s+1}=0$ .....(xxii),

$\tilde{\mu}_{2s}=(2s-1)(2s-3)\ldots 3\,.\,1\,.\,\sigma^{2s}$ .....(xxiii),

which determine the moments of the binomial to an infinite power plotted to vanishingly small elements. These are of course the well-known moment-coefficients of the normal curve.

Equation (v) shows us that it must be possible to express μ_{s+2} in terms of $c_1=nq+r(p-q)$, $c_2=rqp(n-r)$ and n, by eliminating all the μ's but μ_{s+2} and μ_0 by determinants. I have not, however, so far succeeded in reaching any useful general result. I should be glad to hear of progress in this direction, even for example, in the simple case of the binomial. It would enable us to deal more effectively than at present with some of the probable errors of random sampling from limited populations.

PROBABILITY INTEGRALS FOR THE POINT BINOMIAL.

By Dr BURTON H. CAMP.

(1) The probability that, in selecting n samples, the number of successes will exceed a given number is equal to the sum of the terms of a point binomial up to a certain term, provided that the sampling is such that the probability of a success does not vary from one sample to another. This sum is what is meant by the "probability integral" for the binomial. It is known that the normal curve, as fitted to the point binomial by De Moivre, so that the means, standard deviations, and total areas of the two curves agree, is not a good enough fit to give this integral very accurately, even when n is large, except near the mode, especially when the binomial has even a moderate degree of skewness. When n is small the individual terms may of course be computed and added together, and when the probability of a success is very small and n large, the Poisson series is available, but in the remaining cases there is no good method. It is the object of this paper to derive approximations that will be helpful in these cases.

(2) It is desirable to consider the problem in a slightly more general form first. Let $f(\xi)$ be any frequency function, and let $f \geqq 0$, if $a \leqq \xi \leqq b$; $f = 0$ elsewhere.

Case I (*f continuous*). Let

$$\int_a^b f(\xi)\, d\xi = 1, \quad A = \int_a^\tau f(\xi)\, d\xi, \quad a \leqq \tau \leqq b.$$

It is required to find the value of A. We shall use in essence Laplace's method of evaluating an integral, except that we shall so fit the normal curve to $f(\xi)$ that the two curves coincide at the stump τ of the tail instead of at the mode of $f(\xi)$. Make the transformation $\xi = \tau - v$, so that $f(\xi) = f(\tau - v) = \phi(v)$. Then

$$A = \int_0^{\tau - a} \phi(v)\, dv \quad \text{.....(i).}$$

Let $$\log_e \phi(v) = \psi(v) \cong \psi(0) + v\psi'(0) + \tfrac{1}{2}v^2\psi''(0),$$

approximately *, the accents indicating derivatives. Thus we may replace $\phi(v)$ by $e^{-c_0 - c_1 v - c_2 v^2}$ if

$$-c_0 = \psi(0) = \log_e \phi(0),$$

$$-c_1 = \psi'(0) = \phi'(0)/\phi(0),$$

$$-c_2 = \tfrac{1}{2}\psi''(0) = \tfrac{1}{2}\{\phi(0)\,\phi''(0) - (\phi'(0)^2\}/\phi^2(0);$$

* We shall use the symbol "$\cong$" to mean, "equals approximately."

and we then have

$$A \cong \int_0^{\tau-a} e^{-c_0-c_1v-c_2v^2}\,dv = \phi(0)\,e^{\frac{c_1^2}{4c_2}}\int_0^{\tau-a} e^{-c_2\left(v+\frac{c_1}{2c_2}\right)^2}\,dv$$

$$= \sigma\phi(0)\,e^{\frac{x_1^2}{2}}\int_{x_1}^{x_2} e^{-\frac{x^2}{2}}\,dx, \text{ where } \sigma = \frac{1}{\sqrt{2c_2}},\ x_1 = c_1\sigma, \text{ and}$$

$$x_2 = \frac{1}{\sigma}(\tau - a + c_1\sigma^2) \quad \text{.....(ii).}$$

It will be found that in most cases x_2 may be replaced by infinity without appreciable error. Then our expression for A takes the convenient form

$$A \cong \sigma\phi(0)\,\frac{\frac{1}{2}(1-\alpha_{x_1})}{z_{x_1}} \quad \text{.....(iii)}$$

in Sheppard's notation*, $\phi(0)$ being the ordinate, $f(\tau)$ at the stump τ. Equation (iii) may be stated in the following words: The area of the tail of any frequency curve equals, approximately, the ordinate at the stump, multiplied by the squared standard deviation of the normal curve which fits the frequency curve at the stump, divided by the abscissa of the mean of the tail of this normal curve; where this abscissa is measured from the centre of the normal curve, and this tail also is cut off at the given stump. As will be illustrated numerically in Example 7, the auxiliary normal curve here used may be a very poor fit to that part of the frequency curve which is left after the tail has been cut off. In particular, the two modes may be far apart. It is this fact which differentiates the present method from that of De Moivre.

In certain cases a sufficient approximation is

$$\phi(v) \cong e^{-c_0-c_1v} \quad \text{.....(iv),}$$

leading to a further simplification of (ii) and (iii). For, then,

$$A \cong \phi(0)\int_0^{\tau-a} e^{-c_1v}\,dv = \frac{\phi(0)}{c_1}(1-e^{-c_1(\tau-a)}) \quad \text{.....(ii \textit{bis}),}$$

and if, as before, we neglect the small last term,

$$A \cong \frac{(\phi(0))^2}{-\phi'(0)} \quad \text{.....(iii \textit{bis}).}$$

Case II (*f discontinuous*). In this case it is necessary to find†

$$A = \Sigma_a^{\tau} f(\xi)\,\Delta\xi = \{f(a) + f(a+\Delta\xi) + \ldots + f(\tau)\}\,\Delta\xi.$$

We proceed in a manner analogous to that of Case I, making the normal curve go through the stump and the next two points. Put $\xi = \tau - v$, and invert limits:

$$A = \Sigma_0^{\tau-a}\,\phi(v)\,\Delta v, \quad \Delta v = \Delta\xi.$$

* *Tables for Statisticians and Biometricians*, 2nd Edition, Part I. 1924, pp. 2—11.

† It should be observed that, in the notation to be used, Σ_a^{τ} is a sum which *includes* the values at the limits a and τ. It is not necessary to write the increment $\Delta\xi$, when, as in our application, it equals unity, but this will be done because sometimes it is not equal to unity.

Let $\log_e \phi(v) = \psi(v) \cong \psi(0) + v\Delta_0\psi + \frac{v(v-1)}{2}\Delta_0^2\psi$, thus replacing $\phi(v)$ by $e^{-c_0-c_1v-c_2v^2}$, where $-c_0 = \psi(0) = \log_e \phi(0)$, $-c_1 = \Delta_0\psi - \frac{1}{2}\Delta_0^2\psi$, and $-c_2 = \Delta_0^2\psi/2$. Since $\Delta_0\psi = \log_e \phi(\Delta v)/\phi(0)$, and $\Delta_0^2\psi = \log_e \phi(0)\phi(2\Delta v)/(\phi(\Delta v))^2$, it is convenient to write

$$Q = \frac{\phi(\Delta v)}{\phi(0)} = \text{ratio of terms at the stump, } R = \sqrt{Q\frac{\phi(\Delta v)}{\phi(2\Delta v)}},$$

$$-c_1 = \log_e QR, \quad c_2 = \log_e R \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(v)}.$$

Then

$$A \cong \Sigma_0^{\tau-a} e^{-c_0-c_1v-c_2v^2}\Delta v = \phi(0)e^{\frac{c_1^2}{4c_2}}\Sigma_0^{\tau-a} e^{-c_2\left(v+\frac{c_1}{2c_2}\right)^2}\Delta v$$

$$= \sigma\phi(0)e^{\frac{x_1^2}{2}}\Sigma_{x_1}^{x_2} e^{-\frac{x^2}{2}}\Delta x, \quad \Delta x = \sqrt{2c_2}\,\Delta v = \frac{\Delta v}{\sigma}, \quad \Delta v = \Delta\xi \ldots\text{(v } bis\text{)},$$

where x_1 and x_2 are as in (ii).

To evaluate (v *bis*) it is usually necessary to replace the sum by an integral by means of the Euler-Maclaurin Theorem. We have, then, in Sheppard's notation:

$$A \cong \sigma\phi(0)\frac{\frac{1}{2}(1-\alpha_{x_0}) - E}{z_{x_1}}, \quad x_0 = x_1 - \tfrac{1}{2}\Delta x, \quad x_2 \cong \infty$$

as before, and

$$E \cong \frac{x_0 z_{x_0}(\Delta x)^2}{24}\left(1 + \frac{7(3-x_0^2)(\Delta x)^2}{240} + \frac{31(15-10x_0^2+x_0^4)(\Delta x)^4}{40320}\right) \text{ (vi)*}.$$

As in Case I, it will sometimes be sufficient to omit the term in c_2v^2, and we then arrive at the sum of a geometric series:

$$A \cong \phi(0)\Sigma_0^{\infty} e^{-c_1v}\Delta v = \frac{\phi(0)\Delta v}{1-Q^{\Delta v}}, \quad \Delta v = \Delta\xi, \quad Q = \frac{\phi(\Delta v)}{\phi(0)} \quad \ldots\text{(vi } bis\text{)}.$$

(3) *The Point Binomial.* Consider now the point binomial $(p+q)^n$, where $p+q=1$. The $(t+1)$th term may be written

$$u(t) = \frac{n!}{s!\,t!}p^s q^t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(vii)},$$

where $s = n-t$. We shall call $u(t)$ the "point binomial function." Let it be required to find the sum of the first $(t+1)$ terms,

$$S_{t+1}(p) = u(0) + u(1) + \ldots + u(t) \ldots\ldots\ldots\ldots\ldots\ldots\text{(viii)}.$$

It is obvious that, if it were necessary to find instead the sum of the last $(t+1)$ terms, this would become the sum of the first $(t+1)$ terms by an interchange of the values for p and q. It is also obvious that the more general point binomial $(\alpha+\beta)^n$ can be reduced to the one being considered by division of each term of the expansion by the value of $(\alpha+\beta)^n$ if $\alpha+\beta \neq 0$. Thus our restrictions have not in fact involved loss of generality. We shall now develop two general methods of approximating to $S_{t+1}(p)$. The first gives better results when the stump is near

* In case tables for $\frac{1}{2}(1-\alpha_x)/z_x$ are available a more convenient form for A is provided by

$$A \cong \phi(0)\{\sigma\tfrac{1}{2}(1-\alpha_{x_1})/z_{x_1} + (\tfrac{1}{2}+E')\Delta v\},$$

where
$$E' = \tfrac{1}{12}x_1(\Delta x)\left\{1 + \tfrac{1}{60}(\Delta x)^2(3-x_1^2) + \frac{(\Delta x)^4}{2520}(15-10x_1^2+x_1^4)\right\}.$$

the mode, and when the binomial is only moderately skew. The second involves somewhat less computation, is best at considerable distances from the mode, and is not much affected by increases in the skewness.

First Method. The theory is quite simple and consists merely in applying (vi) and (vi *bis*) to the point binomial function (vii). The results are, in the order suitable for computation*:

$$Q=\frac{t}{s+1}\frac{p}{q},\quad R=\sqrt{\frac{t}{s+1}\frac{s+2}{t-1}},\quad c_1=-\log_e QR,\quad c_2=\log_e R,$$

$$\Delta x=\frac{1}{\sigma}=\sqrt{2c_2},\quad x_1=c_1\sigma,\quad x_0=x_1-\tfrac{1}{2}\Delta x,$$

$$E\cong\frac{z_{x_0}x_0(\Delta x)^2}{24}\left(1+\frac{7\,(3-x_0^2)\,(\Delta x)^2}{240}+\frac{31\,(\Delta x)^4}{40320}\,(15-10x_0^2+x_0^4)\right),$$

$$u_t=\frac{n\,!}{s\,!\,t\,!}p^sq^t,\quad S_{t+1}(p)\cong\sigma u(t)\,\frac{\frac{1}{2}(1-\alpha_{x_0})-E}{z_{x_1}}\qquad\text{.....(ix).}$$

By (vi *bis*): $$S_{t+1}(p)\cong\frac{u(t)}{1-Q}\qquad\text{.....(ix \textit{bis}).}$$

It is worth noting that the simple geometric series (ix *bis*), although not a close approximation, except when the stump is far from the mode, is, of the various approximations in this paper, much the easiest for the computer, and should therefore be used when only a rough idea of the size of the sum is needed. The right-hand side is the sum of the infinite series: $u(t)(1+Q+Q^2+Q^3+\ldots)$, and the true value of S is the sum of the finite series: $u(t)(1+Q+QQ_1+QQ_1Q_2+\ldots)$, where $Q_1=u(t-2)/u(t-1)$ and is less than Q, and $Q_2=u(t-3)/u(t-2)$ and is less than Q_1, etc. It follows that in the use of (ix *bis*) the error, $u(t)/(1-Q)-S$, is always positive, and one may form a rough idea as to its size from the known value of Q. Sometimes, when a sufficiently good result cannot be obtained in this way, it pays to compute first a few terms of the tail, and then apply the formula to the remainder. Thus, if three extra terms be computed, the approximation would be:

$$S_{t+1}\cong u_{t-3}\left(\frac{1}{1-\dfrac{t-3}{s+4}\dfrac{p}{q}}+\frac{s+3}{t-2}\frac{q}{p}+\frac{s+3}{t-2}\frac{s+2}{t-1}\frac{q^2}{p^2}+\frac{s+3}{t-2}\frac{s+2}{t-1}\frac{s+1}{t}\frac{q^3}{p^3}\right)\ldots\text{(ix \textit{ter}).}$$

It is only when Q is small that either (ix) or (ix *bis*) is valuable.

Example 1. Find the sum of the first 261 terms of the binomial $(\frac{1}{3}+\frac{2}{3})^{450}$. The stump is approximately $4\sigma'$ distant from the mode, where $\sigma'=\sqrt{npq}$ and is the standard deviation of the point binomial.

By (ix): $t=260,\quad s=190,\quad n=450,\quad p=\frac{1}{3},\quad q=\frac{2}{3},$

$\log_{10}Q=-\cdot167\ 090\ 015,\quad \log_{10}R=0{\cdot}001\ 970\ 723,\quad c_1=0{\cdot}380\ 2012,$

$c_2\dagger=0{\cdot}004\ 537\ 757,\quad \Delta x=0{\cdot}095\ 2655,\quad x_1=3{\cdot}990\ 965,\quad x_0=3{\cdot}943\ 332,$

$E=0{\cdot}000\ 000\ 250,\quad u(260)=1{\cdot}7101\times10^{-5},\quad \frac{1}{2}(1-\alpha_{x_0})=0{\cdot}000\ 040\ 178,$

$z_{x_1}=0{\cdot}000\ 138\ 750,\quad S_{t+1}(p)=5{\cdot}1659\times10^{-5}.$

* If tables for $\frac{1}{2}(1-\alpha_x)/z_x$ are available it is better to use the equivalent form

$$S_{t+1}(p)\cong u(t)\,\{\sigma\tfrac{1}{2}(1-\alpha_{x_1})/z_{x_1}+\tfrac{1}{2}+E'\},$$

where E' has the value given in the footnote p. 165.

† It was necessary to use nine place logarithms in order to obtain seven significant figures in c_2.

The true value is $5{\cdot}1662 \times 10^{-5}$. The result is, with this small degree of skewness, as good as will be obtained by the second method to be developed. The distance from the mode is favourable to the use of the rougher approximations (ix *bis*) and (ix *ter*). The results are:

by (ix *bis*) $5{\cdot}355 \times 10^{-5}$; by (ix *ter*) $5{\cdot}213 \times 10^{-5}$.

(4) *Second Method: Special Case.* It is known* that the first $(t+1)$ terms of Poisson's expansion,

$$S_{t+1}(m) = e^{-m}\left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \ldots + \frac{m^t}{t!}\right) \ldots\ldots\ldots\ldots\ldots\ldots\text{(x)},$$

may be expressed by means of a definite integral:

$$1 - I(m, t) = 1 - \frac{1}{\Gamma(t+1)}\int_0^m e^{-v} v^t \, dv \ldots\ldots\ldots\ldots\ldots\ldots\text{(xi)},$$

a fact which becomes immediately obvious on integration of the latter by parts. In the special instances where the Poisson series may be used to advantage, we have, therefore, a tabulation of its exact sum, to any number of terms, in the *Tables of the Incomplete* Γ*-function*. Before taking up the case of the general point binomial, let us consider an example of this special type.

Example 2. Find the sum of the first 12 terms of the point binomial

$$(113/114 + 1/114)^{662}.$$

Here $m = 5{\cdot}80702$, $S_{12}(m) = 1 - I(5{\cdot}80702,\ 11)$. The tables give $I(u,\ t)$, where $u = m/\sqrt{t+1} = 1{\cdot}676\ 342$. Interpolating by central differences we find:

$$I(1{\cdot}676\ 342,\ 12) = (\cdot 23658)(\cdot 011\ 6083) + (\cdot 76342)(\cdot 017\ 7043)$$
$$- \tfrac{1}{6}(\cdot 76342)(\cdot 23658)\{(1{\cdot}23658)(17583) + (1{\cdot}76342)(21335)\},$$

neglecting δ^4, whence $S_{12}(m) = 0{\cdot}983\ 917$.

In order to test this result by means of the tables† which give the individual terms of the Poisson expansions, it is necessary to have a means of interpolating into those tables, for they give the terms for $m = 5{\cdot}8$, and for $m = 5{\cdot}9$, but not for $m = 5{\cdot}800\ 702$. Since

$$dS_{t+1}(m) = e^{-m}\left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \ldots + \frac{m^{t-1}}{(t-1)!}\right) dm - e^{-m}\left(1 + \frac{m}{1!} + \frac{m^2}{2!} + \ldots + \frac{m^t}{t!}\right) dm$$
$$= -e^{-m}\frac{m^t}{t!}\, dm,$$

we have approximately, as an interpolation formula:

$$S_{t+1}(m + \delta m) \cong S_{t+1}(m) - e^{-m}\frac{m^t}{t!}\,\delta m,$$

that is: The sum of $(t+1)$ terms of Poisson's series for $(m + \delta m)$ equals the corresponding sum for m, less δm times the $(t+1)$th term for m. The use of this method of interpolation yields $S_{t+1}(5{\cdot}80702) = 0{\cdot}984\ 050 - 0{\cdot}000\ 133 = 0{\cdot}983\ 917$, agreeing exactly with the result above. The true value is 0·984 350.

* *Tables of the Incomplete* Γ*-function*, H.M. Stationery Office, 1921, Introduction.

† *Tables for Statisticians and Biometricians*, 2nd Edition, Part I. 1924, pp. 114—115.

(5) *Second Method: General Case.* A relation for the general point binomial may be found by integration by parts of the incomplete B-function, $B(s, t+1)$, where by definition

$$B_p(s, t+1) = \int_0^p \xi^{s-1}(1-\xi)^t \, d\xi \quad \text{.....(xii),}$$

and

$$B(s, t+1) = B_1(s, t+1) = \frac{t!\,(s-1)!}{n!}, \quad n = s+t \quad \text{.....(xiii).}$$

It is desirable, however, to obtain this relation a little differently because by so doing we can exhibit a general method which has been found to be of value in transforming other sums.

Since $S_{t+1}(0) = 0$, if $t < n$, we have obviously:

$$S_{t+1}(p) = \int_0^p \frac{dS}{dp}\, dp = \int_0^p dp \left\{ \Sigma_0^t \frac{n!}{s!\,t!} (sp^{s-1}q^t - tp^s q^{t-1})\, \Delta t \right\}, \text{ if } t < n.$$

This sum is:

$$np^{n-1}q^0 + \frac{n(n-1)}{1!}p^{n-2}q^1 + \frac{n(n-1)(n-2)}{2!}p^{n-3}q^2 + \ldots + \frac{n!}{(s-1)!\,t!}p^{s-1}q^t$$
$$- \frac{n}{1!}p^{n-1}q^0 - \frac{n(n-1)}{1!}p^{n-2}q^1 - \ldots - \frac{n!}{s!\,(t-1)!}p^s q^{t-1},$$

which equals

$$\frac{n!}{(s-1)!\,t!}p^{s-1}q^t.$$

So* $$S_{t+1}(p) = \frac{n!}{(s-1)!\,t!}\int_0^p \xi^{s-1}(1-\xi)^t\, d\xi = \frac{B_p(s, t+1)}{B(s, t+1)}, \text{ if } t < n \quad \text{...(xiv).}$$

We shall now approximate† to the integral in (xiv) to obtain the value of the sum desired. The reader should notice that, though we have expressed the sum function as an integral, the process is not at all analogous to a quadrature. We do not draw a certain curve through the successive points and integrate it over an interval of the t-axis. The sum required is a function of p as well as of t, and the integration is over an interval of the p-axis.

Put $\xi = p - v$, then:

$$S_{t+1}(p) = \frac{n!\,p^{s-1}q^t}{(s-1)!\,t!}\int_0^p \left(1 - \frac{v}{p}\right)^{s-1}\left(1 + \frac{v}{q}\right)^t dv \quad \text{.....(xv).}$$

Put $\phi(v) = \left(1 - \frac{v}{p}\right)^{s-1}\left(1 + \frac{v}{q}\right)^t$, and we have an integral of the form (i). Moreover, $\log_e \phi(v)$ permits an expansion in powers of v which is very rapidly convergent as long as v is considerably less than p and q, and so we may expect the approximating normal curve used with (i) to fit very well when v is near zero, and, since

* This equation furnishes a simple proof that the complete B-function has the value given in (xiii), for $S_{t+1}(p) = 1$ when $p = 1$. It also shows that, were the incomplete B-function tabulated, we would have here an exact evaluation of the sum of the first $(t+1)$ terms of any point binomial, including the special case treated above, just as the tabulation of the incomplete Γ-function affords an exact evaluation of the sum of $(t+1)$ terms of a Poisson series.

† Our approximation will suppose s and t large. For other approximations see H. E. Soper, *Tracts for Computers*, No. 7, Cambridge University Press, 1921.

$\phi(v)$ diminishes rapidly as v increases, for the sort of values of the constants to be considered, it does not much matter whether the fit is good far from zero or not; that is, the absolute error there will be very small whether the proportional error is small or large. Also, we may as well consider that it is the function under the integral sign of (xiv),

$$\xi^{s-1}(1-\xi)^t,$$

that we are thus fitting with a certain normal curve, and it will be noticed that this function does not involve p. The portion of this function which we are going to call its tail when making the fit does depend on p, but not the function itself. We may, therefore, expect, as was indicated above, that the goodness of fit by this method will not vary greatly with p, that is with the skewness of the binomial*, although it may with the relative values of t and s, that is, with the relative numbers of terms of the binomial taken and left. As compared with the first method, this one has the marked advantage that, since we are approximating to an integral instead of a sum, there is no Euler-Maclaurin correction necessary; and the computation is in other respects somewhat lighter. By (iii),

$$S_{t+1}(p)=\frac{\sigma n!\,p^{s-1}q^t}{(s-1)!\,t!}\,\frac{\frac{1}{2}(1-\alpha_{x_1})}{z_{x_1}},\quad c_1=\frac{s-1}{p}-\frac{t}{q},\quad x_1=c_1\sigma,\quad \frac{1}{\sigma^2}=\frac{s-1}{p^2}+\frac{t}{q^2}\quad\text{(xvi)},$$

and this approximation presupposes that our initial problem is so stated that c_1 is positive. Let us consider the reason for this last statement. The mode of the normal curve which approximates to $\phi(v)$ is at the point $v=-c_1\sigma^2$. To obtain a good approximation to the area desired it is clear that we should like to have this come out negative, that is c_1 positive, for the mode of our normal curve would then be outside our interval of integration, and we should be fitting the tail of our given function with the tail of a normal curve, not with the other part which contains the mode. Now, our problem can always be stated so that $c_1 \geqq 0$. For, suppose $c<0$, i.e. $(s-1)/p<t/q$. Restate the problem so as to find, not the first $(t+1)$ terms of the binomial $(p+q)^n$, but the first s terms of the binomial $(q+p)^n$. Our new value of c_1 is then $t/q-(s-1)/p$, which is by assumption positive.

Example 3. Solve Example 1 by (xvi). As before, $t=260$, $s=190$, $p=\frac{1}{3}$, $q=\frac{2}{3}$; and now

$$c_1=177,\ c_2=1143,\quad 1/\sigma=47{\cdot}81213,\quad x_1=3{\cdot}701\ 989,$$
$$\tfrac{1}{2}(1-\alpha_{x_1})=0{\cdot}000\ 106\ 9584,\quad z_{x_1}=0{\cdot}000\ 421\ 6646,$$
$$S_{t+1}=\frac{450!}{189!\,260!}\,\frac{2^{260}}{3^{449}\sqrt{2c_2}}(0{\cdot}253\ 657)=(0{\cdot}000\ 203\ 869)(0{\cdot}253\ 657)=5{\cdot}1713\times10^{-5}.$$

Had (xv) been used instead of (xvi), we should have had $x_2=c_1\sigma+p/\sigma=21{\cdot}6$, and this would not have affected the result up to about the 111th decimal place.

Recapitulating our results for Example 1 in order of goodness, we have: True value $5{\cdot}166\times10^{-5}$; by (ix) $5{\cdot}166\times10^{-5}$; by (xvi) $5{\cdot}171\times10^{-5}$; by (ix *ter*) $5{\cdot}213\times10^{-5}$; by (ix *bis*) $5{\cdot}355\times10^{-5}$; by the "modal" normal curve, i.e. the normal curve as fitted by De Moivre, $3{\cdot}91\times10^{-5}$. It is to be noted that the stump of the binomial is about $4\sigma'$ from its mode.

* As a test of this, cf. Example 6 and, the very extreme case, Example 8.

Example 4. For the same binomial, $(\frac{1}{3}+\frac{2}{3})^{450}$, find the sum (*a*) of the first 271 terms, (*b*) of the first 281 terms.

(*a*) Stump approximately $3\sigma'$ from the mode; true value 0·001 791, by (xvi) 0·001 794; by the modal* normal curve 0·001 589.

(*b*) Stump approximately $2\sigma'$ from the mode; true value 0·02644; by (ix) 0·02643; by (xvi) 0·02653; by the modal* normal curve 0·02559; by (ix *bis*) 0·03013. It might have been expected that the first method would give the best results here, because as we approach the mode of the binomial function the nearer it is to the normal function. It would not, however, have been as well to have used the modal normal curve here.

Example 5. For the binomial $(\frac{2}{3}+\frac{1}{3})^{450}$, find the sum (*a*) of the first 111 terms, (*b*) of the first 121 terms. We are thus considering the other tail of the binomial of Examples 1, 3, and 4.

(*a*) Stump approximately $4\sigma'$ from the mode; true value $2{\cdot}5752\times10^{-5}$; by (xvi) $2{\cdot}5742\times10^{-5}$.

(*b*) Stump approximately $3\sigma'$ from the mode; true value 0·001 352; by (xvi) 0·001 351.

Example 6. The point binomials $(\frac{1}{3}+\frac{2}{3})^{90}$, $(\frac{1}{9}+\frac{8}{9})^{90}$. This example and the next are offered to help the reader to form a judgment as to how the tests are affected by changing p and n.

(*a*) Find the sum of the first 46 terms of $(\frac{1}{3}+\frac{2}{3})^{90}$. The stump is at about $3{\cdot}2\sigma'$ from the mode. True value 0·000 7926; by (ix *ter*) 0·000 7942; by (xvi) 0·000 7960; by (ix *bis*) 0·000 8193.

(*b*) Find the first 73 terms of $(\frac{1}{9}+\frac{8}{9})^{90}$. The stump is at about $2{\cdot}5\sigma'$ from the mode. True value 0·009 5614; by (ix *ter*) 0·009 577; by (xvi) 0·009 608; by (ix *bis*) 0·009 954; by Poisson 0·014 277.

Example 7. Solve Example 2 by the general method (xvi). This is clearly an extreme case, in that q is very small. The stump is at about $2{\cdot}4\sigma'$ from the mode. True value 0·98435; by (xvi) 0·98372. This example furnishes a good illustration of a remark made earlier to the effect that our method of fitting a normal curve to the tail of a frequency curve does not imply a fit elsewhere. The function fitted is

$$\phi(v)=\left(1-\frac{v}{p}\right)^{s-1}\left(1+\frac{v}{p}\right)^{t},$$

and its mode (not to be confused with the mode of the binomial) is at the point $v=-c_1pq/(n-1)$. The mode of the normal curve is at the point

$$v=-c_1\sigma^2=\frac{-c_1p^2q^2}{(s-1)\,q^2+tp^2}.$$

These two modal values are equal when $p=q$, but not generally. In this example they are, respectively, $2{\cdot}982\sigma$ and $1{\cdot}579\sigma$ from the stump.

* Cf. Example 3.

(6) *Other Methods.* The reader will perhaps have noticed that our choice of an approximating curve was rather arbitrary, and have wondered whether some curve other than the normal curve, or the exponential, might not have done as well. There are of course a very large variety of familiar curves, having the same general form for some distance, which might have yielded good approximations which would also have been simple from the computer's standpoint. Among those which we have tried, the following tetrachoric function is worth noting:

$$\phi(v) \cong (c_1 + 2c_2 v)\, e^{-c_0 - c_1 v - c_2 v^2}.$$

This should yield a simple result because it can be integrated.

Applying this method to (xv), we have

$$S_{t+1}(p) \cong \frac{n!}{(s-1)!\,t!}\,\frac{p^{s-1}q^t}{c_1}\,(1 - e^{-c_1 p - c_2 p^2}) \cong \frac{n!}{(s-1)!\,t!}\,\frac{p^{s-1}q^t}{c_1},$$

usually; and

$$H = \frac{t}{q} - \frac{s-1}{p},\quad 2K = \frac{t}{q^2} + \frac{s-1}{p^2},\quad \alpha > 0,\quad 2K\alpha^2 + H\alpha - 2 = 0,\quad c_1 = \frac{1}{\alpha} - H,\quad c_2 = \frac{c_1}{2\alpha}$$

...(xvii).

Example 8. Solve Example 4 (*b*) by this method. True value 0·02644; by (xvii) 0·02663, not as good a result as that obtained by (xvi), and the computation is somewhat more troublesome.

I have to thank Professor K. Pearson for aid at various points, and in particular for communicating to me his unpublished theorem that the sum of the first p terms of a binomial is expressible in terms of the Incomplete B-function.

ON A CERTAIN DOUBLE HYPERGEOMETRICAL SERIES AND ITS REPRESENTATION BY CONTINUOUS FREQUENCY SURFACES.

By KARL PEARSON, F.R.S.

(1) In previous papers* I have given the chief statistical constants of a simple hypergeometrical series, namely the mean ν_1', the standard deviation σ, the moment coefficients about the mean μ_2, μ_3, μ_4, μ_5, and the corresponding moment ratios $\beta_1 = \mu_3^2/\mu_2^3$, and $\beta_2 = \mu_4/\mu_2^2$. I have long been convinced of the importance of the *double* hypergeometrical series for describing bivariate skew frequency, but the difficulty of integrating the differential equations to the corresponding skew surface has for many years impeded progress.

In my paper on Skew Variation of 1894† I gave the contour curves of the solid histogram which arises from the correlation between the numbers of a given suit, which two players of whist can hold, and I expressed the opinion, that such a surface could not be adequately described by a Gaussian bi-variate frequency surface. This opinion was, I think, correct, but I was possibly too dogmatic in the view that these ovals could not possibly be expressed *approximately* by ellipses. In fact it was a doubt as to this possibility which led me to the non-skew bi-variate frequency surface discussed in the last volume of *Biometrika*. In the course of the present paper I shall show that a certain double hypergeometrical series of some statistical importance satisfies some of the chief conditions for the application of the non-skew surface, and approximately some of the remaining conditions; those conditions can, perhaps, not be wholly satisfied owing to the discreteness, or non-continuity, of the double hypergeometrical series.

Let us consider first the following problem: A limited population of size N contains m marked individuals; a sample of size n is found to contain s of these marked individuals. Supposing a second sample of size n' to be extracted, the first sample not being replaced, what are the probabilities of 0, 1, 2 ... s' ... n' (or $m - s$ if $n' \geqq m - s$) marked individuals occurring in n'? If we make $N = 4m = 4n = 4n' = 52$, we reduce the problem to that of the numbers of cards of the same suit in two players' hands at whist‡.

* *Phil. Mag.* Feb. 1899, p. 239; *Biometrika*, 1906, Vol. v. p. 174.

† *Phil. Trans.* Vol. 186 (1895), A, p. 411 and Plate 9.

‡ In 1892 I collected the records of 25,000 deals at whist, the cards being actually played. It was found on comparison with theory that the shuffling was not "perfect" and there was a quite significant preponderance of average hands, due to the segregation of cards of the same suit in tricks, see Table III. A sample of 3400 deals of non-played hands was in good accordance with theory, Table IV. It may be of some interest to the reader, who is a whist player, if I place these results at the end of this paper.

It is clear that if we use s and s' as variates, we obtain for the probability that the doublet (s, s') will arise a double or solid histogram, the briquettes of which approximate to a frequency surface. We seek first our marginal totals. The chances of drawing 0, 1, 2 ... s ... marked individuals are given by the successive terms of the series

$$\frac{N-m}{N}\frac{N-m-1}{N-1}\frac{N-m-2}{N-2}\cdots\cdots\frac{N-m-n+1}{N-n+1}$$
$$\times\left(1+n\frac{m}{N-m-n+1}+\frac{n(n-1)}{1.2}\frac{m(m-1)}{(N-m-n+1)(N-m-n+2)}\cdots\right.$$
$$\left.+\frac{\lfloor n}{\lfloor s\,\lfloor n-s}\frac{m(m-1)\ldots\ldots(m-s+1)}{(N-m-n+1)(N-m-n+2)\ldots(N-m-n+s)}+\ldots\right)$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)},$$

the series running out either on the n or the m factor. A little consideration will show that the marginal total for the second sample will be precisely of the same form except that n' replaces n and s' replaces s. The mean of the first sample is nm/N and of the second $n'm/N$ marked individuals. Let us write $q=m/N$, $p=(N-m)/N$. Then the moment coefficients are*

$$\mu_2=\sigma_1^2=npq\left(1-\frac{n-1}{N-1}\right),\quad \mu_3=npq\,(p-q)\left(1-\frac{n-1}{N}\right)\left(1-\frac{2\,(n-1)}{N-2}\right),$$
$$\mu_4=npq\left(1-\frac{n-1}{N-1}\right)\left\{1-\frac{6\,(n-1)}{N-2}\left(1-\frac{n-2}{N-3}\right)\right.$$
$$\left.+3pq\,(n-2)\left(1-\frac{n-1}{N-2}\left[\frac{n-10}{n-2}+\frac{9}{N-3}\right]\right)\right\}\ \ldots\ldots\ldots\ldots\text{(ii)},$$

for the first sample, and like values with an interchange of n with n' for the second sample.

Should the first and second samples be of the same size (or $n'=n$), then the three moment coefficients are the same for the two samples, and β_1 and β_2 will be the same. This occurs in the whist illustration to which we have previously referred.

We have for the first sample:

$$\beta_1=\frac{p-q}{npq}\frac{\left(1-\frac{2\,(n-1)}{N-2}\right)^2}{1-\frac{n-1}{N-1}}\ \ldots\ldots\ldots\ldots\ldots\text{(iii)},$$

and for the second:

$$\beta_1'=\frac{p-q}{n'pq}\frac{\left(1-\frac{2\,(n'-1)}{N-2}\right)^2}{1-\frac{n'-1}{N-1}}\ \ldots\ldots\ldots\ldots\ldots\text{(iv)}.$$

It follows therefore if neither p nor q be small, and N fairly large as compared with $n+n'$, that β_1 and β_1' will vary inversely with n and n', and accordingly for fair sized samples β_1 and β_1' will be small.

* *Biometrika*, Vol. v. p. 174.

In the case of whist, where $q=\frac{1}{4}$, $p=\frac{3}{4}$, $n=13$, $N=52$, we find:

$$\mu_2 = 1{\cdot}863{,}9706, \quad \mu_3 = {\cdot}484{,}63235, \quad \mu_4 = 10{\cdot}052{,}4125,$$

leading to:

$$\beta_1 = \beta_1' = {\cdot}036{,}2667, \quad \beta_2 = \beta_2' = 2{\cdot}893{,}2915.$$

Accordingly even for $n=13$, the value of β_1 is in this case small, while the deviation of β_2 from 3 is considerably greater. It is conceivable that the general effect on the contour lines of putting $\beta_1=0$ might not be so considerable as supposing $\beta_2=3$.

We now proceed to consider the array of second samples corresponding to a definite number s of marked individuals occurring in the first sample.

The population now consists of $N-n$ individuals of whom $m-s$ are marked. We have then for probabilities:

$$q_s = \frac{m-s}{N-n}, \qquad p_s = \frac{N-n-m+s}{N-n},$$

and accordingly the means of the arrays are to be found from

$$\bar{y}_s = {}_s\nu_1' = n' \frac{m-s}{N-n} \quad \text{.....(v).}$$

Similarly the means the other way are given by:

$$\bar{x}_{s'} = {}_{s'}\nu_1' = n \frac{m-s'}{N-n'} \quad \text{.....(vi).}$$

Both these equations are *linear*, or the system has double linear regression. The regression coefficients are $-\frac{n'}{N-n}$ and $-\frac{n}{N-n'}$ respectively, or

$$r^2_{ss'} = \frac{nn'}{(N-n)(N-n')},$$

which gives us since the correlation is *negative**

$$r_{ss'} = -\sqrt{\frac{nn'}{(N-n)(N-n')}} \quad \text{.....(vii).}$$

This result may be of use in the problem of first and second samples from a limited population.

Since we are taking a sample of size n' from a limited population of $N-n$, the variance $\sigma^2_{s'.s}$ will be given by:

$$\sigma^2_{s'.s} = n' p_s q_s \left(1 - \frac{n'-1}{N-n-1}\right)$$

$$= n' \frac{m-s}{N-n} \frac{N-n-(m-s)}{N-n} \left(\frac{N-n-n'}{N-n-1}\right) \quad \text{.....(viii).}$$

* In the case of whist, the correlation between the number of cards of the same suit in two hands is $-\frac{1}{3}$ and the mean number in partner's hand is $\frac{1}{3}(13-s)$, or knowing your own s, it is easy to predict your partner's probable cards. Many persons have played whist all their lives without knowing this simple rule, i.e. subtract your number of one suit from 13 and divide by three and this obviously represents the likely number of the same suit in anybody else's hand.

This may be written

$$\sigma^2_{s'.s} = n' \frac{N-n-n'}{N-n-1} \left\{ \tfrac{1}{4} - \left(\frac{m-s}{N-n} - \tfrac{1}{2} \right)^2 \right\} \quad \text{.....(ix).}$$

Similarly:

$$\sigma^2_{s.s'} = n \frac{N-n-n'}{N-n'-1} \left\{ \tfrac{1}{4} - \left(\frac{m-s'}{N-n'} - \tfrac{1}{2} \right)^2 \right\} \quad \text{.....(x).}$$

Thus $\sigma_{s'.s}$ and $\sigma_{s.s'}$ are given by portions of ellipses, or the variances by parabolae. The scedasticity both ways is accordingly parabolic.

The actual distribution of the terms of the array is:

$$\frac{N-n-(m-s)}{N-n} \frac{N-n-1-(m-s)}{N-n-1} \frac{N-n-2-(m-s)}{N-n-2} \cdots \frac{N-n-n'+1-(m-s)}{N-n-n'+1}$$
$$\times \left\{ 1 + n' \frac{m-s}{N-n-n'+1-(m-s)} \right.$$
$$+ \frac{n'(n'-1)}{1.2} \frac{(m-s)(m-s-1)}{(N-n-n'+1-(m-s))(N-n-n'+2-(m-s))} + \cdots + \frac{\lfloor n'}{\lfloor s \, \lfloor n'-s'}$$
$$\times \frac{(m-s)(m-s-1) \ldots\ldots (m-s-s'+1)}{(N-n-n'+1-(m-s))(N-n-n'+2-(m-s)) \ldots (N-n-n'+s'-(m-s))}$$
$$\left. + \ldots\ldots \right\} \text{.....(xi).}$$

Of course to obtain the general frequency of the doublet (s, s'), we must multiply the above result by the chance of s occurring in the first sample of n, or the total chance $z_{s,s'}$ will be given by:

$$z_{s,s'} = \frac{\lfloor n \, \lfloor n'}{\lfloor s \, \lfloor n-s \, \lfloor s' \, \lfloor n'-s'} \frac{\lfloor N-n-n'}{\lfloor N} \frac{\lfloor m}{\lfloor m-s-s'} \frac{\lfloor N-m}{\lfloor N-m-n-n'+s+s'}$$
$$\text{.....(xii).}$$

In the particular case of whist, $N = 52$, $m = n = n' = 13$, and the chance of s and s' cards of the same suit in two players' hands is given by*:

$$z_{s,s'} = \frac{(13\,!)^2}{\lfloor s \, \lfloor 13-s \, \lfloor s' \, \lfloor 13-s'} \frac{\lfloor 26}{\lfloor 52} \frac{\lfloor 13}{\lfloor 13-s-s'} \frac{\lfloor 39}{\lfloor 13+s+s'} \text{.....(xiii).}$$

Here the double hypergeometrical series is limited by the three lines $s = 0$, $s' = 0$, $s + s' = 13$, and in this triangle all the frequency should lie. We should not expect that any continuous surface fitted to this distribution would accurately obey this condition but very little of the volume ought to lie outside the contour which just touches these lines. Given s cards of one suit in one player's hand the distribution of cards in a second player's hand follows the series†

$$\frac{(26+s)(25+s)(24+s) \ldots (14+s)}{39 \;.\; 38 \;.\; 37 \;\ldots\; 27} \left\{ 1 + 13 \frac{13-s}{14+s} + \frac{13.12}{1.2} \frac{(13-s)(12-s)}{(14+s)(15+s)} \right.$$
$$\left. + \frac{13.12.11}{1.2.3} \frac{(13-s)(12-s)(11-s)}{(14+s)(15+s)(16+s)} + \text{etc.} \right\} \quad \text{.....(xiv).}$$

* The theoretical values in our Table III are calculated from this formula.

† The Whist Player's Companion, Table V, is derived from this series.

(2) It will be clear that a double hypergeometrical series of the type considered belongs to a class of frequency distribution in which both regressions are linear, and both scedasticities parabolic. Professor Narumi has shown on certain assumptions that only one type of frequency surface satisfies these conditions. It is the non-skew frequency surface discussed on pp. 231—244 of the preceding volume. It seems probable therefore that we cannot have a *continuous* frequency surface, for which β_1 and β_1' differ from zero and for which β_2 does not equal β_2', which combines double linear regression with double parabolic scedasticity. It is obvious however that this is possible in the case of a *discontinuum* like our double hypergeometrical series.

We can, however, endeavour to fit our non-skew surface to data in which $\beta_2 = \beta_2'$, for example when the sizes of the successive samples n and n' are equal, and when β_1, β_1' are so small as to be negligible, say under ·05. In such cases our surface will satisfy precisely the fundamental conditions requisite for the double hypergeometrical series. We can illustrate the degree of closeness of fit of this surface to the double hypergeometrical series in the case of the whist data. The form of the surface, with the mean as origin, is*

$$z = \frac{N}{2\pi\sigma_1\sigma_2}\frac{1}{\sqrt{1-r^2}}\frac{n-1}{n-2} \times \frac{1}{\left\{1 + \frac{1}{2(n-2)}\frac{1}{1-r^2}\left(\frac{x^2}{\sigma_1^2} - \frac{2rxy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2}\right)\right\}^n},$$

where
$$n = \frac{3(\beta_2 - 2)}{\beta_2 - 3} \quad \text{.....(xv).}$$

In the case of the whist hands:

$$\beta_1 = \beta_1' = \cdot 036{,}2667 \text{ and will be neglected,}$$
$$\beta_2 = \beta_2' = 2\cdot 893{,}2914,$$
$$r = -\tfrac{1}{3}, \quad \sigma_1^2 = \sigma_2^2 = 1\cdot 863{,}9706.$$

Hence we find $n = -25\cdot 113{,}974$ and the equation to the surface for a population of 100,000 deals becomes:

$$z = 8722\cdot 423\,\{1 - \cdot 0111{,}29875\,(x^2 + \tfrac{2}{3}xy + y^2)\}^{25\cdot 113{,}974} \quad \text{.....(xvi).}$$

If we suppose the axes turned through 45°, we find

$$z = 8722\cdot 423\left\{1 - \frac{X^2}{(8\cdot 207{,}1595)^2} - \frac{Y^2}{(11\cdot 609{,}1517)^2}\right\}^{25\cdot 113{,}974}.$$

This shows that the surface is limited by the ellipse

$$\frac{X^2}{(8\cdot 207{,}1595)^2} + \frac{Y^2}{(11\cdot 609{,}1517)^2} = 1,$$

and not by the three lines (origin at zero cards of the suit), $x' = 0$, $y' = 0$, $x' + y' = 13$, as the *discontinuum*.

* n is here a mere symbol for a power, and not to be confused with the n of the first sample. See *Biometrika*, Vol. xv. p. 232.

It is clear that the contour lines of the surface are a system of ellipses concentric with the above and similar to it, their equation being

$$1 = \frac{X^2}{(\chi \times 8{\cdot}20716)^2} + \frac{Y^2}{(\chi \times 11{\cdot}60915)^2},$$

with χ as parameter. The centre is on the bisector of the angle and at the point which is the mean of the whole distribution, i.e. at 3·25 cards in both hands, or at a distance along this bisector of $3{\cdot}25\sqrt{2}$ or 4·59619 from zero cards.

The contour line which touches the card axes is *

$$1 = \frac{X^2}{(2{\cdot}6530)^2} + \frac{Y^2}{(3{\cdot}7528)^2},$$

which corresponds to $\chi = {\cdot}323{,}2602$.

The volume outside this contour is found by Equation (xiv) of the paper previously cited to be

$$N \times {\cdot}056011,$$

or about 5·6 of the frequency lies outside the touching contour. Not all this frequency, however, is irrelevant, but only that between the bounding ellipse and the lines $x = -3{\cdot}75$ and $y = -3{\cdot}75$† below the card axes. The volume of this portion was kindly found for me by Mr J. O. Irwin, who reducing the integral from a double to a single one then determined the value of the latter by quadratures. It is only ·005N or a half per cent. This indicates that the extension of the surface beyond the bounding lines of the discontinuum produces no serious statistical error.

I now proceeded to calculate the contour lines of the surface for z values equal to 8000, 7000, 6000, 5000, 4000, 3000, 2000, 1000, 500, 100, 10, 1, ·001 and 0. The semi-axes, the minor along the bisector X and the major perpendicular to it, are for

$z=$	8000	7000	6000	5000	4000	3000	2000	1000
Minor	·4811	·7664	·9980	1·2149	1·4349	1·6739	1·9587	2·3572
Major	·6806	1·0842	1·4117	1·7185	2·0296	2·3678	3·7706	3·3371

$z=$	545·567	500	100	10	1	·001	0
Minor	2·6530	2·6922	3·3135	3·9898	4·5194	5·6312	8·2072
Major	3·7528	3·8081	4·6870	5·6436	6·3927	7·9655	11·6092

These ellipses were drawn and we thus reached the contour lines of the non-skew surface. The contour lines of the whist double hypergeometrical series were drawn

* It corresponds to $z = 545{\cdot}567$.

† The trapezette which contains the frequencies corresponding to zero stretches from $-{\cdot}5$ to $+{\cdot}5$ or to $-3{\cdot}25 - {\cdot}5$ from origin, i.e. $-3{\cdot}75$.

about thirty years ago and published in 1894. In a somewhat primitive manner by mere *linear* interpolation, the contours were obtained from the briquette values without any attempt at smoothing. In redoing them now, I have adopted a more elaborate procedure. Four series of sections of the whist solid histogram were taken: (i) the vertical series through 0, 1, 2, ... 13, (ii) the similar horizontal series, (iii) the diagonal series parallel to the axis of symmetry, (iv) the diagonal series perpendicular to

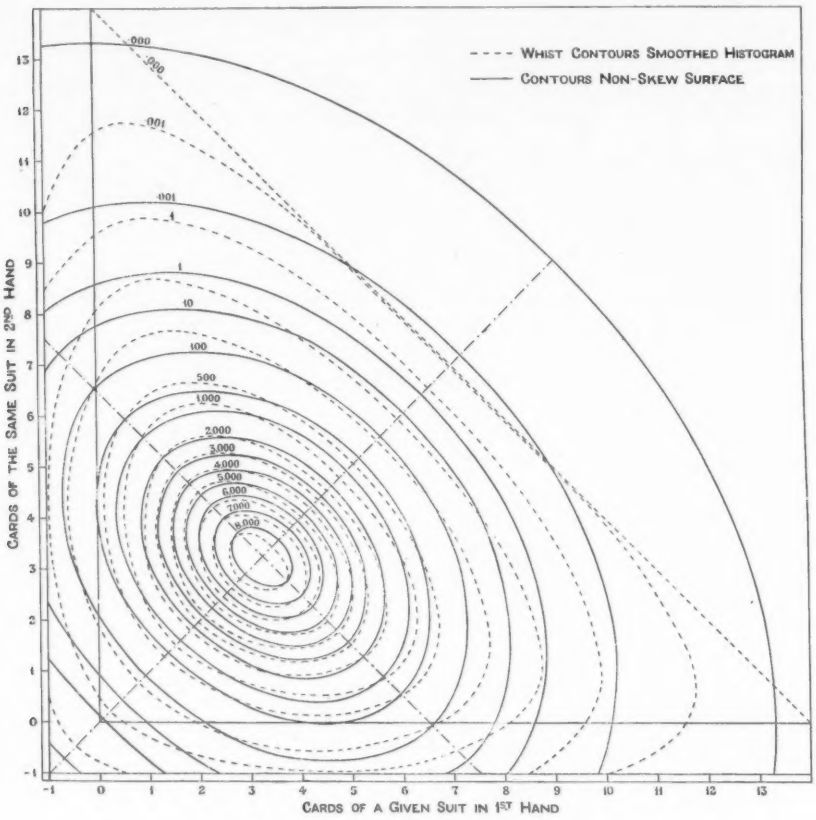

Fig. 1. ACTUAL AND THEORETICAL LINES.
Illustrating the divergence of the Non-skew Correlation Surface from the contour lines of the Whist Double Hypergeometrical Series.

the axis of symmetry. The discontinuum thus gave four series of discontinuous points for each section and these were splined up to form continuous curves. Each of the four series of curves was then cut by a system of lines at the several contour heights. This gave the coordinates of a number of points on each contour. But these points did not form a continuous curve. This is due to two sources,

(i) there are no actual contours to a discontinuum, we cannot expect the four series therefore to give smooth results in full agreement, (ii) there is a considerable personal judgment involved in splining a curve through a system of by no means very close points. Owing to one or other or both these sources a system was obtained such as is represented in Fig. 2 for the contour 2000. This system was replaced by a smooth curve, supposed to stand for the contour of the *discontinuum*.

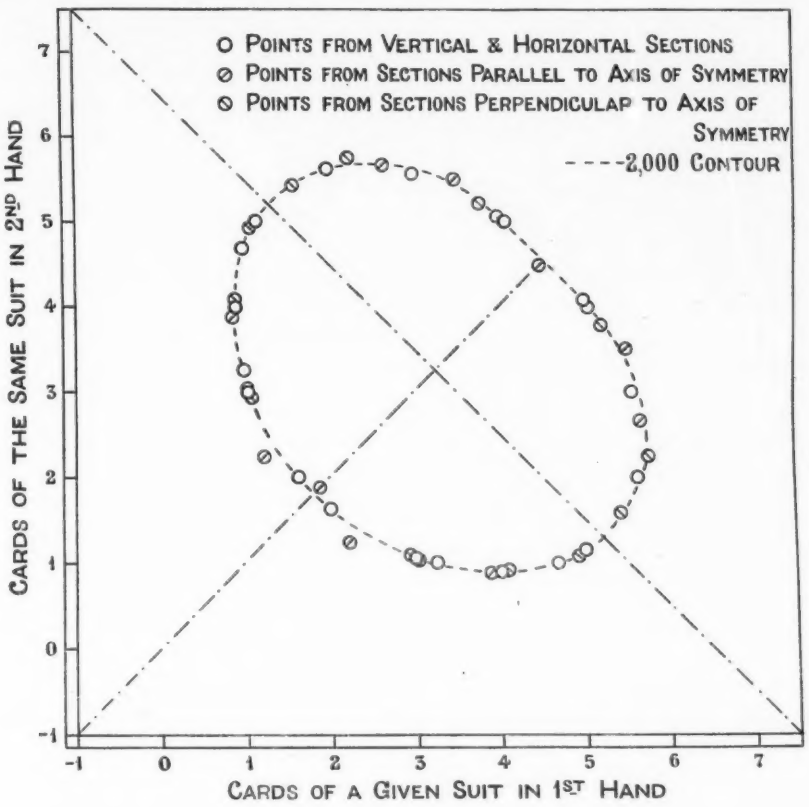

Fig. 2. SAMPLE OF CONTOUR LINES.

Illustrating the manner in which a contour line was obtained from the Discrete Double Hypergeometrical Series.

It is these contours which in Fig. 1 are represented by broken lines and superposed on them are the ellipses of the non-skew frequency surface. Allowing for the fact that the contour lines beyond 1 correspond to a very small fraction of the surface we see that there is a considerable approximation of the contours of the

non-skew surface to the whist contours, but the fit is by no means ideal. The whist contours diverge from the ellipse by a flattening of the contours above the mean and an extension of them below along the axis of symmetry; the whist contours are more "cancrine"—crablike—in form than truly elliptic. Undoubtedly such ellipses would describe more adequately the correlation of the numbers of marked individuals in two successive samples of the same size drawn from a limited population, if the size of the samples and of the sampled population were more considerable than is the case in hands at whist.

We have to remember that our surface gives the true regression lines, the true correlation, and a parabolic scedasticity, if not the true one, of the actual material. What it fails to do, while providing the exact β_2, is to give the skewness characterised by $\beta_1 = \cdot 036{,}2667$. It is difficult to be sure that the divergence is not due to our endeavouring to replace a discontinuum by a surface, especially if we accept Narumi's proof of the limitation of linear regression and parabolic variance to my non-skew surface. At the same time it is possible that some other surface—for example the Filon-Isserlis, which would give the exact β_1—may provide a nearer approximation to the actual whist contours, while failing to provide β_2 or the true type of scedasticity.

We shall now proceed to make a comparison between the Filon-Isserlis Surface and the Non-skew Surface.

(3) *The Filon-Isserlis Surface.* It must be noted first how the methods of fitting the Non-skew Surface and the Filon-Isserlis Surface differ. In the Non-skew Surface we put $\beta_1 = 0$ and fit by the mean, the correlation $(-\frac{1}{3})$ and β_2. In the Filon-Isserlis Surface we disregard β_2 altogether and fit by $\beta_1 = \cdot 036{,}2667$, by the mean and the correlation $(-\frac{1}{3})$. Accordingly the β_2 will not agree with the observed value. Dr Isserlis very kindly calculated the appropriate surface and found for its equation

$$z = z_0 \left(1 + \frac{x}{9{\cdot}426{,}0683}\right)^{35{\cdot}012{,}9824} \left(1 + \frac{y}{9{\cdot}426{,}0683}\right)^{35{\cdot}012{,}9824} \times \left(1 - \frac{x+y}{19{\cdot}121{,}3529}\right)^{71{\cdot}025{,}9647} \quad \text{.....(xvii),}$$

where $z_0 = 8978{\cdot}98$, say $8979{\cdot}0$,

and the origin is at the mode, or 3·1827 cards. The ordinates of this surface were calculated by Miss Stoessiger, the four series of sections as for the double hypergeometrical series were plotted from these ordinates, and Miss McLearn then determined as in the previous case the contour lines. These are shown in Fig. 3. The contours are certainly more cancrine than the ellipses of the Non-skew Surface, but it would be difficult to assert from these contours that the Filon-Isserlis Surface is markedly superior to the Non-skew Surface; both are clearly not very satisfactory. We are compelled therefore to consider other characteristics of the two surfaces.

We shall first throw the surface from the Isserlis into the Filon form. This is done by writing:

$$x' = 9{\cdot}426{,}0683 + x, \quad y' = 9{\cdot}426{,}0683 + y,$$

whence we find:

$$z = z_0' \left(\frac{x'}{37{\cdot}973{,}4895}\right)^{35{\cdot}012{,}9824} \left(\frac{y'}{37{\cdot}973{,}4895}\right)^{35{\cdot}012{,}9824} \times \left(1 - \frac{x'+y'}{37{\cdot}973{,}4895}\right)^{71{\cdot}025{,}9647} \quad \text{.....(xviii).}$$

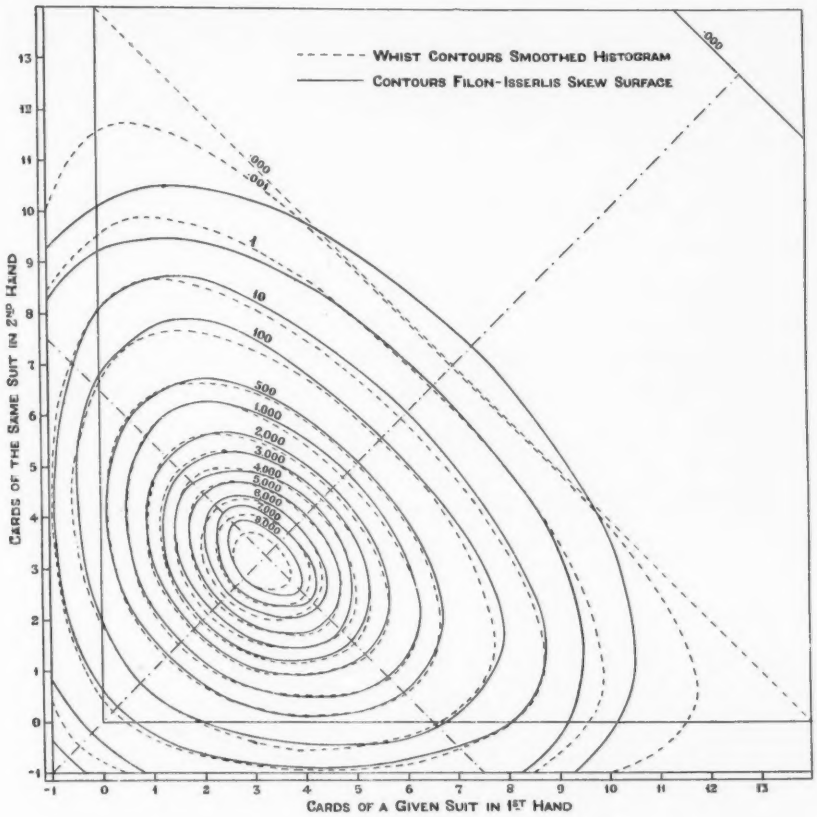

Fig. 3.

In other words the b of the Filon form is 37·973,4895. We are now therefore in a position to use the results in the "Notes on Skew Frequency Surfaces"*.

* *Biometrika*, Vol. xv. pp. 222—230.

Equation (xii) of that paper gives the scedasticity:

$$\sigma_{x_y} = (b - y') \frac{\sqrt{(p_1+1)(q+1)}}{(p_1+q+2)\sqrt{p_1+q+3}} \quad \text{.....(xix)}$$

$$= (37{\cdot}973{,}4895 - y') \times {\cdot}045{,}1443.$$

The origin of y' here is 9·426,0683 from the mode which is 3·182,6959 from the zero of any suit. Hence

$$\sigma_{x_y} = 1{\cdot}288{,}753 - {\cdot}045{,}1443\, y \quad \text{.....(xx)},$$

y being the number of cards in one hand. The scedasticity is accordingly linear, and the values of it are calculated from this formula and given in the third column of Table I (p. 184).

The regression line is given by Equation (viii) of the same paper, i.e.

$$\bar{x}_y' = (b - y') \frac{p_1+1}{p_1+q+2} \quad \text{.....(xxi)}$$

$$= (37{\cdot}973{,}4895 - y') \times {\cdot}333{,}3333,$$

or transferring to the origin of zero cards:

$$\bar{x}_y + 6{\cdot}243{,}3724 = \tfrac{1}{3}\,(31{\cdot}730{,}1171 - y),$$

or,
$$\bar{x}_y = \tfrac{1}{3}\,(13 - y) \quad \text{.....(xxii)},$$

is the equation to the regression line, as it should be.

Combining Equations (ii) and (iii) of the same article we find for the marginal total for x:

$$z_x = \frac{N}{b} \frac{\Gamma(p_1+p_2+q+3)}{\Gamma(p_1+1)\,\Gamma(p_2+q+2)} \left(\frac{x'}{b}\right)^{p_1} \left(1 - \frac{x'}{b}\right)^{p_2+q+1} \quad \text{.....(xxiii)}.$$

Substituting the numerical values we find for the desired equation*

$$z_x = 29150{\cdot}101 \left(\frac{x'}{37{\cdot}973{,}4895}\right)^{35{\cdot}012{,}9824} \left(1 - \frac{x'}{37{\cdot}973{,}4895}\right)^{107{\cdot}038{,}9471} \quad \text{...(xxiv)}.$$

The mode of this marginal total is given by

$$b \frac{p_1}{p_1+p_2+q+1} = 9{\cdot}359{,}7118,$$

while the mode of the whole surface is at

$$b \frac{p_1}{p_1+p_2+q} = 9{\cdot}426{,}0683;$$

or, reducing to the zero of cards, at 3·116,3394 and 3·182,6959 respectively. The means of the whole surface and of the marginal total curve are the same, i.e. 3·250,0000. Thus the mode of the surface is almost midway between the mode and the mean of the marginal total curve.

* *Tracts for Computers*, No. VIII, containing the logarithms of the Γ-functions from 2 to 1200, will be found useful for such calculations.

It is a point of some interest to note that the mode of a skew surface will not generally coincide with the modes of the two marginal totals, while its mean will coincide with their means. The equation to the line of modes is given as (xi) in the article referred to, i.e.

$$x'_{m_1 y} = \frac{p_1}{p_1 + q}(b - y') \quad \text{.....(xxv),}$$

or, making the numerical substitutions and referring to the zero of cards as origin, we find

$$x_{m_1 y} = \cdot 3301{,}8983\ (12\cdot 821{,}684 - y).$$

Before discussing these results further we may consider the corresponding quantities for the non-skew surface, already discussed in this paper. This surface being symmetrical there is no distinction between the line of means and that of modes, and the regression line is as before, the origin here being zero cards:

$$\bar{x}_y = \tfrac{1}{3}(13 - y).$$

The scedasticity is given*, however, by

$$\sigma_{x_y}{}^2 = \sigma_1{}^2(1 - r^2)\left\{1 + \frac{1}{2n+3}\left(1 - \frac{y^2}{\sigma_2{}^2}\right)\right\} \quad \text{.....(xxvi),}$$

where the origin is at the card mean 3·2500 and

$$n = 3\,(\beta_2 - 2)/(3 - \beta_2) = 25\cdot 113{,}974, \quad \sigma_1{}^2 = \sigma_2{}^2 = 1\cdot 863{,}9706.$$

Substituting we find:

$$\sigma_{x_y}{}^2 = 1\cdot 687{,}9853 - \cdot 0163{,}8869\, y^2,$$

or referred to the zero of cards:

$$\sigma_{x_y}{}^2 = 1\cdot 687{,}9853 - \cdot 0163{,}8869\,(y - 3\cdot 25)^2 \quad \text{.....(xxvii).}$$

The values of σ_{x_y} are given in the fifth column of Table I (p. 184) for each value of y.

The equation to the marginal total is†

$$z_x = \frac{N}{\sqrt{2\pi}\,\sigma_1}\,\frac{1}{\sqrt{n+2}}\,\frac{\Gamma(n+2)}{\Gamma(n+1\cdot 5)}\left(1 - \frac{x^2}{2\,(n+2)\,\sigma_1{}^2}\right)^{n+\frac{1}{2}} \quad \text{.....(xxviii),}$$

the origin of x being at the mean 3·25 cards. Taking as before $N = 100{,}000$ deals we find on substitution, and using zero cards for origin‡:

$$z_x = 28812\cdot 1928\left(1 - \frac{(x - 3\cdot 25)^2}{101\cdot 079{,}3028}\right)^{25\cdot 613{,}974} \quad \text{.....(xxix).}$$

We are now in a position to compare the skew and non-skew surfaces under two further aspects, (i) the standard deviations of the arrays, and (ii) the distribution of the marginal totals.

(i) *Standard Deviations of Arrays.* Table I provides the values, actual values from Equation (ix) and the values for the skew and non-skew surfaces as given by

* See "On Non-skew Frequency Surfaces," *Biometrika*, Vol. xv. pp. 231—244.

† *Biometrika*, Vol. xv. p. 240, Equation (xxxi).

‡ The actual modal frequency at 3 cards is 28633.

Equations (xx) and (xxvii). It is well during this comparison for the reader to bear in mind the following constants:

Constants	Actual Data	Skew Surface	Non-Skew Surface
$\sigma_1=\sigma_2$	1·365,273	1·365,273	1·365,273
$\beta_1=\beta_1'$	·036,2667	·036,2667	·000,000
$\beta_2=\beta_2'$	2·893,2915	3·013,2282*	2·893,2915

TABLE I. *Standard Deviations of Arrays.*

Cards in First Hand	Variability of same suit in Second Hand				
	Actual	Skew Surface	Δ	Non-Skew-Surface	Δ
0	1·4059	1·4324	+·0265	1·2308	−·1751
1	1·3765	1·3873	+·0108	1·2669	−·1096
2	1·3421	1·3421	·0000	1·2893	−·0528
3	1·3023	1·2970	−·0053	1·2988	−·0035
4	1·2566	1·2518	−·0048	1·2957	+·0391
5	1·2043	1·2067	+·0024	1·2798	+·0755
6	1·1445	1·1616	+·0171	1·2506	+·1061
7	1·0761	1·1164	+·0403	1·2073	+·1312
8	·9971	1·0713	+·0742	1·1481	+·1510
9	·9048	1·0261	+·1213	1·0706	+·1658
10	·7947	·9810	+·1863	·9706	+·1759
11	·6578	·9358	+·2780	·8388	+·1810
12	·4652	·8907	+·4255	·6582	+·1930
13	·0000	·8456	+·8456	·3606	+·3606
.....	.....	.....	.....	.....	.....
13·3987	—	·8276	—	·0000	—
Square Root Mean Square Deviation from Actual	—	$S(\Delta^2/14)=$·0676 ..... ·2600		$S(\Delta^2/14)=$·0259 ..... ·1609	

As the distributions in all the arrays have their theoretical and not observational values, they have not been weighted. While the skew surface is distinctly better than the non-skew surface for small first hands, it is very much worse for many cards of the same suit in the first hand, and the square root mean square deviation from the actual is finally better for the non-skew surface than the skew surface; the parabolic scedasticity of the latter, however, is not that of the actual data. It is clear that the skew surface would give the better result, if we confined our attention to the small runs of one suit, which, of course, are on the whole more frequent. Neither surface comes out of this test very satisfactorily. While the variation of the array of cards in the second hand actually vanishes for 13 cards in the first hand it does not vanish till about 13·4 cards in the non-skew surface,

* Obtained from the equation (*Biometrika*, Vol. xv. p. 227)

$$6(\beta_2-\beta_1-1)/(3\beta_1-2\beta_2-6)=p_1+p_2+q+3$$

by remembering that β_1 has the value ·036,2667.

while in the skew surface, when it ought to be zero, it is still $\frac{1}{32}$ about of its maximum value. In fact while the skew surface fails to give curved scedasticity by its straight line, the non-skew surface fails, because its parabola is not the right one.

(ii) *Distribution of the Marginal Totals.*

The labour of determining the actual goodness of fit for the whole of these surfaces being excessive, and the contour lines giving us no clear cut discrimination between the two, we have confined our attention to the marginal totals. The following table gives the actual results to the nearest unit in 100,000 deals, and those obtained from plotting the curves in Equations (xxiv) and (xxix) and integrating them by aid of a planimeter.

TABLE II. *Distribution of Marginal Totals.*

Cards in First Hand	Frequency of Cards as given by Surfaces		
	Actual	Skew Surface	Non-Skew Surface
0	1279·1	1680	2166
1	8006·2	7930	7948
2	20587·3	20458	19225
3	28632·9	28369	27827
4	23860·8	23664	24479
5	12469·2	12355	13282
6	4156·4	4332	4231
7	881·7	1013	762
8	116·7	173	80
9	9·3	27	?
10	0·4	?	?
11	[·0009]	?	?
12	[·0000]	?	?
13	[·0000]	?	?
χ^2	—	1·5513	5·7754
P	—	·98	·56

1008·1 | 1213 | 842

Here to get comparative values of P, we base χ^2 on 1000, not 100,000 deals. Of course what number we choose is entirely arbitrary as all the results are theoretical. Both curves give bad results at the tails, but there is no doubt that the skew surface does give the marginal frequencies better than the non-skew surface. In other words β_1, even when it is relatively small, is of more importance than β_2 for marginal frequency.

But taken all round as far as these data are concerned neither surface is really adequate and this, I think, is not solely because we are replacing a discrete series by a continuous surface. I do not believe (as I have said before) that we shall obtain really satisfactory bi-variate frequency surfaces until we have enough constants at our disposal to provide for σ_x, σ_y, r_{xy}, ${}_x\beta_1$, ${}_x\beta_2$, ${}_y\beta_1$ and ${}_y\beta_2$. To determine such a surface, if only empirically, is the most urgent task before mathematical statisticians at the present time. All we can say as a result of the present enquiry is that to neglect either value, β_1 or β_2, does not really lead to good approximations.

TABLE III.

Whist Correlation.

Same suit in Two Hands.

Theory Heavy Type. Experiment Light Type. 25,000 returns.

First Hand.

Second Hand.

No.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
0	0 **0**	3 **5**	22 **24**	38 **60**	76 **88**	65 **79**	26 **44**	12 **16**	4 **3**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
1	5 **5**	43 **51**	159 **212**	380 **457**	531 **572**	358 **433**	153 **202**	54 **58**	4 **10**	0 **1**	0 **0**	0 **0**	0 **0**	0 **0**
2	20 **24**	183 **212**	746 **748**	1458 **1372**	1590 **1444**	926 **910**	323 **347**	49 **79**	7 **10**	0 **1**	0 **0**	0 **0**	0 **0**	0 **0**
3	42 **60**	360 **457**	1451 **1372**	2297 **2118**	2059 **1853**	934 **953**	222 **289**	34 **50**	4 **5**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
4	54 **88**	542 **572**	1588 **1444**	2048 **1853**	1497 **1324**	505 **542**	101 **126**	14 **16**	1 **1**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
5	45 **79**	378 **433**	906 **910**	926 **953**	506 **542**	139 **170**	25 **28**	3 **2**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
6	34 **44**	166 **202**	312 **347**	255 **289**	96 **126**	21 **28**	2 **3**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
7	10 **16**	47 **58**	69 **79**	34 **50**	16 **16**	2 **2**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
8	5 **3**	2 **10**	9 **10**	4 **5**	0 **1**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
9	0 **0**	0 **1**	0 **1**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
10	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
11	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
12	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
13	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**

Experimental Excess and Defect limits enclosed by thick rule.

This comparison of theoretical and actually played hands shows how very imperfect even among habitual whist players shuffling is.

TABLE IV.

Whist Correlation, Perfect Shuffling.

Theory Heavy Type. Experiment Light Type. 3400 returns in permilles.

First Hand.

Second Hand. No.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
0	0 **0**	0 ·2	1·8 ·95	1·2 2·4	4·4 3·5	6·8 3·2	1·5 1·8	·6 ·6	·3 ·1	0 **0**	0 **0**	0 **0**	0 **0**	0 **0**
1	·3 ·2	1·8 2·1	8·8 8·5	16·5 18·3	23·5 22·9	17·4 17·3	5·88 3·1	·3 2·3	·9 ·4	0 **0**	0 **0**	0 **0**	0 **0**	
2	·6 ·95	9·1 8·5	34·1 29·9	54·1 54·9	62·6 57·8	39·4 36·4	13·5 13·9	3·5 3·2	1·2 ·4	0 **0**	0 **0**	0 **0**		
3	2·6 2·4	18·8 18·3	47·9 54·9	79·7 84·7	72·6 74·1	33·2 38·1	8·8 11·6	1·5 2	·3 ·2	0 **0**	0 **0**			
4	3·5 3·5	24·4 22·9	58·8 57·8	75·6 74·1	56·2 53	24·7 21·7	3·2 5	·3 ·6	·3 **0**	0 **0**				
5	1·5 3·2	18·2 17·3	37·1 36·4	36·2 38·1	25 21·7	7·6 6·8	·9 1·1	0 ·1	0 **0**					
6	1·2 1·8	8·5 8·1	13·5 13·9	10·6 11·6	5·3 5	1·5 1·1	0 ·1	0 **0**						
7	·6 ·6	3·3 2·3	2·6 3·2	1·8 2	·6 ·6	0 ·1	0 **0**							
8	0 ·1	·6 ·4	0 ·4	0 ·2	0 **0**	0 **0**								
9	0 **0**	·6 **0**	·3 **0**	0 **0**	0 **0**									
10	0 **0**	0 **0**	0 **0**	0 **0**										
11	0 **0**	0 **0**	0 **0**											
12	0 **0**	0 **0**												
13	0 **0**													

This Table indicates a reasonable accordance between theory and "perfect shuffling."

TABLE V.

Whist Player's Companion.

Constitution of Second Player's Hand per 100 deals.

First Player's Known Hand.	*0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
0	·13	1·55	7·42	18·70	27·50	24·75	13·90	4·86	1·04	·13	·01	·00	·00	·00
1	·25	2·57	10·59	22·85	28·56	21·65	10·10	2·89	·49	·05	·00	·00	·00	0
2	·46	4·12	14·54	26·66	28·06	17·68	6·73	1·53	·20	·01	·00	·00	0	
3	·84	6·39	19·17	29·59	25·89	13·32	4·04	·70	·07	·00	·00	0		
4	1·47	9·58	24·21	31·09	22·19	9·08	2·11	·26	·02	·00	0			
5	2·54	13·90	29·19	30·58	17·37	5·44	·91	·07	·00	0				
6	4·28	19·46	33·35	27·86	12·09	2·72	·29	·01	0					
7	7·06	26·21	35·74	27·79	7·12	1·03	·05	0						
8	11·42	33·76	35·22	16·14	3·23	·22	0							
9	18·18	41·09	30·82	9·04	·87	0								
10	28·45	46·23	22·19	3·13	0									
11	43·86	45·61	10·53	0										
12	66·67	33·33	0											
13	100	0												

Examples of Use. (i) Dealer has 5 trumps, what is the chance that his partner will have 4? Number in row 5 and column 4 is 17·37. Hence it will occur about 17 times in 100 deals and the odds against it are 83 to 17.—(ii) I hold 3 hearts, how many is my partner likely to hold? Largest number in row 3 is 29·59 and falls in column 3. My partner probably holds 3 hearts also but the odds against it are about 70 to 30. He is more likely to hold 4 than 2 and twice as likely to hold 5 as 1.—(iii) Leader has 6 diamonds, what chances are there of opponents trumping on first round? Number in row 6 and column 0 is 4·28, doubling for two opponents we have 8·56. Thus opponents can trump about twice on 17 occasions.

I have to thank not only Dr Isserlis for providing the equation to the fitted skew surface, Miss Stoessiger for computing its contours, and Miss M^{c}Learn for drawing the figures, but also Miss M. Moul for calculating the ordinates of the marginal totals for both surfaces, plotting the curves and integrating them mechanically.

MISCELLANEA.

I. On Bayes' Formula.

By W. BURNSIDE.

Denote by p_A the probability that, when a trial is made, the result satisfies condition A; by p_{AB} the probability that the result satisfies both conditions A and B; by $p_{(A)B}$ the probability that if the result satisfies condition A it also satisfies condition B.

Denote also by $A_1, A_2, \ldots A_s$ a set of s conditions such that every result of the trial satisfies one of them and no result of the trial satisfies two of them.

Then the formula
$$p_{(B)A_i} = \frac{p_{A_i} p_{(A_i)B}}{\sum_1^s p_{A_i} p_{(A_i)B}}, \qquad (i = 1, 2, \ldots s)$$
has been described as Bayes' formula*.

The formula is not an immediate consequence of the "equal probability" assumption on which the values of $p_A, p_{AB}, \ldots$ rest: and the further assumptions necessary to establish the formula may quite conceivably be contested.

It seems however from Professor Pearson's paper (*Biometrika*, Vol. XIII. pp. 1—16) that the point at issue between statisticians is not so much the validity of the formula as the use that may be made of it when the numbers p_{A_i} $(i = 1, 2, \ldots s)$ are not definitely known.

When Bayes' formula is applied to the statistical problem stated in the first paragraph of Professor Pearson's paper, p_{A_i} is the (unknown) probability that in the "total population of occurrences" the "event" has a given frequency, say q_i; and $p_{(A_i)B}$ is the probability that, when the event has this frequency q_i in the total population of occurrences, the event shall occur just p times in a sample of n trials. Thus $p_{(A_i)B}$ is a definite fraction depending only on q_i, n and p. The value of p_{A_i} has no relation to or effect upon the value of $p_{(A_i)B}$.

The unknown numbers p_{A_i} may be taken all equal to each other (Professor Pearson's "equal distribution of ignorance"), or p_{A_i} may be taken as some particular continuous function of q_i (as Professor Pearson suggests on p. 6). When Bayes' formula is used to calculate $p_{(B)A_i}$, the number substituted for $p_{(A_i)B}$ is the same on the second supposition about p_{A_i} as it is on the first.

In the investigation on p. 5 of Professor Pearson's paper, an application of Bayes' formula is made to a case in which a change in p_{A_i} necessarily carries with it a change in $p_{(A_i)B}$; in other words this investigation is concerned with a case in which the numerical value of $p_{(A_i)B}$ depends not only on the nature of the condition A_i, but also on the probability p_{A_i} that condition A_i may be satisfied.

It has been seen that, for the statistical problem stated at the beginning of the paper, the numerical value of $p_{(A_i)B}$ depends in no way on the value of p_{A_i}.

There is therefore no reason for supposing that any conclusions drawn from the investigation on p. 5 will hold with respect to the statistical problem stated at the beginning of Professor Pearson's paper.

* The formula is so described by Poincaré, *Calcul des Probabilités*, 2me Edition, p. 155.

II. Note on Bayes' Theorem.

By KARL PEARSON, F.R.S.

Dr Burnside, I venture to think, does not realise either the method in which I approach Bayes' Theorem, or the method in which Bayes actually approached it himself. Bayes did not approach it from the standpoint of *chances* at all, nor do I. Both of us suppose that when a certain variate x, which must lie between two values, say 0 and a, exceeds a certain value ξ, an occurrence will take place; if it does not exceed ξ the occurrence will not happen. The best illustration is perhaps immunity; if an individual's immunity is less than a certain but unknown value ξ, then he succumbs to the disease; if on the other hand it exceeds this value he escapes it. I think a large number of phenomena of which we have only experiential knowledge can be fit ed to a similar scheme.

Now *a priori* we do not know what ξ may be, nor what its *possible* values are. There is of course only one actual value of ξ, but there may be many *a priori* possible values, which of course can only lie between 0 and a. Let us suppose $y=\phi(\xi)$ to be the frequency curve of these *a priori* possible values of ξ.

Next we have to consider the frequency curve of values of x in the population under investigation. This again is unknown to us. Let us assume the frequency curve to be $Nf(x)$, where N is the total population. Then the chance of the limiting value being between ξ and $\xi+d\xi$ is $\phi(\xi)\,d\xi$*, and the chance of an occurrence is $\int_0^\xi f(x)\,dx$, $=P_\xi$, say. Accordingly the chance of p occurrences and q non-occurrences for a given value of ξ is

$$\frac{\lfloor p+q}{\lfloor p \lfloor q} P_\xi^p (1-P_\xi)^q \phi(\xi)\,d\xi \text{ in } p+q \text{ trials.}$$

If now with Bayes we take the probability of a given value of ξ to be proportional to the probability that the observed event would follow on that value, the probability of the constitution being ξ is

$$\frac{P_\xi^p (1-P_\xi)^q \phi(\xi)\,d\xi}{\int_0^a P_\xi^p (1-P_\xi)^q \phi(\xi)\,d\xi} \quad \text{.....(i),}$$

Or, the chance of a (p, q) sample being followed by an (r, s) sample, given a definite ξ-value of the determining variate (e.g. the immunity), is

$$\frac{\lfloor r+s}{\lfloor r \lfloor s} \frac{P_\xi^{p+r}(1-P_\xi)^{q+s}\phi(\xi)\,d\xi}{\int_0^a P_\xi^p (1-P_\xi)^q \phi(\xi)\,d\xi}.$$

Hence the total chance of (r, s) following (p, q) is

$$C_{(p,q)(r,s)}=\frac{\lfloor r+s}{\lfloor r \lfloor s} \frac{\int_0^a P_\xi^{p+r}(1-P_\xi)^{q+s}\phi(\xi)\,d\xi}{\int_0^a P_\xi^p (1-P_\xi)^q \phi(\xi)\,d\xi} \quad \text{.....(ii).}$$

How does this value generalise the original value of Bayes? In the first place he takes $\phi(\xi)=1/a$, or assumes that *all* values of ξ are equally likely. In the second place he assumes that $f(x)=\frac{1}{a}$ or that his frequency distribution of x (e.g. the grades of immunity in the population) is a horizontal straight line—both very improbable distributions for characters in any population. That *a priori* all values of x and all values of ξ are equally probable is contrary to almost universal experience, and it has been this point in Bayes' deduction which has been most vigorously attacked. Our very wide experience is against variate frequencies following

* Provided we take $\int_0^a \phi(\xi)\,d\xi=1$, which we are at liberty to do.

rectangles as distributions. To make this assumption is to suppose that the bulk of frequency distributions fall into the point $\beta_1=0$, $\beta_2=1{\cdot}8$—or the rectangular point—of the β_1, β_2 quadrant; it is an assumption still more arbitrary than the assumption that all frequencies are Gaussian. We cannot distribute our ignorance equally (or as Bayes put it, place the ball with equal probability anywhere on the billiard table), when we have the knowledge that frequency distributions are no more rectangular than they are triangular as Lagrange originally supposed. It was to get over this difficulty of Bayes' treatment that I wrote my paper of which Dr Burnside, who does not seem to have read Bayes, disapproves.

Let us now go a step farther and noting that

$$dP_\xi = f(\xi)\, d\xi,$$

write our result

$$C_{(p,q)(r,s)} = \frac{\lfloor r+s}{\lfloor r \quad \lfloor s} \frac{\int_0^1 P_\xi^{p+r}(1-P_\xi)^{q+s} \dfrac{\phi(\xi)}{f(\xi)}\, dP_\xi}{\int_0^1 P_\xi^{p}(1-P_\xi)^{q} \dfrac{\phi(\xi)}{f(\xi)}\, dP_\xi} \qquad \text{.....(iii),}$$

since when $\xi=a$, $P_\xi=1$.

Now it is quite clear that until we settle some relation between $\phi(\xi)$ and $f(\xi)$ we cannot determine $C_{(p,q)(r,s)}$. We are, however, entirely ignorant of both of them *a priori*. The reasonableness of any value of ξ is only deducible *a posteriori* from the (p, q) result. We do not know in the given example what is the plausible distribution of limiting immunities, or the distribution of intensities of immunity in the population at large. In our ignorance any existing immunity might *a priori* be equally likely to be the limiting immunity. If we make this assumption $\phi(\xi)=f(\xi)$, and accordingly if we write $z=P_\xi$,

$$C_{(p,q)(r,s)} = \frac{\lfloor r+s}{\lfloor r \quad \lfloor s} \frac{\int_0^1 z^{p+r}(1-z)^{q+s}\, dz}{\int_0^1 z^{p}(1-z)^{q}\, dz},$$

which is Bayes' Theorem, without the assumption that both frequencies are rectangles. To remove this assumption was one of the purposes of my original paper. If the reader says we have still made an assumption, I readily admit it, but it does not seem an assumption which directly contradicts a fact of experience, i.e. that frequency distributions do not follow rectangles. Of course our variate—the immunity—has only a single value, but we do not *a priori* know that value, and assume in our complete ignorance that the immunity of any individual in the community may have the limiting intensity. This assumption has not got an obvious fact of experience against it, and without some assumption of the kind we can get no measure of future probability from past experience. Every practical man is constantly arguing from past experience to future probability. He needs it for every action of his life, and some measure of it is essential for all business conduct. The deduction of it ultimately turns on the values we assign to $\phi(\xi)$ and $f(\xi)$. If Bayes' Theorem does not give us reasonable results, then we must select a better value of the ratio $\phi(\xi)/f(\xi)$ than unity, but at present it has not been demonstrated to lead to results contrary to experience; it has been solely criticised on the ground that equal distribution of ignorance is not logical.

It will be seen that Dr Burnside's formula quoted from Poincaré does not correspond to my (i). There is the absence of the term $\phi(\xi)\, d\xi$ or as Bayes put it of $d\xi/a$. Bayes' and my developments naturally introduce this term, because we are considering excess of one variate (x) over a second (ξ) as the determining factor of occurrence. Condorcet and Laplace introduced the $d\xi/a$ following actually Bayes; but their reasons for introducing it are somewhat obscurely stated. With Bayes the origin of it is perfectly clear, and while he deals essentially with variates, his successors have, in my opinion, confused the issue by speaking of the "equal distribution of chances." Dr Burnside cites as Bayes' formula, what is only an element in Bayes' Theorem, and he does so on the strength of Poincaré, who in all probability had not studied Bayes' original work. In short Bayes' assumption as to the *a priori* probabilities of a given set of observations having arisen

from various *a priori* possibilities is an element on which he bases his theorem—in my view a minor element—it is *not* his theorem. My point is how he applies this element to deduce future occurrences and this is the stage where the "equal distribution of ignorance" arises. Bayes in his theorem makes $P_\xi = \frac{\xi}{a}$ and therefore $\binom{\xi}{a}^p \left(1-\frac{\xi}{a}\right)^q$ is a function of ξ, and this view is adopted by Condorcet and Laplace. A change in ξ modifies this probability and I see no reason why it should not do so. Opinions may differ as to the reasonableness of taking $\phi(\xi) = f(\xi)$, but the assumption here made is well defined and can be discussed, it is not a limitation of frequency like Bayes' assumption. It is summed up in the question: Is it reasonable, if all we can hypothecate of an occurrence is that it will happen when a certain variate exceeds a certain unknown value, to assume that this unknown value is equally likely to be any one of the existing values? My own cogitations seem to indicate that the moment I begin to make ϕ more restricted than f I am consciously or unconsciously appealing to some other experience I have had (of other limiting immunities, for example), while in fact all such experience should be excluded by the statement of the theorem, which limits our knowledge to the (p, q) observation alone. If the same result has been reached physically or chemically under the same conditions two or three times, it has been suggested that our conviction of its future repetition would be ridiculously small as determined by Bayes' Theorem. This is wholly beside the mark, because we have, and are really unconsciously arguing from, a far vaster experience, namely that physical and chemical experiments made under the same conditions do repeat themselves. For example, if we knew that limiting immunities were in our past experience close to the modal values of immunity in the general population, we should certainly give to f a more restricted variation than to ϕ. It cannot be too generally recognised that it is the basis of Bayes' Theorem to assume no knowledge beyond the (p, q) observation.

One further matter; the chance of an individual occurring with character x is according to Bayes dx/a and the chance of an occurrence is ξ/a for a given value of ξ. That individuals of all characters should be equally frequent in the population is the point to which I, and many others, have taken exception. The chance that the discriminating value should be ξ is according to Bayes' $d\xi/a$ and here again exception must be taken. Clearly since all values of ξ are equally likely to occur the frequency curve of ξ and therefore of the chance of an occurrence is a straight line—all chances are equally likely *a priori*. It is not so much the equality of the chances as the equality of the frequency of individuals from which it is deduced, to which I take exception. Now

$$P_\xi = \int_0^\xi f(x)\,dx, \quad \text{or} \quad \delta P_\xi = f(\xi)\,d\xi.$$

Again the frequency of a value of ξ between ξ and $\xi+\delta\xi = \phi(\xi)\,d\xi$, and P_ξ is fixed by ξ. Hence if $F(P_\xi)$ be the frequency curve of P_ξ, we have:

$$\text{Frequency of } P_\xi \text{ between } P_\xi \text{ and } P_\xi + \delta P_\xi = F(P_\xi)\,\delta P_\xi = \phi(\xi)\,d\xi = \frac{\phi(\xi)}{f(\xi)}\,\delta P_\xi,$$

and accordingly:

$$F(P_\xi) = \phi(\xi)/f(\xi).$$

This is the frequency distribution of chances. If $\phi(\xi)/f(\xi)$ be a constant we reach the conclusion that the frequency distribution of chances will be a horizontal straight line, or as in Bayes' Theorem all chances are equally likely*. Where then have we advanced beyond Bayes? It seems to me that we have advanced very considerably. We do *not* assume in our proof (i) that all individuals whatever their character are equally likely, (ii) that all values of the limiting character are equally likely. We assume a general frequency distribution more in accord with universal experience. And we show (iii) that to reach Bayes' Theorem it is needful to assume these two frequency distributions to be the same. The equality of the chances is not something which flows either from an assumption of linear distributions of frequency (Bayes) or from an appeal to a somewhat vague experience (Edgeworth). It flows from any frequency

* I owe to Miss Ethel Newbold this insight into the exact relation between the two hypotheses.

distributions whatever provided they are the same for the general population sampled and the *a priori* possible values of the critical variate. Further the proof turns on the reasonableness of the *a priori* hypothesis that the critical value is equally likely to be any one of the existing character (e.g. immunity) values. The equality of the chances is then not a fundamental axiom, but a deduction from the above hypothesis. The hypothesis by which Bayes reached equality of the chances, i.e. rectangular distribution of the variates, is far from general enough. Condorcet and Laplace in adopting Bayes' "equality of the chances" do not seem to see the nature of the process by which he had reached it; they introduce their dP apparently as a means of passing from the discontinuous probability $P^p(1-P)^q$ to an integrable form $P^p(1-P)^q dP$. They are really starting from equality of chances and not from Bayes' idea of excess of a variate over a critical value.

Whatever criticism may be made of the above method of proof, it appears to me that the generalised form (iii) of Bayes' Theorem indicates very clearly what assumption must be made to reach Bayes' own result.

III. Note on the Simplification of the Calculation of Abruptness Coefficients to Correct Crude Moments.

By FRANK SANDON, M.A. Cantab.

In connection with the application, in some recent work dealing with exponential curves, of the formulae given in *Biometrika*, 1919, Vol. XII. p. 239, it was found that the calculations could be considerably abbreviated. The formulae concerned are:—

If $\frac{n_1}{N}=n_1'$, $\frac{n_2}{N}=n_2'$, $\frac{n_3}{N}=n_3'$, $\frac{n_4}{N}=n_4'$ and $\frac{n_5}{N}=n_5'$,

$$a_1=-\tfrac{1}{60}(137n_1'-163n_2'+137n_3'-63n_4'+12n_5'),$$
$$a_2=+\tfrac{1}{12}(45n_1'-109n_2'+105n_3'-51n_4'+10n_5'),$$
$$a_3=-\tfrac{1}{4}(17n_1'-54n_2'+64n_3'-34n_4'+7n_5'),$$
$$a_4=+(3n_1'-11n_2'+15n_3'-9n_4'+2n_5'),$$
$$a_5=-(n_1'-4n_2'+6n_3'-4n_4'+n_5'),$$

and similar expressions (of opposite sign) for the b's at the far end; then, when h is the working interval, h_0 the subrange interval at the beginning of the curve ($x=x_0$) and h_p at the end ($x=x_p$), and further if

$$a_s'=a_s\times\left(\frac{h}{h_0}\right)^s, \quad b_s'=b_s\times\left(\frac{h}{h_p}\right)^s,$$

we have the corrections given by

$$\mu_1'=\nu_1'+\frac{h}{12}\left(a_1'-\frac{1}{60}a_3'+\frac{1}{2520}a_5'\right)+\frac{h}{12}\left(b_1'-\frac{1}{60}b_3'+\frac{1}{2520}b_5'\right),$$

$$\mu_2'=\nu_2'-\frac{h^2}{12}+h^2\Big[\frac{x_p'}{6}\left(b_1'-\frac{1}{60}b_3'+\frac{1}{2520}b_5'\right)+\frac{x_0'}{6}\left(a_1'-\frac{1}{60}a_3'+\frac{1}{2520}a_5'\right)$$
$$+\frac{1}{120}\left(b_2'-\frac{5}{126}b_4'\right)-\frac{1}{120}\left(a_2'-\frac{5}{126}a_4'\right)\Big],$$

$$\mu_3'=\nu_3'-\frac{h^2}{4}\nu_1'+h^3\Big[-\frac{1}{40}\left(a_1'-\frac{5}{63}a_3'+\frac{1}{240}a_5'\right)-\frac{1}{40}\left(b_1'-\frac{5}{63}b_3'+\frac{1}{240}b_5'\right)+\frac{x_p'}{40}\left(b_2'-\frac{5}{126}b_4'\right)$$
$$+\frac{x_p'^2}{4}\left(b_1'-\frac{1}{60}b_3'+\frac{1}{2520}b_5'\right)-\frac{x_0'}{40}\left(a_2'-\frac{5}{126}a_4'\right)+\frac{x_0'^2}{4}\left(a_1'-\frac{1}{60}a_3'+\frac{1}{2520}a_5'\right)\Big],$$

$$\mu_4'=\nu_4'-\frac{h^2}{2}\nu_2'+\frac{7}{240}h^4+h^4\Big[\frac{1}{126}\left(a_2'-\frac{7}{80}a_4'\right)-\frac{1}{126}\left(b_2'-\frac{7}{80}b_4'\right)-\frac{x_0'}{10}\left(a_1'-\frac{5}{63}a_3'+\frac{1}{240}a_5'\right)$$
$$-\frac{x_p'}{10}\left(b_1'-\frac{5}{63}b_3'+\frac{1}{240}b_5'\right)-\frac{x_0'^2}{20}\left(a_2'-\frac{5}{126}a_4'\right)+\frac{x_p'^2}{20}\left(b_2'-\frac{5}{126}b_4'\right)$$
$$+\frac{x_0'^3}{3}\left(a_1'-\frac{1}{60}a_3'+\frac{1}{2520}a_5'\right)+\frac{x_p'^3}{3}\left(b_1'-\frac{1}{60}b_3'+\frac{1}{2520}b_5'\right)\Big].$$

If $h=h_0=h_p$, and we put

$$Na_1=A_1, \text{ etc.},$$
$$Nb_1=B_1, \text{ etc.},$$

then writing

$$+\frac{1}{12}\left(A_1-\frac{1}{60}A_3+\frac{1}{2520}A_5\right)=K_1,$$
$$-\frac{1}{120}\left(A_2-\frac{5}{126}A_4\right)=K_2,$$
$$-\frac{1}{40}\left(A_1-\frac{5}{63}A_3+\frac{1}{240}A_5\right)=K_3$$
$$+\frac{1}{126}\left(A_2-\frac{7}{80}A_4\right)=K$$

and similarly the expressions in B of opposite sign$=k_1$, k_2, k_3 and k_4, the equations for the corrections are

$$\mu_1'=\nu_1'+\frac{hK_1}{N}-\frac{hk_1}{N},$$

$$\mu_2'=\nu_2'-\frac{h^2}{12}+h^2\left[\frac{2x_p'k_1}{N}+\frac{2x_0'K_1}{N}+\frac{K_2}{N}+\frac{k_2}{N}\right],$$

$$\mu_3'=\nu_3'-\frac{h^2}{4}\nu_1'+h^3\left[\frac{K_3}{N}-\frac{k_3}{N}+\frac{3x_0'K_2}{N}+\frac{3x_p'k_2}{N}+\frac{3x_0''^2K_1}{N}-\frac{3x_p''^2k_1}{N}\right],$$

$$\mu_4'=\nu_4'-\frac{h^2}{2}\nu_2'+\frac{7}{240}h^4+h^4\left[\frac{K_4}{N}+\frac{k_4}{N}+\frac{4x_0'K_3}{N}-\frac{4x_p'k_3}{N}+\frac{6x_0''^2K_2}{N}+\frac{6x_p''^2k_2}{N}+\frac{4x_0''^3K_1}{N}-\frac{4x_p''^3k_1}{N}\right].$$

Now $$A_1=-\frac{1}{60}\{137n_1-163n_2+137n_3-63n_4+12n_5\}, \text{ etc.}$$

$$\therefore\quad K_1=+\frac{1}{12}\left\{-\frac{1}{60}(137n_1-163n_2+137n_3-63n_4+12n_5)\right.$$
$$-\frac{1}{60}\times-\frac{1}{4}(17n_1-54n_2+64n_3-34n_4+7n_5)$$
$$\left.+\frac{1}{2520}\times-(n_1-4n_2+6n_3-4n_4+n_5)\right\}.$$

$$\therefore\quad -K_1=\frac{1}{60480}\{11153n_1-12566n_2+10176n_3-4586n_4+863n_5\}$$
$$=\cdot1844\,0806\,8783\,n_1-\cdot2077\,7116\,4021\,n_2+\cdot1682\,5396\,8253\,n_3$$
$$-\cdot0758\,2671\,9576\,n_4+\cdot0142\,6917\,9894\,n_5.$$

Similarly $$-K_2=\cdot0302\,5793\,6507\,n_1-\cdot0720\,5687\,8306\,n_2+\cdot0679\,5634\,9206\,n_3$$
$$-\cdot0324\,4047\,6190\,n_4+\cdot0062\,8306\,8783\,n_5.$$

$$K_3=\cdot0487\,5496\,0317\,4\,n_1-\cdot0415\,4761\,9047\,n_2+\cdot0259\,6230\,1587\,n_3$$
$$-\cdot0098\,0158\,7301\,n_4+\cdot0016\,3194\,4444\,n_5.$$

$$K_4=\cdot0276\,7857\,1428\,n_1-\cdot0644\,5105\,8201\,n_2+\cdot0590\,2777\,7777\,n_3$$
$$-\cdot0274\,8015\,8730\,n_4+\cdot0052\,2486\,7724\,n_5.$$

Similarly since $$k_1=-\frac{1}{12}\left(B_1-\frac{1}{60}B_3+\frac{1}{2520}B_5\right),$$

where $$b_1=+\frac{1}{60}(137m_1'-163m_2'+137m_3'-63m_4'+12m_5'),$$

the last five subranges being m_5, m_4, m_3, m_2 and m_1, m_1 finishing at $x=x_p$,

we have $$-k_1=\cdot1844m_1-\cdot2077m_2+\cdot1682m_3-\cdot0758m_4+\cdot0142m_5, \text{ etc.}$$

Thus by putting on the calculating machine each of these multipliers in turn and multiplying by the integral factors n_1, n_2, n_3, n_4, n_5 (or similarly by the m's) we have very quickly K_1, K_2, K_3 and K_4 (or the k's). If next we put on $1/N$ and multiply by each K in turn, we have the various values ready for substitution in the simplified expressions for μ_1', μ_2', μ_3' and μ_4'.

Special Case. If $m_1=m_2=m_3=m_4=m_5=0$,

then $b_1=b_2=b_3=b_4=b_5=0$

and $k_1=k_2=k_3=k_4=0$.

Accordingly: $\mu_1'=\nu_1'+h\dfrac{K_1}{N}$,

$$\mu_2'=\nu_2'-\frac{h^2}{12}+h^2\left[\frac{K_2}{N}+2\nu_0'\frac{K_1}{N}\right],$$

$$\mu_3'=\nu_3'-\frac{h^2}{4}\nu_1'+h^3\left[\frac{K_3}{N}+3x_0'\frac{K_2}{N}+3x_0'^2\frac{K_1}{N}\right],$$

$$\mu_4'=\nu_4'-\frac{h^2}{2}\nu_2'+\frac{7}{240}h^4+h^4\left[\frac{K_4}{N}+4x_0'\frac{K_3}{N}+6x_0'^2\frac{K_2}{N}+4x_0'^3\frac{K_1}{N}\right].$$

But in this case (*J*-shaped or sharp-rising curves) we can usually choose our origin at will and put $x_0'=0$.

Thus we have $\mu_1'=\nu_1'+h\dfrac{K_1}{N}$,

$$\mu_2'=\nu_2'-\frac{h^2}{12}+h^2\frac{K_2}{N},$$

$$\mu_3'=\nu_3'-\frac{h^2}{4}\nu_1'+h^3\frac{K_3}{N},$$

$$\mu_4'=\nu_4'-\frac{h^2}{2}\nu_2'+\frac{7}{240}h^4+h^4\frac{K_4}{N}.$$

If we can also put $h=1$ (and thus $h_0=1$, for it must be remembered that the above simplifications are only possible when $h=h_0$), we then have

$$\mu_1'=\nu_1'+\frac{K_1}{N},$$

$$\mu_2'=\nu_2'-\cdot08\dot{3}+\frac{K_2}{N},$$

$$\mu_3'=\nu_3'-\cdot25\nu_1'+\frac{K_3}{N},$$

$$\mu_4'=\nu_4'-\cdot5\nu_2'+\cdot02916\dot{6}+\frac{K_4}{N}.$$

These formulae can usually be employed and will be found to reduce very considerably the labour commonly entailed in correcting the moments.

The working procedure is then as follows:

From the frequency tables calculate the crude moments (not moment coefficients). These are, in the ordinary way, $2N\nu_1'$, $4N\nu_2'$, $8N\nu_3'$ and $16N\nu_4'$, for we usually, in cases involving abruptness coefficients, work with frequencies assumed concentrated at abscissae 1, 3, 5, 7, 9, ... Then from these calculate $N\nu_1'$, $N\nu_2'$, $N\nu_3'$ and $N\nu_4'$. Calculate also $-K_1$, $-K_2$, $+K_3$ and $+K_4$ from u_1, u_2, u_3, u_4 and u_5. We then have $N\mu_1'$, $N(\mu_2'+\cdot08\dot{3})$, $N\mu_3'$ and $N(\mu_4'-\cdot02916\dot{6})$ giving μ_1', $\mu_2'+\cdot08\dot{3}$, μ_3' and $\mu_4'-\cdot02916\dot{6}$ and thus finally the moment coefficients about the origin.

IV. Note on the Approximations to the Probable Error of a Coefficient of Correlation.

By EGON S. PEARSON.

The following note deals with an arithmetical difficulty of methods of approximation, which may frequently arise, and against which we appear to have *a priori* no safeguard.

The most general value for the standard deviation, $\sigma_{r_{xy}}$, of a correlation coefficient is

$$\sigma_{r_{xy}}=\frac{r_{xy}}{\sqrt{N}}\left\{\frac{\bar{p}_{22}-\bar{p}_{11}^{\,2}}{\bar{p}_{11}^{\,2}}+\tfrac{1}{4}\frac{\bar{p}_{40}-\bar{p}_{20}^{\,2}}{\bar{p}_{20}^{\,2}}+\tfrac{1}{4}\frac{\bar{p}_{04}-\bar{p}_{02}^{\,2}}{\bar{p}_{02}^{\,2}}+\tfrac{1}{2}\frac{\bar{p}_{22}-\bar{p}_{20}\bar{p}_{02}}{\bar{p}_{20}\bar{p}_{02}}-\frac{\bar{p}_{31}-\bar{p}_{11}\bar{p}_{20}}{\bar{p}_{11}\bar{p}_{20}}-\frac{\bar{p}_{13}-\bar{p}_{11}\bar{p}_{02}}{\bar{p}_{11}\bar{p}_{02}}\right\}^{\frac{1}{2}} \quad \text{...(i)},$$

where in the usual notation the $\bar{p}$'s are the product moments about the mean of the sampled population*. If we assume linearity of regression but not homoscedasticity, we may write

$$\frac{\bar{p}_{31}}{\bar{p}_{11}\bar{p}_{20}}=\beta_2, \quad \frac{\bar{p}_{13}}{\bar{p}_{11}\bar{p}_{02}}=\beta_2',$$

and more approximately

$$\bar{p}_{22}=\sigma_x^{\,2}\sigma_y^{\,2}\{1-r_{xy}^{\,2}+\tfrac{1}{2}r_{xy}^{\,2}(\beta_2+\beta_2')\},$$

where β_2 and β_2' are the second β's for the x and y variables respectively. We then reach the approximate but more readily calculated form for the standard deviation

$$\sigma_{r_{xy}}=\frac{1-r_{xy}^{\,2}}{\sqrt{N}}\left\{1-\tfrac{1}{4}(\beta_2-3+\beta_2'-3)\frac{r_{xy}^{\,2}}{1-r_{xy}^{\,2}}\right\}^{\frac{1}{2}} \quad \text{.....(ii)}.$$

Finally if we assume that there is homoscedasticity in both directions as well as linearity of regression, or in other words that our frequency surface is a normal surface so that $\beta_2=3=\beta_2'$, we obtain the simple relation

$$\sigma_{r_{xy}}=\frac{1-r_{xy}^{\,2}}{\sqrt{N}} \quad \text{.....(iii)}.$$

These equations have been given in *Biometrika* (Vol. IX. pp. 4—6), but the following numerical example brings out certain points of interest. The data chosen are the 2922 observations of Barometric Heights recorded at Southampton and Laudale originally used by Pearson and Lee in *Phil. Trans.* A. Vol. 190 (1897), p. 423†. The frequency constants have been found to be as follows:

$\bar{p}_{20}=\sigma_x^{\,2}$	$=10{\cdot}562938,$	$\bar{p}_{02}=\sigma_y^{\,2}$	$=15{\cdot}462905.$
$\bar{p}_{31}$	$=361{\cdot}329845,$	$\bar{p}_{13}$	$=500{\cdot}535855.$
β_1	$={\cdot}17114\pm{\cdot}03677,$	β_1'	$={\cdot}22454\pm{\cdot}03013.$
β_2	$=3{\cdot}61203\pm{\cdot}18663,$	β_2'	$=3{\cdot}19495\pm{\cdot}09794.$
$\eta_{y.x}$	$={\cdot}780819,$	$\eta_{x.y}$	$={\cdot}783647.$
$r_{xy}=+{\cdot}780225,$	$\bar{p}_{11}=9{\cdot}971435,$	$\bar{p}_{22}=399{\cdot}245042.$	

Sheppard's corrections have been used throughout in calculating the moments and product moments.

From these figures we see

(1) That the distribution is skew, the values of β_1 differing quite significantly from zero;

(2) That the regressions of y upon x and of x upon y are very nearly linear having regard to the probable error of r_{xy} given below.

* In practice of course we rarely know the constants of the sampled population but must make use of those for the sample as the best available approximation.

† The table of frequencies has been given again in Mr E. C. Rhodes paper "On a certain Skew Correlated Surface," *Biometrika*, Vol. XIV. pp. 372, 373.

We should therefore expect that Equation (ii) would provide a more satisfactory approximation to the value of $\sigma_{r_{xy}}$ (as found from (i)) than Equation (iii). The values actually found are

from (i) $\sigma_{r_{xy}} = \cdot 007395$,

(ii) $\sigma_{r_{xy}} = \cdot 005995$,

(iii) $\sigma_{r_{xy}} = \cdot 007238$,

so that (iii) is considerably closer to (i) than is (ii). To understand this apparent anomaly we must examine the terms of Equation (i) separately. We find:

	(i)	(ii)	(iii)
$+\dfrac{\bar{p}_{22}-\bar{p}_{11}^2}{\bar{p}_{11}^2}$	3·015357	3·046200	2·642709
$+\frac{1}{4}\dfrac{\bar{p}_{40}-\bar{p}_{20}^2}{\bar{p}_{20}^2}=\frac{1}{4}(\beta_2-1)$	·653007	·653007	·500000
$+\frac{1}{4}\dfrac{\bar{p}_{04}-\bar{p}_{02}^2}{\bar{p}_{02}^2}=\frac{1}{4}(\beta_2'-1)$	·548738	·548738	·500000
$+\frac{1}{2}\dfrac{\bar{p}_{22}-\bar{p}_{20}\bar{p}_{02}}{\bar{p}_{20}\bar{p}_{02}}$	·722176	·731563	·608751
$-\dfrac{\bar{p}_{31}-\bar{p}_{11}\bar{p}_{20}}{\bar{p}_{11}\bar{p}_{20}}$	−2·430532	−2·61203	−2·000000
$-\dfrac{\bar{p}_{13}-\bar{p}_{11}\bar{p}_{02}}{\bar{p}_{11}\bar{p}_{02}}$	−2·246283	−2·19495	−2·000000
Totals	+·262463	+·172528	+·251460
$\sigma_{r_{xy}}$	·007395	·005995	·007238
Probable Error of r_{xy}	±·0050	±·0040	·0050

Column (i) gives the full values, columns (ii) and (iii) the values modified in accordance with the assumptions of Equations (ii) and (iii). It will be seen that term for term (ii) is in all cases a far more satisfactory approximation to (i) than (iii), yet because the last two terms are negative, (iii) gives by chance a remarkably accurate result on taking the sum. This phenomenon is one which may always occur when combining a number of positive and negative terms for each of which a separate approximation has been obtained; the series of worse approximations will sometimes give a better result, but in the long run the more accurate method will pay.

The difference between the results of methods (i) and (ii) arises mainly from the approximation to $\bar{p}_{31}$ and to a lesser extent $\bar{p}_{13}$, made in the latter case. If we ask whether the regression is sufficiently nearly linear to have justified these approximations, we must compare

$$r_{xy} = \cdot 7802 \pm \cdot 0050, \quad \eta_{yx} = \cdot 7808, \text{ and } \eta_{xy} = \cdot 7836,$$

and clearly cannot assert that the regression is not linear. The array-means in our sample do not of course lie exactly on the regression straight line, and these small differences, which barely affect the value of σ_{y_x} $(=\eta_{y_x}\sigma_y)$, become greatly increased in the 4th order product-moments $\bar{p}_{31}$ and $\bar{p}_{13}$. But even then, the difference between

$$\bar{p}_{31} = 361{\cdot}33 \pm 29{\cdot}49 \text{ and } \beta_2\bar{p}_{11}\bar{p}_{20} = 380{\cdot}45$$

is less than the probable error of the former*. Or again the probable error of $\bar{p}_{31}$ if divided by $\bar{p}_{11}\bar{p}_{20}$ or 105·327650 gives a quantity of order ·28 which is of the same order as the difference

* This probable error is only approximate, having been obtained on the assumption that the distribution is normal.

between the most discrepant terms of (i) and (ii). In fact in view of the magnitude of the probable errors of these high moments ($\bar{p}_{22}$, $\bar{p}_{31}$, $\bar{p}_{13}$) we cannot assert that (i) is really more correct than (ii). On the other hand the differences of the components of (i) and (iii) are usually significant and we cannot legitimately argue that (iii)—i.e. with the normal values of the high moments—may always be used in the expression for the probable error of r. Of course for all practical purposes such values as ·005 and ·004 are identical, but notwithstanding it is sad that the approximate method (ii) does not give a closer approach to (i) than (iii), although for aught we can say to the contrary it may be closer to the true value—the value based on the moments in the population we are sampling from—than either of them.

V. On the Mean-Error of Frequency Distributions.

By KARL PEARSON, F.R.S.

In the *Sitzungsberichte der Berliner mathematischen Gesellschaft*, February 28, 1923, S. 21–32, Professor L. v. Bortkiewicz discusses the relation of the mean error to the standard deviation in certain forms of frequency curves under the title: "Ueber eine verschiedenen Fehlergesetzen gemeinsame Eigenschaft." It is quite possible to get a still more general result than he has obtained if we start from the differential equation to my types of frequency curves. Let l_1 to l_2 be the frequency range, the origin being at the mean. Then the differential equation in question is:

$$\frac{1}{y}\frac{dy}{dx} = -\frac{x+a}{c_0+c_1x+c_2x^2} \quad \text{.....(i),}$$

where $$c_0=\frac{4\beta_2-3\beta_1}{2(5\beta_2-6\beta_1-9)}\sigma^2, \quad c_1=a=\frac{\sqrt{\beta_1}(\beta_2+3)}{2(5\beta_2-6\beta_1-9)}\sigma, \quad c_2=\frac{2\beta_2-3\beta_1-6}{2(5\beta_2-6\beta_1-9)},$$

σ being the standard deviation and β_1, β_2 the usual statistical constants μ_3^2/μ_2^3 and μ_4/μ_2^2*. Multiply up and and integrate from $x=0$ to l_2:

$$\int_0^{l_2}(c_0+c_1x+c_2x^2)\,dy = -\int_0^{l_2}(x+a)\,y\,dx \quad \text{.....(ii),}$$

or integrating by parts:

$$\Big[(c_0+c_1x+c_2x^2)y\Big]_0^{l_2} - c_1\int_0^{l_2}y\,dx - 2c_2\int_0^{l_2}xy\,dx = -a\int_0^{l_2}y\,dx - \int_0^{l_2}xy\,dx.$$

The term between limits vanishes at $x=l_2$ for all my types, even for J- and U-curves. Hence, if we remember that $c_1=a$, we have:

$$c_0y_0=(2c_2-1)\int_0^{l_2}xy\,dx \quad \text{.....(iii).}$$

But if e_m be the mean error and N the total frequency

$$Ne_m=2\int_0^{l_2}xy\,dx.$$

Accordingly on substituting for c_0 and c_2

$$\frac{\sigma^2 2y_0}{Ne_m}=\frac{6(\beta_2-\beta_1-1)}{4\beta_2-3\beta_1} \quad \text{.....(iv).}$$

It is therefore possible to find the ordinate of the frequency curve at the mean and so its absolute constant without Γ-, B-, or $G(r, \nu)$-functions, by simply determining the mean deviation and the standard deviation.

* Pearson, "On the Mathematical Theory of Errors of Judgement" *Phil. Trans.* Vol. 198 A (1902), pp. 274–9.

v. Bortkiewicz discusses the value of $\sigma^2 2y_0/(Ne_m)$ for nine cases of frequency distribution including certain of my types, deducing it for each curve separately. The above result will cover all his cases and several others.

For the normal curve ($\beta_2=3$, $\beta_1=0$) and for the curve of Type III ($2\beta_2-3\beta_1-6=0$), $\frac{\sigma^2 2y_0}{Ne_m}=1$, but when β_1 and β_2 are independent no such simple value can be attributed to it.

For the point binomial $(p+q)^n$ and its limit in Poisson's series $S\left(e^{-m}\frac{m^n}{n!}\right)$, special conditions as to y_0 must be made as they are not distributions satisfying the differential equation (i). These are dealt with by v. Bortkiewicz. To what extent in practice that author's theorem or rather its more general form given in (iv) would be of service for determining y_0 remains to be ascertained by practical work. In long series subjected to grouping e_m is not easy to determine. We need not only a correction for grouping, but a method of dealing with the division of the group in which the mean occurs. For determining e_m from a knowledge of y_0 and σ, the formula would be probably of greater value. But the usefulness of e_m, if σ be ascertained, is not very great and would possibly only arise in certain very special problems.

A more interesting extension of the problem is to the determination, not of the mean error, but of the centroid of the 'tail' of any frequency distribution, and accordingly of any block of the frequency curve as the difference between two tails. Returning to equation (ii) we can generalise it as

$$\int_x^{l_2}(c_0+c_1x+c_2x^2)\,dy=-\int_x^{l_2}(x+a)\,y\,dx,$$

or
$$\left[(c_0+c_1x+c_2x^2)\,y\right]_x^{l_2}-c_1\int_x^{l_2}y\,dx-2c_2\int_x^{l_2}xy\,dx=-\int_x^{l_2}(x+a)\,y\,dx \quad \text{.....(v).}$$

But the expression between brackets vanishes if l_2 corresponds to the tip of the tail and hence as before we have:

$$(c_0+c_1x+c_2x^2)\,y=(1-2c_2)\int_x^{l_2}xy\,dx.$$

Now let all the tail beyond the ordinate y at x be given by $N\frac{1}{2}(1-\alpha_x)$, and $\bar{x}$ be the distance of the centroid of the tail from the mean of the whole frequency, then we have:

$$\frac{c_0+c_1x+c_2x^2}{1-2c_2}\,y=\bar{x}\times N\tfrac{1}{2}(1-\alpha_x).$$

Let us write $z_x=\frac{\sigma}{N}y$, then we have:

$$\frac{c_0+c_1x+c_2x^2}{1-2c_2}\,\frac{z_x}{\frac{1}{2}(1-\alpha_x)}=\frac{\bar{x}}{\sigma},$$

or,

$$\frac{\bar{x}}{\sigma}=\frac{4\beta_2-3\beta_1+\sqrt{\beta_1}(\beta_2+3)\left(\frac{x}{\sigma}\right)+(2\beta_2-3\beta_1-6)\left(\frac{x}{\sigma}\right)^2}{6(\beta_2-\beta_1-1)}\times\frac{z_x}{\frac{1}{2}(1-\alpha_x)} \quad \text{.....(vi)}$$

This is a still wider generalisation of the problem.

(i) If we have a Normal curve: $\beta_1=0$, $\beta_2=3$, and accordingly:

$$\frac{\bar{x}}{\sigma}=\frac{z_x}{\frac{1}{2}(1-\alpha_x)} \quad \text{.....(vii),}$$

a very familiar property, z_x and $\frac{1}{2}(1-\alpha_x)$ being tabled in the *Tables for Statisticians.*

(ii) If we have a curve of Type III, then

$$\frac{\bar{x}}{\sigma}=(1+\tfrac{1}{2}\sqrt{\beta_1}x)\,\frac{z_x}{\frac{1}{2}(1-\alpha_x)} \quad \text{.....(viii),}$$

a novel result showing how easy it is to write down the mean of any portion of a Type III curve.

(iii) In any case whatever, all that we have to do is to substitute the numerical values of β_1 and β_2 and we have a formula which can be readily applied to the determination of $\bar{x}$. If we require the mean of the trapezette of frequency from the ordinate y_1 at x_1 to the ordinate y_2 at x_2, we easily find by integrating from x_1 to x_2 that

$$\frac{\bar{x}_{12}}{\sigma}=\frac{\left\{4\beta_2-3\beta_1+\sqrt{\beta_1}(\beta_2+3)\frac{x_1}{\sigma}+(2\beta_2-3\beta_1-6)\frac{x_1^2}{\sigma^2}\right\}z_1-\left\{4\beta_2-3\beta_1+\sqrt{\beta_1}(\beta_2+3)\frac{x_2}{\sigma}+(2\beta_2-3\beta_1-6)\frac{x_2^2}{\sigma^2}\right\}z_2}{6(\beta_2-\beta_1-1)\{\frac{1}{2}(1-a_1)-\frac{1}{2}(1-a_2)\}} \quad \text{...(ix).}$$

For the Normal curve $\beta_1=0$, $\beta_2=3$ this reduces to

$$\frac{\bar{x}_{12}}{\sigma}=\frac{z_1-z_2}{\frac{1}{2}(1-a_1)-\frac{1}{2}(1-a_2)}, \text{ a well known result} \quad \text{.....(x).}$$

For Type III curve it reduces to

$$\frac{\bar{x}_{12}}{\sigma}=\frac{z_1-z_2+\frac{1}{2}\sqrt{\beta_1}\left(\frac{x_1z_1-x_2z_2}{\sigma}\right)}{\frac{1}{2}(1-a_1)-\frac{1}{2}(1-a_2)} \quad \text{.....(xi).}$$

Finally if we take* $r=\dfrac{6(\beta_2-\beta_1-1)}{3\beta_1-2\beta_2+6}$, we can put (ix) in the form:

$$\frac{\bar{x}_{12}}{\sigma}=\frac{(r+1)(z_1-z_2)+\frac{1}{2}\sqrt{\beta_1}(r+2)\left(\frac{x_1z_1-x_2z_2}{\sigma}\right)-\frac{x_1^2z_1-x_2^2z_2}{\sigma^2}}{r\{\frac{1}{2}(1-a_1)-\frac{1}{2}(1-a_2)\}} \quad \text{.....(xii).}$$

If we make r infinite, i.e. $2\beta_2-3\beta_1-6=0$, this reduces to (xi), and if in addition $\beta_1=0$, to (x).

When r is finite but $\beta_1=0$, we have:

$$\frac{\bar{x}_{12}}{\sigma}=\frac{2\beta_2(z_1-z_2)-(3-\beta_2)\frac{z_1x_1^2-z_2x_2^2}{\sigma^2}}{3(\beta_2-1)\{\frac{1}{2}(1-a_1)-\frac{1}{2}(1-a_2)\}} \quad \text{.....(xiii),}$$

providing for the whole series of symmetrical frequency curves.

These formulae seem to me likely to be of service in determining scales to represent frequency in the case of skew distributions with broad-category data.

The Study of Palaeopathology. Notices.

Palaeopathology. By R. L. Moodie, Ph.D. University of Illinois Press, 1923. In this volume the author has collected together in the first ten chapters most of what is known about the pathological anatomy of fossil animals and plants, and in the last four chapters he deals with the pathology of primitive man. Most of the evidence of fossil pathology has been obtained from vertebrate remains found in Wyoming and elsewhere and is necessarily almost entirely confined to the pathology of bones. As might be expected, the preserved parts of earlier invertebrate animals give us very little satisfactory evidence either for or against the existence of disease amongst them. The antiquity of suppurative processes, and therefore of pyogenic bacteria, is demonstrated by the fractured vertebral spine of a Permian reptile and the skull of a phytosaurian from the Triassic, both showing necrotic sinuses, and by the fractured humerus of a dinosaur from the Cretaceous of Canada presenting the shell of an enormous subperiosteal abscess. Renault's researches on actual fossilised bacteria found in Permian coprolites are undoubtedly of great interest, but it cannot be said that any of the evidence for the pathogenic nature of these or other forms of fossil bacteria is very convincing; various forms of non-pathogenic micrococci have of course been found even in Pre-Cambrian rocks. The evidence for the activities

* *Phil. Trans.* Vol. 186, A, 1895, p. 368.

of parasites is also very fragmentary and obscure, but the finding of fossil tsetse flies in the Oligocene suggests the possible existence of trypanosomiasis in ungulates at this period.

Exostoses seem to be as ancient as bones themselves, and their tendency when multiple to occur at the "growing ends" of bones, as in man, is exemplified by symmetrical outgrowths at the lower ends of both radii in a dog from the Oligocene (Plate LIII), though for some reason the author states that "*no modern human examples of such things have been met with so far as I can learn*" (p. 247). It is also somewhat strange that the author should omit any notice of the noteworthy exostosis on the reputed femur of *Pithecanthropus erectus*.

The author's curve (Fig. 2) purporting to show the "relative frequency of disease in the different geological periods, based on the meagre evidences at present available," we find difficult to appreciate. Apparently the ordinates are arrived at by counting the number of different diseases of bones known to us which have been identified in fossil remains from the different periods, a number which increases from Permian to Cretaceous, then falls and again rises to a maximum at the present. The fall in the curve he explains by asserting (p. 37) that "*with the close of the Cretaceous and the sudden extinction of large groups of reptiles, the incidence of disease also decreased. It seems quite probable that many of the diseases which afflicted the dinosaurs and their associates became extinct with them.*" The concluding statement may or may not be true, but since we are only considering diseases now known to us we do not see how this could affect the curve. Later we are told that "*the earliest animals were apparently free from disease*" (p. 291), and again in discussing the rôle of disease in the extinction of races, after alluding to the disappearance of eurypterids and trilobites the following statement is made (p. 343): "*these will suffice to show that disease has not always been an important factor since it was almost wholly absent during the periods of the early and middle Palaeozoic.*" But the only evidence we have for these statements is the absence of discoverable lesions in the fossilised parts of these early forms of life. It cannot be argued from this that disease was absent, for the most deadly and important diseases leave no lesions in the hard parts which are only slowly attacked by the more chronic forms; thus it is very doubtful whether, a few million years hence, fossilised man will show the slightest trace of some of the great scourges which have decimated races of humanity*.

Turning to ancient human pathology an interesting example of a lesion resembling Pott's disease in the dorsal vertebrae of Neolithic man is instructive as suggesting the possible antiquity of tuberculosis. The author however doubts the evidence for the existence of syphilis in Neolithic man, and a like caution in other directions would have been commendable. Thus the section on Opisthotonos in vertebrates, containing as it does the suggestions that certain Pterodactyls and the familiar Archaeopteryx may have been wont to die of cerebro-spinal infections or poisoning, whilst a Dinosaur found in the Alberta Chalk with a backward curve of its neck probably died in spasms from eating a poisonous plant, we must confess rather tempted us to laughter.

Though the collection of material with excellent photographic illustrations is undoubtedly of great value, the book is spoilt by much repetition and much fanciful deduction from insufficient evidence. In the present state of this new science the author would have done better to confine himself to a scientific presentation of the facts, which are sufficiently valuable in themselves.

Studies in the Palaeopathology of Egypt. By Sir Marc Armand Ruffer, C.M.G., M.D. University of Chicago Press, 1921. In this admirable volume we have a collection of the writings of the late Sir Marc Armand Ruffer relating to the evidences of disease in Ancient Egypt. The papers have been previously published in various scientific journals, but as several of the latter are not readily accessible, the publication of the papers in a single volume under the direction of Lady Ruffer is of great value. If a study of this book could leave any room for regret, it would be that the purpose of the author to prepare a single work embodying the results of his extensive

* E.g. although there is plenty of evidence of arthritic, tubercular and syphilitic joints in the plague-pit skeletons, we cannot say from the evidence of the skeletons themselves that the owners died of plague. Skeletal remains would hardly give evidence of severe epidemics of typhus or smallpox.

antiquarian studies in Egypt was frustrated by his untimely death. We have no criticism to offer, either of the papers themselves or of the manner in which they have been presented. The reader cannot fail to be impressed by the scientific spirit which dominates them throughout. If the author errs at all, it is on the side of excessive caution, and one feels in one or two instances a sense of surprise when a very careful and detailed description of the lesions found in a specimen is brought to a close without a diagnosis being attempted. Such a spirit is as commendable as it is rare and contrasts strongly with the atmosphere which pervades the work we have previously described.

It is a good many years since the author's researches proving the frequency of arteriosclerosis in Egyptian mummies were published, but we are still taught that syphilis, alcohol, tobacco and the stress of modern life are important factors in the causation of this disease. It is surely time these fictions were discarded and we owned up to our ignorance. The very great prevalence of arthritis deformans in Nubian remains extending over a period of some 8000 years should also dispose of the "cold and wet" hypothesis which so conveniently fills another awkward gap under that most troublesome little heading "aetiology" in medical text-books. Again, the author's study of predynastic skulls shows that dental caries was present in Egypt from very early times; thus 53 per cent. of maxillae and 29 per cent. of mandibles of this period which he examined showed gross lesions of dental disease, the first molars being the most prone to caries, as in modern times. Pyorrhoea and alveolar and perialveolar abscesses were evidently common and it is curious to note the author's statement that no evidence exists that extraction of teeth was ever practised. Neither syphilitic nor rachitic teeth were discovered, nor, we are told, has a single bone dating from ancient Egypt been as yet unearthed which shows undoubted signs of syphilis or rickets, though some evidence of the existence of the latter is furnished by a few tomb paintings. The author attributes to sand erosion the skull-lesions which Fouquet believed to be syphilitic, but it is extremely difficult to believe that the lesions on the skull from El-Amra figured in De Morgan's *Recherches sur les Origines de l'Egypte: Ethnographie Préhistorique et Tombeau Royal de Négadah*, 1897, p. 364, can be due to erosion; they resemble strikingly the incipient syphilitic lesions of crania exhibited at the Royal College of Surgeons.

The paper on the effect of consanguineous marriages in the Royal families of Egypt is of considerable interest to students of heredity, since it proves that the evil results which are supposed to follow persistent inbreeding amongst human beings failed to appear in these lines of kings even though continued for as many as ten generations. The 71 photographic plates which illustrate the volume are faultlessly produced and the book is a fine memorial to a great investigator.

P. S.

VI. Note on the Relationship of the Incomplete B-function to the Sum of the first p terms of the Binomial $(a+b)^n$.

By KARL PEARSON, F.R.S.

The sum of a definite number of terms of a binomial is not infrequently required and is a troublesome business to compute if the binomial power be large. I have pointed out in my Introduction to the *Tables of the Incomplete* Γ-*Function* (p. viii) that the sum of any number of terms of an exponential series may be found from an incomplete Γ-function. In the same way the sum of the first p terms of any binomial can be accurately found from the incomplete B-function, and accordingly would be rapidly found if tables of this function were available*. Considering the case of n only moderately large, the methods discussed by Soper† will apply; if, however, n and p

* I have now considerable hopes that such tables may be ultimately issued.

† *Tracts for Computers*, No. VII.

be considerable then we may obtain an approximate value by the use of an auxiliary normal curve as indicated by Camp on pp. 163—171 of this volume.

My original proof of the relationship was as follows:

Let $x=b/(a+b)$, $q=n+1-p$; then the first p terms of the binomial if written backwards are

$$=(a+b)^n x^{p-1}(1-x)^q\left\{\frac{(p+q-1)!}{(q)!\,(p-1)!}+\frac{1-x}{x}\,\frac{(p+q-1)!}{(q+1)!\,(p-2)!}\right.$$
$$\left.+\left(\frac{1-x}{x}\right)^2\frac{(p+q-1)!}{(q+2)!\,(p-3)!}+\ldots+\left(\frac{1-x}{x}\right)^{p-1}\right\}\ldots\text{(i)}.$$

Now consider $$B_x(p,q)=\int_0^x x^{p-1}(1-x)^{q-1}dx,$$

and $$B(p,q)=(p-1)!\,(q-1)!/(p+q-1)!,$$

where we must use Γ-functions instead of factorials if n and therefore q is fractional.

Now if we integrate $B_x(p,q)$ by parts we have

$$B_x(p,q)=-\int_0^x\frac{x^{p-1}}{q}\,d(1-x)^q$$
$$=-x^{p-1}(1-x)^q\left\{\frac{1}{q}+\frac{(p-1)}{q(q+1)}\,\frac{1-x}{x}+\frac{(p-1)(p-2)}{q(q+1)(q+2)}\left(\frac{1-x}{x}\right)^2+\ldots\right.$$
$$\left.+\frac{(p-1)(p-2)\ldots2}{q(q+1)(q+2)\ldots(q+p-2)}\left(\frac{1-x}{x}\right)^{p-2}\right\}$$
$$+\frac{(p-1)!\,(q-1)!}{(p+q-2)!}\int_0^x(1-x)^{p+q-2}dx.$$

The last line integrates out as

$$-(1-x)^{p+q-1}\frac{(p-1)!\,(q-1)!}{(p+q-1)!}+\frac{(p-1)!\,(q-1)!}{(p+q-1)!}.$$

Accordingly

$$\frac{B_x(p,q)}{B(p,q)}=-x^{p-1}(1-x)^q\left\{\frac{(p+q-1)!}{(q)!\,(p-1)!}+\frac{(p+q-1)!}{(q+1)!\,(p-2)!}\,\frac{1-x}{x}\right.$$
$$\left.+\frac{(p+q-1)!}{(q+2)!\,(p-3)!}\left(\frac{1-x}{x}\right)^2+\ldots+\left(\frac{1-x}{x}\right)^{p-1}\right\}+1,$$

or, by (i), the first p terms of the binomial $(a+b)^n$

$$=(a+b)^n\left\{1-\frac{B_x(p,q)}{B(p,q)}\right\}$$
$$=(a+b)^n\left\{1-\frac{B_{b/(a+b)}(p,n+1-p)}{B(p,n+1-p)}\right\}=(a+b)^n\,\frac{B_{a/(a+b)}(n+1-p,p)}{B(p,n+1-p)}\ldots\text{(ii)};$$

which form of (ii) we should use will depend upon our method of approximating to the incomplete B-function, but we should as a rule select the one that does not integrate through the mode of the integrated curve. Camp has shown how by the choice of a suitable normal curve we obtain a fairly reasonable evaluation of the tail of a B-function. It would be of much interest to discover another incomplete integral function which as in the case of the incomplete Γ- and B-functions would *exactly* represent the sum of the first p terms of the hypergeometrical series $F(\alpha,\beta,\gamma,1)$.

It will be clear that when the tables of the incomplete B-function are published it will be possible to sum any number of terms of a binomial, the power of which falls within the range of the tables.

VII. Portrait of Sir Thomas Browne and his Wife.

(Frontispiece to Volume XV of *Biometrika*.)

I MUCH regret to say that the attribution of the portrait of Sir Thomas Browne and his Wife to the L'Estrange family was an error. The present owner purchased the picture and some other works of art at the same time from a Norwich dealer. Through a mistake on the part of either dealer or owner the source of the picture was stated to be Hunstanton Hall, the seat of the L'Estranges. This information was given by the owner to the Director of the National Portrait Gallery who communicated it to Miss Tildesley, the author of the paper in question. This origin appeared to have some confirmation in the fact that the L'Estranges of the 17th century were friends of Sir Thomas Browne.

It now seems that the dealer purchased the picture at a sale in October, 1922, at Swaffham Prior House. It was entered in the sale catalogue as "Picture of Charles I and his Queen," by Vandyck!—I have been in communication with Mr Charles Allix, the former owner, and he tells me that he believes the picture had been at Swaffham for several generations. The catalogue of the sale shows that it had completely lost its identity. I have so far found no trace of a relationship between the Allixes and Sir Thomas Browne, but it is possible that the distinguished divine of that name, who fled to England on the Revocation of the Edict of Nantes, and founded the English family, may have been personally acquainted with Sir Thomas, or have been one of his admirers.

I may add that Mr Francis Wellesley has most kindly sent me a photograph of the picture after its cleaning; there is no doubt in my own mind that it is and will remain the fundamental and most characteristic portrait of Sir Thomas Browne, and further that it was the source of most of the later portraiture. It would be an interesting antiquarian problem to determine who painted it, how it came into the possession of the Allix family, and why it was lost for generations at Swaffham.

K. P.

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